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Sparse and Low-rank Methods for Structural Identification and SHM

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Abstract

Recent advances in Statistical Learning techniques have enabled the development of new algorithms for monitoring, data cleansing, data compression, structural identification, damage identification, and structural dynamics in general.

This keynote presents novel sparse and low-rank methods to address the inverse problems in structural dynamics, identification, and data-driven health monitoring. In particular, the emerging mathematical tools such as sparse representation (SR) and compressed sensing (CS), as well as the unsupervised multivariate blind source separation (BSS), are used to harness the structural dynamic features and damage information intrinsic within the structural vibration response measurement data, which is found to have sparse and low-rank structure. Data-driven approaches are developed towards rapid, unsupervised, and effective system identification, damage detection, as well as massive SHM data management.

1. Introduction

During service, civil structures are subjected to operational loads and environmental effects, as well as various natural disasters (e.g., earthquakes and hurricanes) and man-made extreme events (e.g., blasts and impacts). Assessing health status and detecting damage of the structure as early as possible is essential to ensure structural integrity. To achieve this goal, structural health monitoring (SHM) systems with an array of networked sensors have been developed to continuously measure structural data for monitoring and assessing structural performance.

Vibration-based measurements (e.g., strains, displacements, and accelerations) and techniques such as modal analysis based system identification and damage detection methods have been widely studied for SHM (Doebling et al. 1996). Traditional modal identification typically complies with the principle of system identification which is based on the relationship of inputs and outputs (Ewins 2000). This corresponds to an ideal situation where excitation to the system can be controlled or measured. For civil structures, typically

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large-scale (e.g., bridges, buildings, dams, etc.), it is extremely difficult or expensive, if not impossible, to apply controlled excitation to conduct input-output modal analysis. Accurate measurement of the ambient excitation (e.g., wind, traffic, etc) to structures is also challenging. Therefore in practical applications, it is often required to identify the structural dynamic properties and health status from only the available structural vibration response measurement data. This is essentially an ill-posed inverse problem, which hardly has analytical solutions. However, with some additional information and appropriate assumptions, one could hope to find solutions that may be sufficient in structural dynamics and health monitoring.

In this context, this paper provides novel sparse and low-rank methods to address the inverse problems of interest where only the structural vibration responses are available. Particularly, the emerging mathematical tools such as sparse representation (SR) (Bruckstein et al. 2008), compressed sensing (CS) (Candes and Wakin 2008), as well as the unsupervised blind source separation (BSS) (Hyvarinen et al. 2001), are used to model and extract the salient structural features and damage information, which are found to have sparse and low-rank structure. The paper describes the recent work by the author and his coworkers on this exciting topic.

2. Definition of sparsity and low-rank

2.1. Sparse representation

Sparsity of a signal $x \in \mathbb{R}^N$ can be defined by the ℓ_0-norm (Bruckstein et al. 2008),

$$\|x\|_{\ell_0} = \#\{i : x_i \neq 0\} \tag{1}$$

simply counting the number of non-zeros in x. A signal x (vector) is K-sparse if it has at most K non-zeros, i.e., $\|x\|_{\ell_0} \leq K$. In analogy, a matrix $\mathbf{X}$ is also said to be sparse if most of its elements are zero. x is also said to be K-sparse (transform sparse) in a domain $\mathbf{\Psi}$ with a representation $\alpha \in \mathbb{R}^N$

$$x = \mathbf{\Psi}\alpha = \sum_{j=1}^{N} \alpha_j \psi_j \tag{2}$$

if $\|\alpha\|_{\ell_0} \leq K$. $\mathbf{\Psi} = [\psi_1, ..., \psi_N]^T \in \mathbb{R}^{N \times N}$ is an orthonormal basis (e.g., sinusoid, wavelet, etc), whose jth row is $\psi_j \in \mathbb{R}^N$ (or $\mathbb{C}^N$on Fourier basis). $\alpha \in \mathbb{R}^N$ is the coefficient sequence of $x \in \mathbb{R}^N$ on $\mathbf{\Psi}$, whose jth element $\alpha_j = \langle x, \psi_j \rangle$ (inner product). This generalization is particularly useful, since, in practice, x is typically sparse in an appropriate domain instead of its original domain.

2.2. Low-rank structure

Structural responses, from potentially hundreds of channels or sensors, can be represented as a data matrix. The multi-channel data matrix is also explicitly exploited and modeled, e.g., by singular value decomposition (SVD) or principal component analysis (PCA) (Jolliffe 1986). The data matrix $\mathbf{X} \in \mathbb{R}^{m \times N}$ with m sensors and N time history sampling points ($m < N$) has an SVD representation

$$\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T = \sum_{i=1}^{r} \sigma_i \mathbf{u}_i \mathbf{v}_i^T \quad (3)$$

where $\mathbf{U} = [\mathbf{u}_1, ..., \mathbf{u}_m] \in \mathbb{R}^{m \times m}$ is an orthonormal matrix associated with the channel (variable) dimension, called left-singular vectors or principal component directions; $\mathbf{\Sigma} \in \mathbb{R}^{m \times N}$ has m diagonal elements σ_i as the ith singular value ($\sigma_1 > ... > \sigma_r > \sigma_{r+1} = ... = \sigma_m = 0$), and $\mathbf{V} = [\mathbf{v}_1, ..., \mathbf{v}_N] \in \mathbb{R}^{N \times N}$ is associated with the time history (measurement) dimension, called the right-singular vector matrix. $\mathbf{X}$ is said to be low-rank if it has only few active (non-zero) singular values ($r \ll \min(m, N)$).

It is well understood that the ith singular value σ_i is related to the energy captured by the ith principal direction of $\mathbf{X}$. In structural dynamics, under some assumption, the principal directions would coincide with the mode directions (Feeny and Kappagantu 1998) with the corresponding singular values indicating their participating energy in the structural responses $\mathbf{X}$, i.e., the structural active modes are captured by r principal components under broadband excitation.

An empirical, but frequently sound, observation is that there are typically only few active modes in the structural vibration responses (Yang and Nagarajaiah 2014a); in other words, few of its singular values are active: r is typically quite small. If the sensor or channel number$m$is reasonably large, then $r \ll \min(m, N) = m$ and $\mathbf{X} \in \mathbb{R}^{m \times N}$ is said to be low-rank. However, this is seldom so for large civil structures, because the sensor number m is not so much more than (often times even less than) the involved r modes; as a result, $r \ll m$ can't be guaranteed for a low-rank representation.

A simple yet effective strategy—rank-invariant matrix reshape (Yang and Nagarajaiah 2014b) has been proposed to guarantee a low-rank representation of structural response data matrix, regardless of the original dimension of $\mathbf{X} \in \mathbb{R}^{m \times N}$. Essentially, mode information (few are active; hence, the rank of the structural response data matrix is small) remains approximately invariant (small) regardless of the reshape of the structural response data matrix.

2.3. Blind source separation (BSS)

BSS as a promising unsupervised multivariate machine learning technique is able to recover the hidden source signals and their characteristic factors using only the measured mixture signals, with high potential in unsupervised learning of the patterns and features hidden in the large-scale multi-channel SHM data set. The linear instantaneous BSS model (Hyvarinen et al. 2001) is expressed as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) = \sum_{i=1}^{n} \mathbf{a}_i s_i(t) \tag{4}$$

where $\mathbf{x}(t) = [x_1(t), ..., x_m(t)]^T$ is the observed mixture vector with m mixture signals, and $\mathbf{s}(t) = [s_1(t), ..., s_n(t)]^T$ is the latten source vector with n sources; $\mathbf{A} \in \mathbb{R}^{m \times n}$ is the unknown constant mixing matrix consisting of n columns with its ith column $\mathbf{a}_i \in \mathbb{R}^m$ associated with $s_i(t)$.

With only $\mathbf{x}(t)$ known, Eq. (4) may not be mathematically solved. To alleviate the problem, most BSS techniques, such as independent component analysis (ICA) (Hyvarinen et al. 2001), second order blind identification (SOBI) (Belouchrani et al. 1997), and complexity pursuit (CP) (Stone 2001), exert a general assumption that the source signals $\mathbf{s}(t)$ are statistically independent (or as independent as possible) at each time instant t, and recover the components $\mathbf{y}(t) = [y_1(t), ..., y_n(t)]^T$ that are as mutually independent as possible

$$\mathbf{y}(t) = \mathbf{W}\mathbf{x}(t) \tag{5}$$

such that $\mathbf{y}(t) = \mathbf{s}(t)$ and $\mathbf{W} = \mathbf{A}^{-1}$. In particular, ICA biases to recover sparse components that are of interest. In Yang and Nagarajaiah (2014c), it is shown that ICA has the ability of extracting sparse component, which is the target structural dynamic and damage features of interest.

3. Sparse/low-rank methods for structural dynamics and SHM

3.1. Sparse clustering of modal expansion

For an n-DOF linear time-invariant system, its equation of motion (EOM) is

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \tag{6}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are constant mass, diagonalizable damping, and stiffness matrices, respectively, and are real-valued and symmetric; $\mathbf{x}(t) = [x_1(t), ..., x_m(t)]^T$ is the system response (displacement) vector and $\mathbf{f}(t)$ is the external force vector. Under broadband excitation, the coupled $\mathbf{x}(t)$ may be expressed as linear combinations of the decoupled modal responses

$$\mathbf{x}(t) = \mathbf{\Phi}\mathbf{q}(t) = \sum_{i=1}^{n} \boldsymbol{\varphi}_i q_i(t) \tag{7}$$

3.1.2. Sparse clustering of modes

A new method, sparse component analysis (SCA) (Gribonval and Lesage 2006), takes advantage of the spectral sparsity and spatially disjoint of the modal responses (Gribonval and Lesage 2006). Transform Eq. (7) into the frequency domain f,

$$\mathbf{x}(f) = \mathbf{\Phi}\mathbf{q}(f) = \sum_{i=1}^{n} \varphi_i q_i(f) \tag{8}$$

Due to the spatially disjoint sparsity of q_j ($j = 1, ..., n$) which is active only at f_k (the modal frequency of the jth mode); elsewhere $f \neq f_k$, $q_j(f) = 0$. Therefore, Eq. (8) becomes

$$\mathbf{x}(f_k) = \boldsymbol{\varphi}_j q_j(f_k) \tag{9}$$

which means that there is only a scale difference, $q_j(f_k)$, between $\mathbf{x}(f_k)$ and $\boldsymbol{\varphi}_j$ (Yang and Nagarajaiah 2013). For the whole $f \in \Omega$, the scatter plot of $\mathbf{x}(f)$ (up to 3-dimension) then reveals all the n directions of the mode shape columns of $\mathbf{\Phi}$ (Fig. 1). In general, the estimated vibration mode matrix $\mathbf{\Phi}$ can automatically be extracted by standard clustering algorithms such as fuzzy-C-means (FCM).

In determined case ($m = n$), time-domain modal responses are readily de-coupled by

$$\mathbf{q}(t) = \mathbf{\Phi}^{-1}\mathbf{x}(t) \tag{10}$$

thereby estimating the modal frequency and damping ratio from $\mathbf{q}(t)$. For underdetermined case ($m < n$) where the sensors are insufficient, $\mathbf{\Phi}$ is rectangular and recovery of $\mathbf{q}(f)$ from the underdetermined Eq. (8) is ill-posed. In Yang and Nagarajaiah (2013a), a sparse recovery technique with ℓ_1-minimization is explored to solve the underdetermined output-only modal identification problem.

To identify highly-damped structures, it is proposed to transform the Eq. (7) to the sparse time-frequency domain using short-time-Fourier-transform (STFT) (Yang and Nagarajaiah 2013b). Furthermore, using a complex-ICA algorithm, STFT-cICA is able to identify structures with complex modes (Nagarajaiah and Yang 2015). Recently, a new BSS based output-only modal identification method, complexity pursuit (CP) which explicitly exploits the data structure of structural responses and modal responses, is found suitable for output-only modal identification of structures with closely-spaced modes, complex highly-damped modes, and in real-time identification of the time-varying cable tension time history; the details are referred to Yang and Nagarajaiah (2013c) and (Yang et al. 2015). Our recent work also shows that output-only modal identification can be performed in a non-uniform low-rate random sampling paradigm based on BSS and compressed sensing (Yang and Nagarajaiah 2015a).

3.4. Data management via low-rank structure

3.4.1. Low-rank structure and ICA multivariate sparse representations for data compression

A relevant observation is that structural vibration responses are typically low-rank by SVD or PCA, i.e., $\mathbf{X} \in \mathbb{R}^{m \times N}$ with small r, since in real world, only a few modes are excited out and present in the structural vibration responses. Therefore, one strategy for data compression is to drop those principal components with significantly small eigen values. As small eigen value indicates small energy of the corresponding principal component, it would cause little data loss by retaining those dominant components with larger Eigen values. Meanwhile, it achieves higher compression by only encoding the retained components. In Yang and Nagarajaiah (2014b), it is derived that ICA naturally yields the optimal (linear) transformation adaptive to data itself for compression in statistical framework. The real-world examples are presented in Yang and Nagarajaiah (2014b).

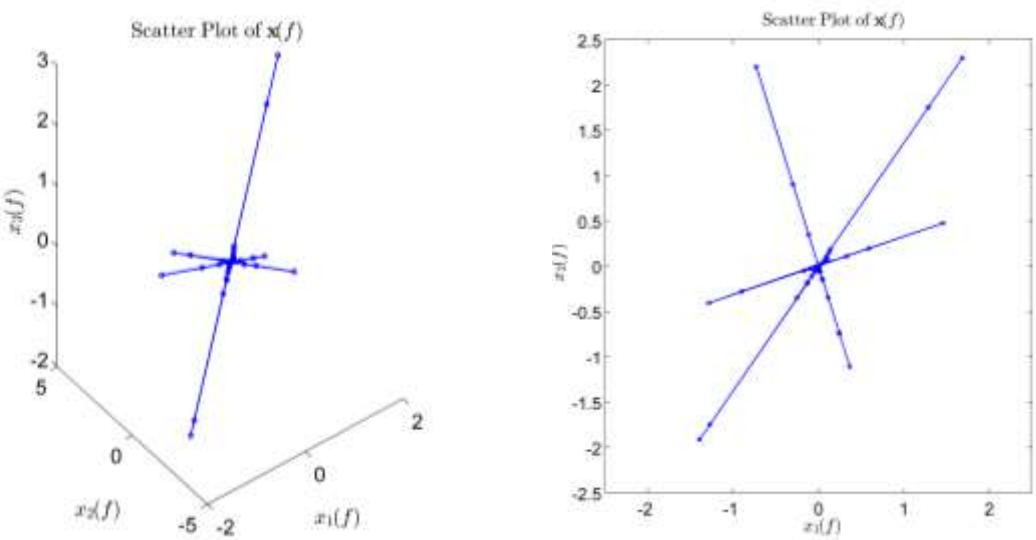

Fig. 1. The scatter plot of the frequency-domain system responses in determined case with three sensors (left) and underdetermined case with two sensors (right).

3.4.2. Significant data compression

Dimension reduction for data compression is most effective when $r \ll m$ ($\mathbf{X} \in \mathbb{R}^{m \times N}$ needs to be as low-rank as possible), i.e., the channel (sensor) number needs to be *much* larger than that of the involved modes. However, it is not satisfied in many situations: for civil engineering structures, typically large-scale, the sensor number m is not so *much* more than the involved r modes; as a result, $r \ll m$ can't be guaranteed for a low-rank representation. A scheme of matrix reshape is proposed to remove this limitation for wider applicability of PCA in multi-channel data compression, as detailed in Yang and Nagarajaiah (2015b), and Fig. 2.

3.4.3. Removing sparse outliers

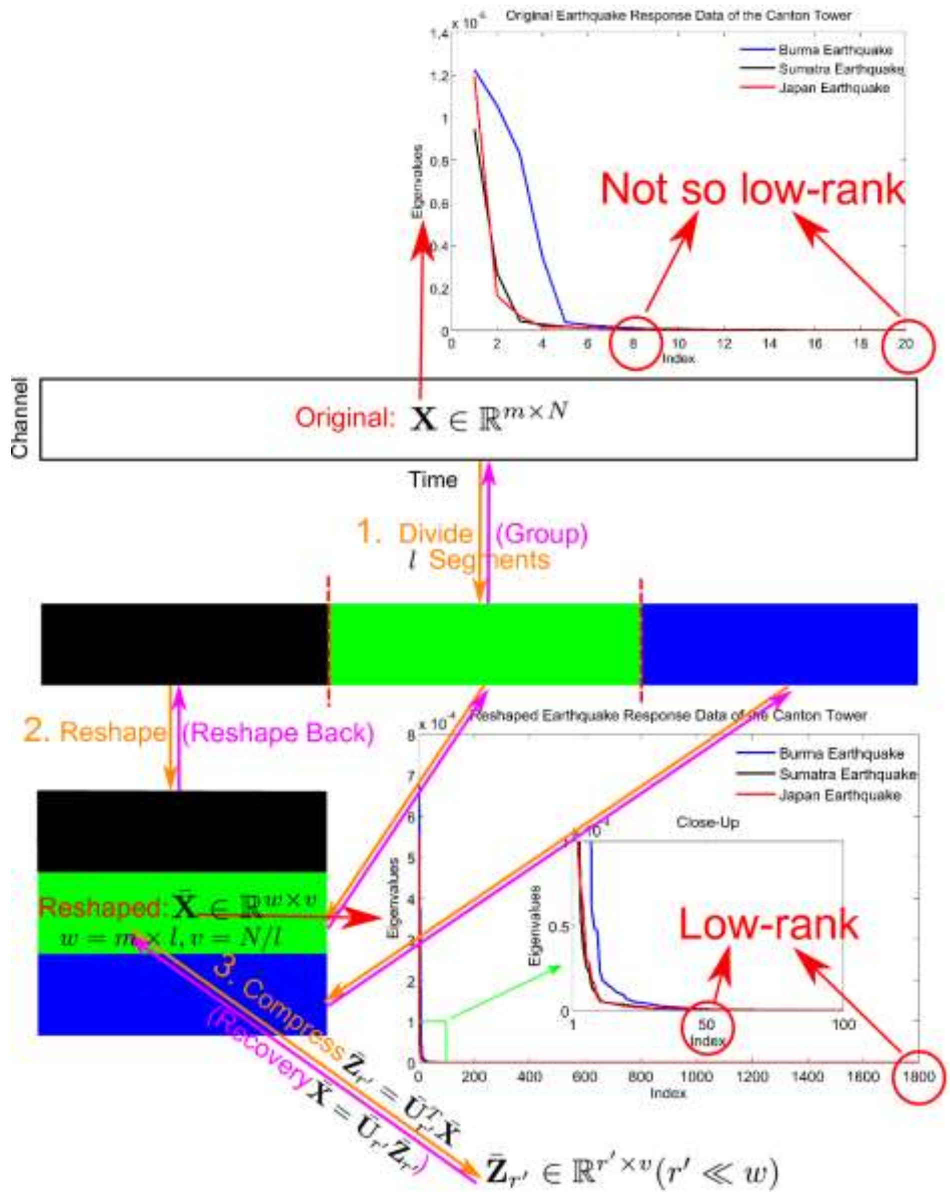

Fig. 2. The data compression scheme with the matrix reshape strategy.

Real-world measured structural response data typically contains considerable noise or errors. Applications of traditional data processing methods can only deal with dense small noise. Robust PCA (Candes et al. 2009), termed PCP, is capable of effectively modeling the noisy data with outliers and thus simultaneously removing both the outliers and dense noise (Yang and Nagarajaiah 2014b). When the original data $\mathbf{X} \in \mathbb{R}^{m \times N}$ are addictively corrupted by both gross errors (outliers) and dense noise,

$$\hat{\mathbf{X}} = \mathbf{X}_0 + \mathbf{N}_0 + \mathbf{Z}_0 \tag{11}$$

where $\mathbf{Z}_0 \in \mathbb{R}^{m \times N}$ has few (sparse) but gross outlier elements with arbitrarily large and located magnitudes, and $\mathbf{N}_0 \in \mathbb{R}^{m \times N}$ is entry-wise i.i.d. small dense noise. PCP aims to recover $\mathbf{X}_0$ by solving the following convex program nuclear-norm-minimization. In Candes et al. (2009), it is shown that real-world structural vibration responses with the gross outliers can be effectively removed and more examples are presented in Yang and Nagarajaiah (2014b).

3.5. Damage detection by sparse signal discovery

3.5.1. ICA for recovery of sparse damage information

The derivation that ICA biases to recover sparse components, which is shown in Section 2.3, can immediately lead to a straightforward application in unsupervised damage identification. Consider transforming structural responses $\mathbf{x}(t)$ carrying the damage information into some wavelet scale l to expose the common spike-like feature (viewed as a latent sparse component $s_j(t)$) hidden within $\mathbf{x}(t)$ and incorporate it into the BSS model,

$$\mathbf{x}^l(t) = \mathbf{A}\mathbf{s}(t) \tag{12}$$

then ICA would blindly recover $y_j(t) = s_j(t)$ (the "interesting" source) with outstanding spike features, directly indicating the damage instant(s). Furthermore, the simultaneously-recovered $\mathbf{a}_j$ conveying the spatial signature of $s_j(t)$ also locates damage location(s). Successful examples are shown in Yang and Nagarajaiah (2014c).

3.5.2. Dynamic imaging for structural surveillance using low-rank plus sparse representation

An unsupervised data-driven framework has been established to automate real-time detection of structural damage by exploiting the fundamental spatiotemporal data structure of the multiple images (video stream) Yang and Nagarajaiah (2015c). If restacking each of N temporal frame of the structure as a long column vector with a resolution of $M = M_1 \times M_2$ pixels (Fig. 3), the multi-frame data matrix $\mathbf{X} \in \mathbb{R}^{M \times N}$ is obtained, whose ith $(i = 1, ..., N)$ column $x_i \in \mathbb{R}^M$ represents the temporal frame at time T_i. PCP is able to blindly decompose $\mathbf{X} \in \mathbb{R}^{M \times N}$ into a superposition of a low-rank matrix $\mathbf{L} \in \mathbb{R}^{M \times N}$ and a sparse matrix $\mathbf{S} \in \mathbb{R}^{M \times N}$ as

$$\mathbf{X} = \mathbf{L} + \mathbf{S} \tag{13}$$

by solving (P_*). $\mathbf{S} \in \mathbb{R}^{M \times N}$ is said to be sparse if it has only few non-zero entries, and $\mathbf{L} \in \mathbb{R}^{M \times N}$ is low-rank in the sense that its SVD has few active singular values.

The $\mathbf{L} + \mathbf{S}$ representation expresses the multiple temporal close-up frames of structures as a superposition of a background component and an innovation component: $\mathbf{L}$ represents the static or slowly-changing correlated background component among the temporal frames, which is naturally low-rank; $\mathbf{S}$ captures the innovation information in each frame induced by the evolutionary damage, which is naturally sparse standing out from the background. See the proposed dynamic imaging framework for local structural assessment in Fig. 3 and Yang and Nagarajaiah (2015c) for more details.

3.5.3. Damage identification via sparse classification

Instead of building and training a parametric classifier in traditional pattern recognition methods, Yang and Nagarajaiah (2014d) proposed a new damage identification method in the classification framework by exploiting the sparsity nature implied in the classification problem itself, via sparse representation classification (SRC) of a test feature in terms of an adaptive reference dictionary (Fig. 4); it is found to be relatively intuitive and efficient.

In the damage identification problem, the features are chosen to be the mode shape columns and are blindly extracted by CP from the system responses of the model. For an n-DOF system, if simulating N different damage classes (with different damage locations and severities), then all the $w = N \times n$ (typically $n \ll w$) mode shape columns are concatenated to yield a reference matrix $\mathbf{\Psi} \in \mathbb{R}^{n \times w}$

$$\mathbf{\Psi} = [\mathbf{\Phi}_1, ..., \mathbf{\Phi}_N] = [\boldsymbol{\varphi}_{1,1}, ..., \boldsymbol{\varphi}_{N,n}] \tag{14}$$

Now suppose the test features $\hat{\mathbf{\Phi}} = [\hat{\boldsymbol{\varphi}}_1, ..., \hat{\boldsymbol{\varphi}}_n] \in \mathbb{R}^{n \times n}$ are extracted from the current structural responses whose damage scenario coincides with one of the damage class of the reference matrix, say, the jth class (but it is of course unknown beforehand), then $\hat{\boldsymbol{\varphi}}_i$ ($i = 1, ..., n$) would equal $\boldsymbol{\varphi}_{j,i}$ up to a scale difference. Expanding $\hat{\boldsymbol{\varphi}}_i$ in terms of the whole reference dictionary,

$$\hat{\boldsymbol{\varphi}}_i = \mathbf{\Psi}\boldsymbol{\alpha}_i = \sum_{k=1}^{N}\sum_{l=1}^{n} \alpha_{k,l}\boldsymbol{\varphi}_{k,l} \tag{15}$$

where $\boldsymbol{\alpha}_i = [0, ..., 0, \alpha_{j,i}, 0, ..., 0]^T \in \mathbb{R}^w$ is its underlying sparse representation whose non-zero element $\alpha_{j,i}$ directly assigns the damage class the test feature belongs to. As introduced above, finding the sparse solution $\boldsymbol{\alpha}_i$ to the (highly) underdetermined linear system of equations Eq. (15) from the knowledge of $\mathbf{\Psi} \in \mathbb{R}^{n \times w}$ and $\hat{\boldsymbol{\varphi}}_i$ can be efficiently accomplished by (P_1)

$$(P_1): \quad \boldsymbol{\alpha}_i^\star = \arg\min \|\boldsymbol{\alpha}_i\|_{\ell_1} \quad \text{subject to} \quad \mathbf{\Psi}\boldsymbol{\alpha}_i = \hat{\boldsymbol{\varphi}}_i \tag{16}$$

SRC directly exploits the essence of the classification problem: the test feature can be most sparsely represented by the reference dictionary. It establishes an underdetermined linear system of equations whose underlying sparse solution can be efficiently recovered to dictate the damage class. Examples are shown in Yang and Nagarajaiah (2014d).

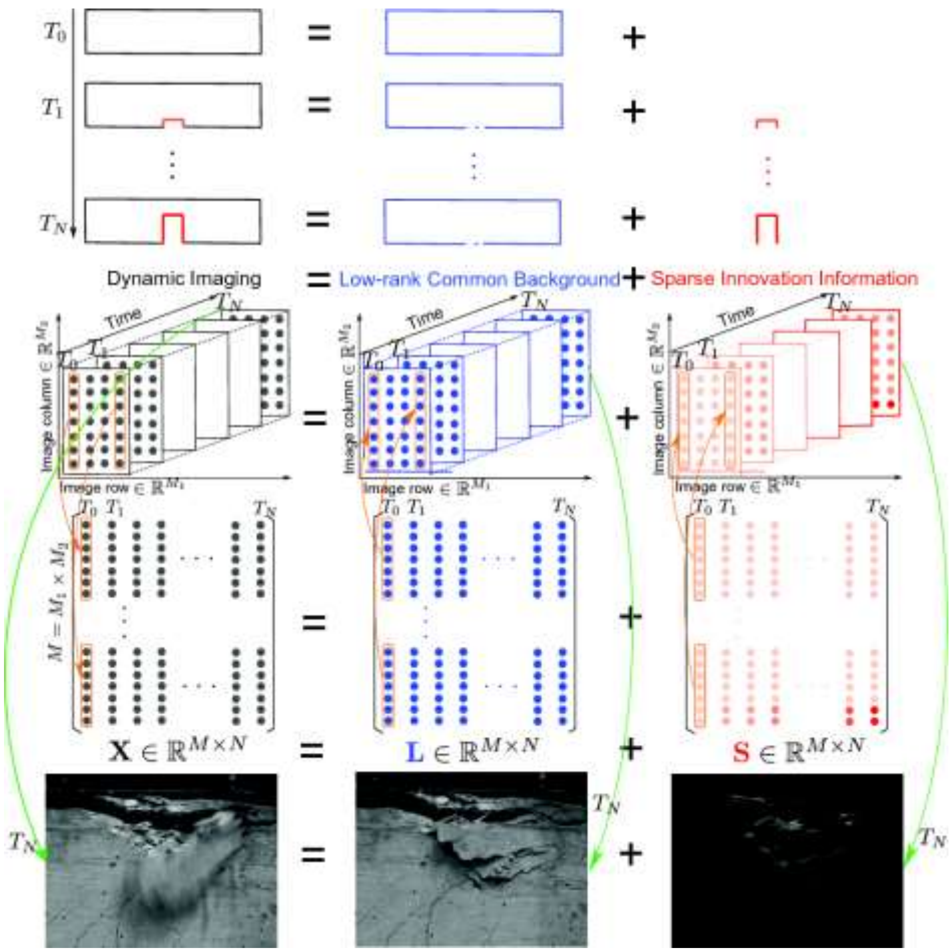

Fig. 3. The dynamic imaging of structures framework based on low-rank plus sparse representation of the multiple temporal images of the structure.

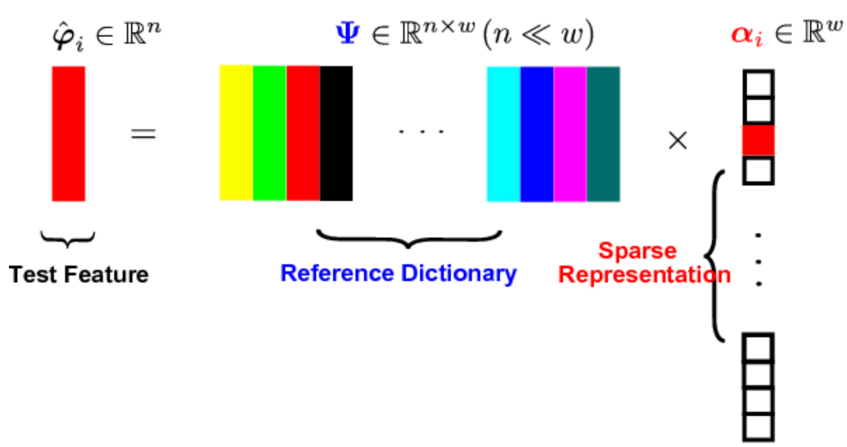

Fig. 4. The sparse representation classification paradigm for damage identification. The test feature $\hat{\varphi}_i \in \mathbb{R}^n$ (red column, e.g., mode shape column) only activiates itself via its

representation $\boldsymbol{\alpha}_i \in \mathbb{R}^w$ (read in its own location, white denotes unactivated zero) in terms of the large reference dictionary $\boldsymbol{\Psi} \in \mathbb{R}^{n \times w}$ ($n \ll w$) (by concatenating all feature columns of all candidate reference damage classes), expressed as a highly underdetermined linear system of equations $\hat{\boldsymbol{\varphi}}_i = \boldsymbol{\Psi}\boldsymbol{\alpha}_i$. The unique non-zero element (red) in $\boldsymbol{\alpha}_i$ (recovered by ℓ_1-minimization) directly dictates which class the test feature belongs to, within the predefined reference dictionary.

Concluding Remarks

This paper briefly describes the most recent developments of novel sparse and low-rank methods for monitoring, structural system identification, and damage detection by the author. This is a new area of research in structural system identification and structural health monitoring that offers many new tools and exciting possibilities for future research. There are many other researchers beginning to work or have already worked in this exciting new area of research, but unfortunately, due to space limitations, further details of the work by others cannot be described in detail, which can be found in a recent publication (Nagarajaiah and Yang 2016).

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