

## **Semiactive Vibration Attenuation of a Flexible Rotor by Squeezing Thin Layers of Normal and Magnetorheological Oils**

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### **ABSTRACT**

The proposed semiactive damping element works on the principle of squeezing two lubricating layers formed by normal and magnetorheological oils. The damping effect is controlled by the change of magnetic flux passing through the film of magnetorheological liquid. In the developed mathematical model the lubricants are represented by Newtonian and Bingham materials. The pressure distribution in the oil layers is governed by the modified Reynolds equations. The mathematical model of the studied element was implemented into the computational procedure for determination of the frequency response of a flexible rotor based on application of a trigonometric collocation method. Results of the simulations show that the maximum damping effect is needed in the range of low rotor velocities while for higher angular speeds the damping should be as small as possible. The proposed damping device produces the damping effect in the whole extent of the running speeds. Adapting its magnitude to the rotor angular velocity makes it possible to extend the speed interval at which the rotating machines can be operated without exceeding the allowable amplitude of the oscillations. Advantage of the proposed damping device is that it does not require an expensive and complicated control system for its operation.

### **1. INTRODUCTION**

Lateral vibrations of rotating machines can be attenuated by adding the damping devices to the constraint elements placed between the rotor and its casing. Practical experience and results of theoretical analyses show that to achieve their optimum performance, the damping effect must be controllable to be possible to adapt its magnitude to the current operating conditions. For this purpose several damping strategies have been developed and tested, among which there are those based on application of controllable classical squeeze film dampers (Mu 1991), electromagnetic

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damping devices (Tonoli 2008), controllable hydrodynamic (Wu 1998) and magnetic bearings (Kasarda 2004).

A wide class of controllable dampers applicable for attenuation of lateral vibration of rotors is represented by magnetorheological semiactive damping systems. A new concept of a controllable damping element utilizing magnetorheological liquid is reported in Zapoměl (2011). In this paper there is analysed its effect on the steady state vibrations of a flexible rotor excited by unbalance forces with the aim to minimize the vibration amplitude and the force transmitted between the rotor and its stationary part. The studied damper works on the principle of squeezing two lubricating layers formed by classical and magnetorheological oils. Its design makes it possible to switch the magnetorheological damping on only if the damping effect produced by the classical oil is insufficient. Another its advantage is that it does not require an expensive and complicated control system for its operation.

## 2. THE INVESTIGATED ROTOR AND ITS COMPUTATIONAL MODEL

The investigated rotor (Fig. 1.) consists of a shaft and of one disc. The shaft is flexible and is supported at both its ends. The rotor turns at constant angular speed and is loaded by its weight and by the disc imbalance. The system is symmetric relative to the disc middle plane perpendicular to the shaft axis. The task is to analyse its steady state vibration in the specified speed interval.

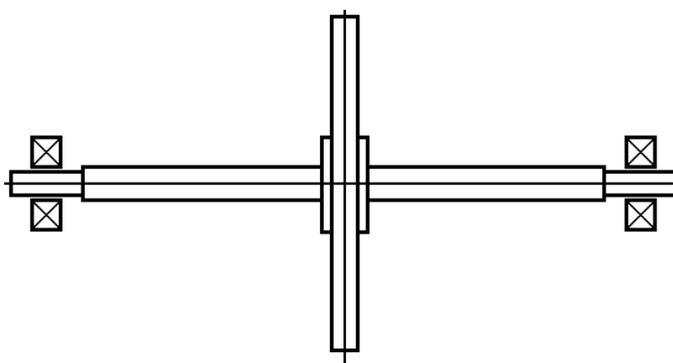


Fig. 1. Scheme of the investigated rotor.

Application of rolling element bearings as the constraint elements arrives at little system damping and at large vibration amplitude if the rotor works close to the critical speed.

To improve the rotor performance a new controllable damping element has been proposed (Fig. 2.). It works on the principle of squeezing two lubricating layers formed by normal and magnetorheological oils. The main parts of the damping element are two fixed and two movable rings. The former are connected with the damper's housing directly, the latter are coupled with the shaft by a rolling element bearing and with the damper's body by a squirrel spring, which enables their vibration in the radial direction and prevents their rotation together with the shaft. The gaps between the fixed and movable rings are filled with classical (inner layer) and magnetorheological oils (outer

layer). The damping device is equipped with an electric coil generating magnetic field passing through the film of magnetorheological liquid. As resistance against its flow depends on magnetic induction, the change of the magnetic flux can be used to control the damping force. The squirrel springs are prestressed, which enables to eliminate their deflection caused by the weight of the rotor.

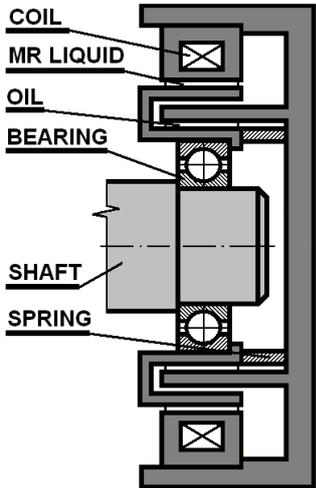


Fig. 2. The proposed damping element.

As the proposed damping device couples the flexible shaft with the stationary part by a squirrel spring, the resonance peak is shifted to lower angular velocities and its magnitude depends on amount of damping produced by squeezing the lubricating layers. If the system were linear, character of the frequency response characteristics would take the form as drawn in Fig. 3.

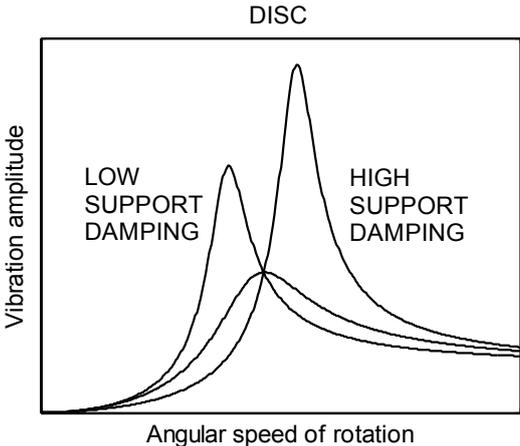


Fig. 3. Frequency response characteristics.

It is evident that a suitably controlled damping effect enables to minimize the maximum amplitude of the rotor vibration (Fig. 3.) and to reduce the speed interval in which the vibrations exceed the allowable values (Fig. 4.).

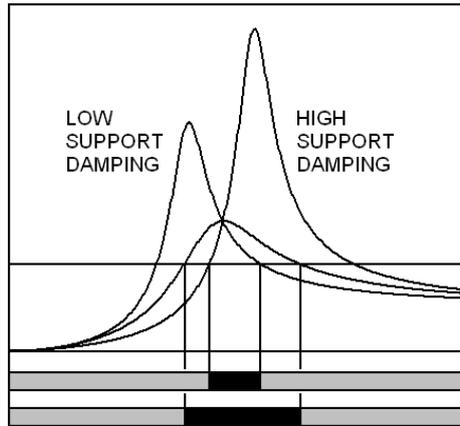


Fig. 4. Allowable speed intervals.

In the computational model of the rotor system the rotor is represented by a flexibly supported Jeffcott one and the damping elements by force couplings.

Taking into account the system symmetry, lateral vibration of the studied rotor is governed by a set of four equations

$$m\ddot{y} = -b_p\dot{y} - k_S y + k_S y_B + m e_T \omega^2 \cos(\omega t + \psi_T) \quad (1)$$

$$m\ddot{z} = -b_p\dot{z} - k_S z + k_S z_B + m e_T \omega^2 \sin(\omega t + \psi_T) - mg \quad (2)$$

$$0 = k_S y - (k_S + 2k_B) y_B + 2 F_{dy} + 2 F_{PSy} \quad (3)$$

$$0 = k_S z - (k_S + 2k_B) z_B + 2 F_{dz} + 2 F_{PSz} \quad (4)$$

$m$  is the disc mass,  $k_S$  is the shaft stiffness,  $k_B$  is stiffness of each squirrel spring,  $b_P$  is the coefficient of the disc external damping,  $e_T$  is eccentricity of the disc centre of gravity,  $g$  is the gravity acceleration,  $y$ ,  $z$ ,  $y_B$ ,  $z_B$  are displacements of the disc and shaft journal centres in the horizontal and vertical directions,  $\omega$  is angular speed of the rotor rotation,  $t$  is the time,  $\psi_T$  is the phase shift of the unbalance excitation,  $F_{PSy}$ ,  $F_{PSz}$  are the  $y$ - and  $z$ - components of the prestress force and  $(\dot{\phantom{x}})$ ,  $(\ddot{\phantom{x}})$  denote the first and second derivatives with respect to time.

### 3. DETERMINATION OF THE DAMPING FORCE MAGNITUDE

Mathematical model of the proposed damping element is based on assumptions of the classical theory of lubrication adapted for short dampers. The normal and magnetorheological oils are represented by Newtonian and Bingham fluids respectively. Then the pressure distribution in the lubricating layers is described by the Reynolds' equations (Zapoměl 2011)

$$\frac{\partial^2 p_{CO}}{\partial Z^2} = \frac{12\eta_{CO}}{h_{CO}^3} \dot{h}_{CO}, \quad (5)$$

$$h_{MR}^3 p_{MR}'^3 + 3(h_{MR}^2 \tau_y - 4\eta_{MR} \dot{h}_{MR} Z) p_{MR}'^2 - 4\tau_y^3 = 0 \quad \text{for } p' > 0, Z > 0, \quad (6)$$

$$h_{MR}^3 p_{MR}'^3 - 3(h_{MR}^2 \tau_y + 4\eta_{MR} \dot{h}_{MR} Z) p_{MR}'^2 + 4\tau_y^3 = 0 \quad \text{for } p' > 0, Z > 0. \quad (7)$$

$p_{CO}$ ,  $p_{MR}$  denote the pressure in the layers of the normal and magnetorheological oils,  $\eta_{CO}$ ,  $\eta_{MR}$  are their dynamic viscosities,  $Z$  is the axial coordinate,  $\tau_y$  represents the yield shear stress and (') denotes the first derivatives with respect to axial coordinate  $Z$ .

The thicknesses of the normal and magnetorheological films  $h_{CO}$ ,  $h_{MR}$  are given by the relations (Krämer 1993)

$$h_{CO} = c_{CO} - e_H \cos(\varphi - \gamma), \quad (8)$$

$$h_{MR} = c_{MR} - e_H \cos(\varphi - \gamma) \quad (9)$$

where  $c_{CO}$ ,  $c_{MR}$  denote the widths of the gaps filled with the normal and magnetorheological oils,  $e_H$  is the journal centre eccentricity,  $\varphi$  is the circumferential coordinate and  $\gamma$  is the angle of the line of centres (Fig. 5).

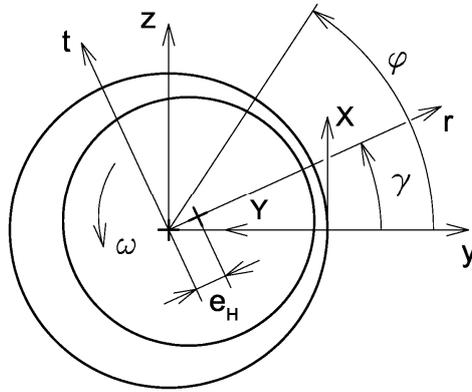


Fig. 5. The damping element coordinate system.

At locations where the thickness of the lubricating films rises with time ( $\dot{h}_{CO} > 0$ ,  $\dot{h}_{MR} > 0$ ) a cavitation is assumed. In these areas the pressure remains constant and equal to the pressure in the ambient space.

Components of the damping force in the  $y$  and  $z$  directions  $F_{dy}$ ,  $F_{dz}$  are then calculated by integration of the pressure distributions around the circumference and along the length of the damping element

$$F_{dy} = -2R_{CO} \int_0^{\frac{L}{2}} \int_0^{2\pi} p_{DCO} \cos \varphi dZ d\varphi - 2R_{MR} \int_0^{\frac{L}{2}} \int_0^{2\pi} p_{DMR} \cos \varphi dZ d\varphi, \quad (10)$$

$$F_{dz} = -2R_{CO} \int_0^{\frac{L}{2}} \int_0^{2\pi} p_{DCO} \sin \varphi dZ d\varphi - 2R_{MR} \int_0^{\frac{L}{2}} \int_0^{2\pi} p_{DMR} \sin \varphi dZ d\varphi. \quad (11)$$

$p_{DCO}$ ,  $p_{DMR}$  are the pressure distributions in the layers of normal and magnetorheological oils taking into account the cavitation,  $L$  is the length of the

damping element and  $R_{CO}$ ,  $R_{MR}$  are the radii of the normal and magnetorheological oil films.

Experiments carried out by a number of researchers and producers of magnetorheological liquids show that dependence of the yielding shear stress on magnetic induction can be approximated by a power function

$$\tau_y = k_y B^{n_y}. \quad (12)$$

$B$  is magnetic induction and  $k_y$  and  $n_y$  are the proportional and exponential material constants.

If the damping device has the simplest design, the circuit conducting the magnetic flux can be considered as a divided core of an electromagnet with two gaps filled with magnetorheological oil. Neglecting dispersion of the magnetic field in the ambient space, magnetic induction in the magnetorheological oil layer is given by the expression

$$B = \mu_0 \mu_r \frac{N_C I}{2 h_{MR}}. \quad (13)$$

$N_C$  is the number coil turns,  $I$  is the electric current and  $\mu_0$  and  $\mu_r$  are the permeability of vacuum and relative permeability of the magnetorheological liquid.

More details on possibilities of determination of magnetic induction in the layer of magnetorheological lubricant can be found in Zapoměl (2010).

#### 4. THE STEADY STATE SOLUTION OF THE GOVERNING EQUATIONS

After some manipulations the governing equations (1) - (4) can be rewritten in a matrix form

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}_0 + \mathbf{f}_C \cos \omega t + \mathbf{f}_S \sin \omega t + \mathbf{f}_H(\mathbf{x}, \dot{\mathbf{x}}). \quad (14)$$

$\mathbf{M}$ ,  $\mathbf{B}$ ,  $\mathbf{K}$  are the mass, external damping and stiffness matrices of the rotor system,  $\mathbf{f}_0$ ,  $\mathbf{f}_C$ ,  $\mathbf{f}_S$  are the vectors of applied forces,  $\mathbf{f}_H$  is the vector of the damping forces in the rotor constraints and  $\mathbf{x}$  is the vector of the rotor generalized displacements.

As evident from Eqs. (5) - (11) components of the damping force depend on displacements and velocities of the rotor journal centres and because of this the governing equation (14) is nonlinear.

A trigonometric collocation method was applied to obtain the steady state response of the investigated rotor system. This requires to approximate the steady state solution of equation (14) by a finite number of terms of a Fourier series

$$\mathbf{x} = \mathbf{x}_0 + \sum_{j=1}^{N_{FS}} \mathbf{x}_{Cj} \cdot \cos\left(j \frac{2\pi}{T} t\right) + \mathbf{x}_{Sj} \cdot \sin\left(j \frac{2\pi}{T} t\right) \quad (15)$$

and to specify  $2N_{FS}+1$  collocation points of time  $t_k$  for  $k = 1, 2, \dots, 2N_{FS}+1$ .  $\mathbf{x}_0$ ,  $\mathbf{x}_{Cj}$ ,  $\mathbf{x}_{Sj}$  are the vectors of the Fourier coefficients and  $T$  is the estimated response period (period of the rotor rotation).

Inserting the assumed solution Eq. (15) and its first and second derivatives with respect to time in governing equation (14) for all collocation points of time yields a set of

nonlinear algebraic equations. The one related to the  $k$ -th collocation point takes the form

$$\begin{aligned} \mathbf{K}\mathbf{x}_0 + \sum_{j=1}^{N_{FS}} \left[ \cos\left(j \frac{2\pi}{T} t_k\right) \left( \mathbf{K} - j^2 \frac{4\pi^2}{T^2} \mathbf{M} \right) - j \frac{2\pi}{T} \sin\left(j \frac{2\pi}{T} t_k\right) \mathbf{B} \right] \mathbf{x}_{Cj} + \\ + \sum_{j=1}^{N_{FS}} \left[ \sin\left(j \frac{2\pi}{T} t_k\right) \left( \mathbf{K} - j^2 \frac{4\pi^2}{T^2} \mathbf{M} \right) + j \frac{2\pi}{T} \cos\left(j \frac{2\pi}{T} t_k\right) \mathbf{B} \right] \mathbf{x}_{Sj} = \\ = \mathbf{r}_k(\mathbf{y}_0, \mathbf{y}_{C1}, \mathbf{y}_{S1}, \mathbf{y}_{C2}, \mathbf{y}_{S2}, \dots, t_k) \end{aligned} \quad (16)$$

$\mathbf{r}_k$  is the corresponding right-hand side vector.

Solving the set of nonlinear algebraic equations (16) gives the unknown values of the Fourier coefficients.

## 5. RESULTS OF THE COMPUTER SIMULATIONS

The investigated rotor is supported by the proposed damping elements at both its ends (Fig. 6.). The goal of the analysis is to study their influence on amplitude of the rotor vibration and on the force transmitted into the rotor casing.

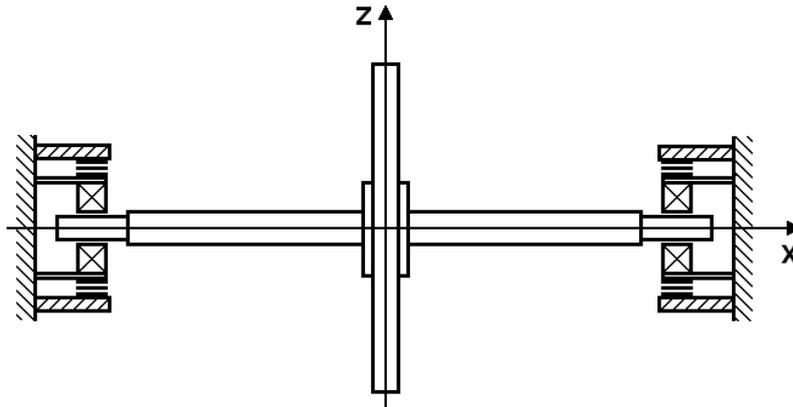


Fig. 6. Scheme of the investigated rotor with the proposed damping elements.

The main parameters of the rotor system are : mass of the disc 130 kg, stiffness of the shaft 7.0 MN/m, stiffness of one squirrel spring 3.0 MN/m, eccentricity of the disc centre of gravity 50  $\mu\text{m}$ , length of the damping element 30 mm, the width and diameter of the damper clearances 0.2 mm, 110 mm (normal oil), 1.0 mm, 150 mm (magnetorheological oil), viscosity of the normal oil 0.004 Pas and viscosity and proportional and exponential material constants of the magnetorheological oil 0.3 Pas,  $2 \times 10^{-5} \text{ N/A}^2$ , 2.

For these parameters the rotor natural frequencies are 158 rad/s and 232 rad/s for undamped and for strongly overdamped supports respectively.

In Fig. 7. there are drawn orbits of the disc and of rotor journal centres for the rotor speed of rotation of 150 rad/s and electric current of 1 A. The orbits are circular and the one corresponding to the disc centre is shifted in the vertical direction due to loading of

the shaft by the disc weight. Fig. 8. and 9. show the frequency response characteristics related to the centres of the disc and of the rotor journals and amplitude of the time variable component of the force transmitted between the rotor and the stationary part in dependence of the rotor angular velocity for several magnitudes of the applied current. It is evident from the results of simulations that the rotor critical velocities are close to the system natural frequencies and that the rising current shifts the resonance peaks to higher velocities. The maximum vibration attenuation and reduction of the transmitted force can be achieved if the damping effect is maximum for velocities smaller than about 200 rad/s. In the region of higher speeds the damping should be as low as possible which can be achieved by switching the applied current off (Fig. 8., 9.).

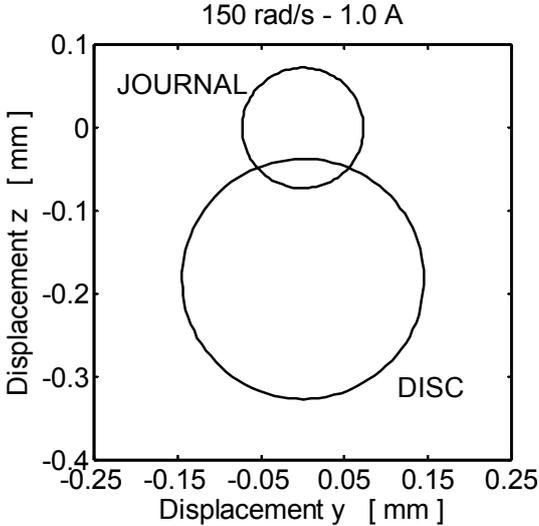


Fig. 7. Orbits of the disc and rotor journal centres.

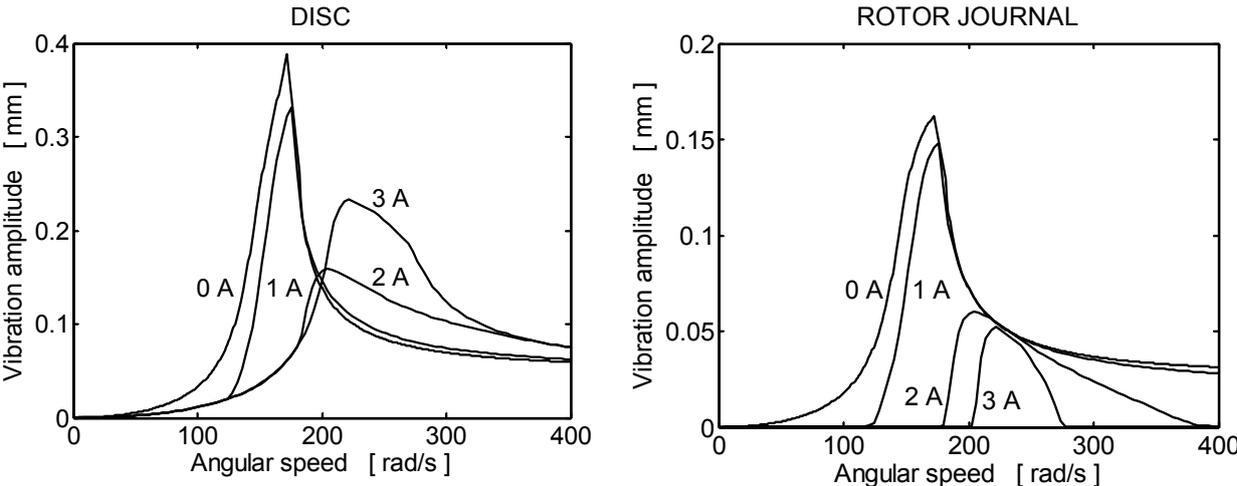


Fig. 8. Response characteristics of the disc and rotor journal centres.

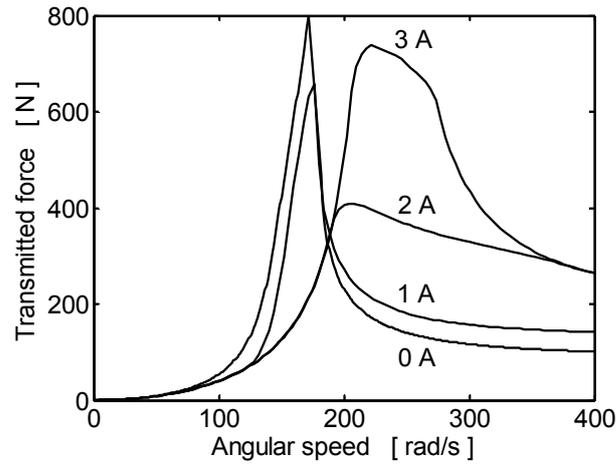


Fig. 9. Force amplitude - rotor angular speed relation.

The force is transmitted between the rotor and the stationary part through the squirrel spring and the layers of lubricating liquids. The time histories of the elastic and damping forces acting on the journal in the horizontal direction for two magnitudes of the applied current and corresponding components of the damping forces transmitted through the layers of normal and magnetorheological oils are drawn in Fig. 10. It is evident that the magnetorheological oil transmits some force also in the case when no current is applied. It is because on these conditions the magnetorheological liquid behaves as normal oil and the damping element works as a classical squeeze film damper with two lubricating layers.

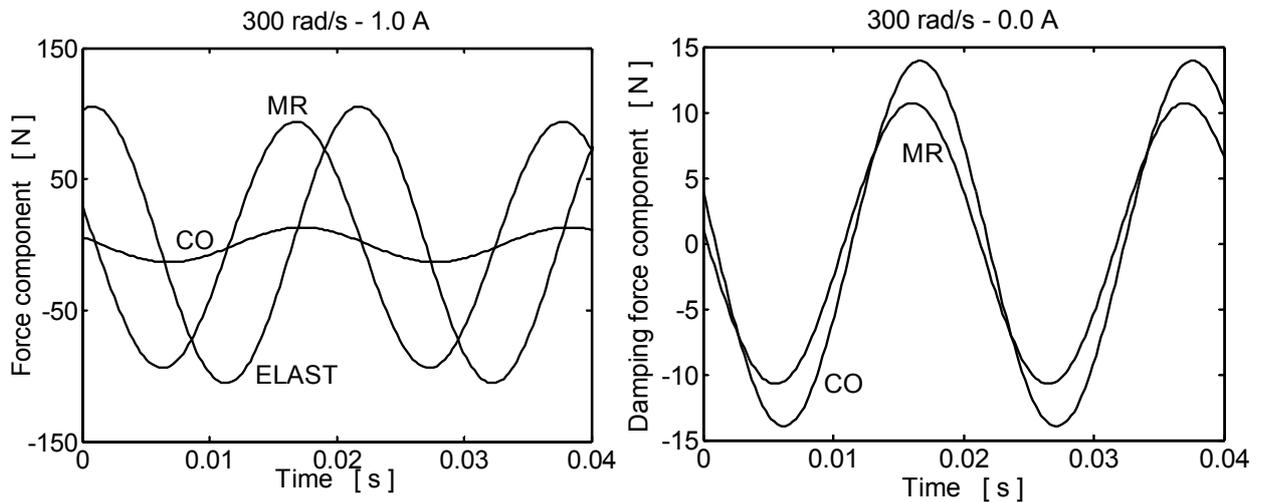


Fig. 10. Time histories of components of the force transmitted into the rotor casing.

## CONCLUSIONS

The studied semiactive damping element works on the principle of squeezing two lubricating films formed by normal and magnetorheological oils and its damping effect is

controlled by the change of magnitude of magnetic flux passing through the layer of magnetorheological liquid. The computational simulations prove that adding this element to the rotor supports makes it possible to extend the speed interval in which the rotor can be operated without exceeding the maximum allowable amplitude of its vibration and magnitude of the force transmitted between the rotor and the stationary part. Advantage of the proposed damping element is that it always produces some amount of damping, which can be increased or decreased in dependence on the angular speed of the rotor rotation. It does not require an expensive and complicated control system for its operation and on the contrary to the classical magnetorheological dampers lower consumption of magnetorheological oil and a simpler design of the hydraulic circuit can be assumed.

**Acknowledgments** The work reported here has been supported by the research projects P101/10/0209, MSM 6198910027 and CZ.1.05/1.1.00/02.0070. The support is gratefully acknowledged.

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