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# An onshore hydro/piezo/electric system and its application to energy harvesting from sea waves

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## ABSTRACT

Sea waves induce significant pressures on coastal surfaces, especially on rocky vertical cliffs or breakwater structures (Peregrine, 2003). In the present work, this hydrodynamic pressure is considered as the excitation acting on a piezoelectric material sheet, installed on a vertical cliff, and connected to an external electric circuit (on land). The whole hydro/piezo/electric system is modeled in the context of linear wave theory. The piezoelectric elements are assumed to be small plates, possibly of stack configuration (Preumont, 2011), under a specific wiring. They are connected with an external circuit, modeled by a complex impedance, as usually happens in preliminary studies (Liang & Liao, 2011). The piezoelectric elements are subjected to thickness-mode vibrations under the influence of incident harmonic water waves. Full, kinematic and dynamic, coupling is implemented along the water-solid interface, using propagation and evanescent modes (Athanassoulis & Belibassakis, 1999). For most energetically interesting conditions the long-wave theory is valid, making the effect of evanescent modes negligible, and permitting us to calculate a closed-form solution for the efficiency of the energy harvesting system, which is dependent on two dimensionless hydro/piezo/electric parameters. The efficiency may become significant (as high as 30 – 50%) for appropriate combinations of parameter values, which, however, corresponds to exotically flexible piezoelectric materials. The existence or the possibility to construct such kind of materials formulate a question to material scientists.

## 1. INTRODUCTION

Ocean waves carry huge amount of energy propagating in a thin layer near the surface of the sea and, eventually, impinging on the coastline. Being a surface phenomenon, sea waves consist one of the most intense natural energy resources. Nowadays this resource has been very well documented throughout the world ocean. See, e.g., Pontes, Athanassoulis *et al*. (1995, 1996), Cavaleri, Athanassoulis, Barstow (1999), Barstow & Mørk *et al*. (2003), Mørk & Barstow *et al*. (2010), which describe the results of three European Commission–funded projects (WERATLAS, EUROWAVES and WORLDWAVES) studying offshore and nearshore wave conditions and wave energy resource.

In the open sea, especially in the northern oceans, the mean wave power may be more than $100\,\mathrm{kW}$ per meter of the wave front ($100\,\mathrm{kW/m}$). Of course, as the waves

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approach the coast, shoaling causes breaking on the free surface and dissipation in the seabed boundary layer, resulting in lower figures for the available mean wave power per wave-front meter. Even thought, when the shoreline has the form of an (almost) vertical cliff, either rocky or manmade, with appreciable depth in front of it, waves impinge on it exerting large pressure loads. The wave climate in such sites has been extensively studied, mainly to provide information for the design of breakwaters or for the study of the erosive effects on natural coasts, as well as for assessing the available wave potential for nearshore and onshore wave energy devices. An extended list of many existing wave energy devices, their physics principles and relevant technological aspects can be found in Cruz (2008) and Khaligh & Onar (2010). In the present paper, we are going to investigate if it is possible to take off wave power directly through a piezoelectric material placed on the cliff.

Piezoelectricity, known since 1880 thanks to the experimental work by the brothers Pierre and Jacques Curie, has been intensively exploited in the recent years for designing energy harvesting devices, mainly in microscale. See, e.g., the recent review articles Sodano *et al.* (2004), Anton & Sodano (2007), Priya (2007), and the books by Priya & Inmam (2009), Erturk & Inman (2011) .

The subject of direct piezoelectric conversion of ocean wave energy is rather undeveloped. The main reason for this seems to be the very low frequency regime of sea waves (below to 0.5 Hz). Early concepts of piezoelectric wave harvesters, based on piezoelectric films or ropes made of Polyvinylidene fluoride (PVDF) (Taylor & Burns, 1983, Haeusler & Stein, 1985), have not been practically implemented. The concept of a floating wave carpet, proposed by Koola & Ibragimov (2003) could be interesting when combined with an appropriate modeling and analysis of a flexible piezo-electric material. Murray & Rastegar (2009) proposed a two-stage piezoelectric wave energy harvester, consisting of a primary, low frequency, subsystem (e.g., a heaving buoy), which excites a secondary subsystem vibrating at its natural frequency, the latter being orders of magnitude higher than the frequency of the primary subsystem. The aforementioned piezo-electric wave energy harvesters, as well as other existing variants of them, all belong to the classes of point absorbers or attenuators. The goal of the present paper is to investigate a terminator-type hydro/piezo/electric system that could extract electric energy from the direct impact of sea waves, impinging upon a vertical cliff.

The structure of the paper is as follows: In Sec. 2 the whole system, consisting of three distinct subsystems, is described in detail. In Sec. 3 the thickness mode of the piezoelectric subsystem is modeled and studied. In Sec. 4 the hydrodynamic problem is formulated and a complete modal representation of the wave potential in the vicinity of the vertical cliff is given. Results from Sec. 3 and 4 are exploited in Sec. 5, where the coupling of the two subsystems is implemented through the interfacial, fluid–solid, matching conditions. In the same section an approximate (yet accurate) closed form expression is obtained for the wave reflection coefficient, which controls the energetic coupling of the three subsystems. Finally, in Sec. 6, the ohmic resistance of the external circuit optimizing the efficiency of the hydro/piezo/electric harvester is found. The optimized efficiency is calculated analytically and investigated numerically. It is shown that efficiency may become significant (as high as 30 – 50%) for appropriate combinations of two dimensionless hydro/piezo/electric parameters. To practically exploit this high efficiency new piezoelectric materials are needed, exhibiting much higher flexibility than the usual ones, and high values of the energy conversion factor. The possibility of manufacturing such kind of materials remains an open question.

## 2. SYSTEM CONFIGURATION

Since the present paper aims at a preliminary assessment of a piezoelectric energy harvesting system, we focus on the basic physics facts, disregarding many technical details.

### *2.1 The hydrodynamic subsystem: sea waves impinging into the cliff*

Waves impinging into the cliff produce a complicated, nonlinear, slightly dissipative, impact phenomenon, resulting in the development of a fluctuating hydrodynamic pressure pattern on the fluid-solid interface. Realistic, wind-generated, sea waves are usually modeled as random waves, characterized by means of their spectrum. The angular frequencies $\omega$ may range from $\approx 0.314\,\text{rad/sec}$ to $\approx 3.14\,\text{rad/sec}$ (corresponding to periods $2\,\text{sec} \le T \le 20\,\text{sec}$), the actual range being strongly case and site dependent. For reasons explained above, we shall restrict ourselves to a reasonably convenient mathematical formulation of the hydrodynamic problem, namely the linear water-wave theory; see, e.g., Wehausen & Laitone (1960), Sec. 11. We shall also make the assumptions that the vertical cliff has an appreciable horizontal extent and the front of the incident wave is almost aligned to it, which permit us to treat the hydrodynamic problem as two-dimensional (2D). In addition, to simplify the hydrodynamic analysis, we assume that the seabed is horizontal. A vertical section of the fluid domain $\Omega$ is shown in Fig. 1. In the same figure it is also shown the Cartesian coordinate system used in the hydrodynamic analysis. The not shown $y$ axis (perpendicular to the paper) extends along the horizontal dimension of the vertical cliff.

Under the assumption of linearity, the superposition principle is valid, which permits us to synthesize any (linear) wave pattern from the monochromatic (frequency domain) solution. Thus, focusing on the monochromatic case, we can assume that the velocity field is derived by a velocity potential $\Phi^f(x,z;t)$, which is expressed in terms of the complex phasor $\Phi^f(x,z;\omega)$ by means of the equation

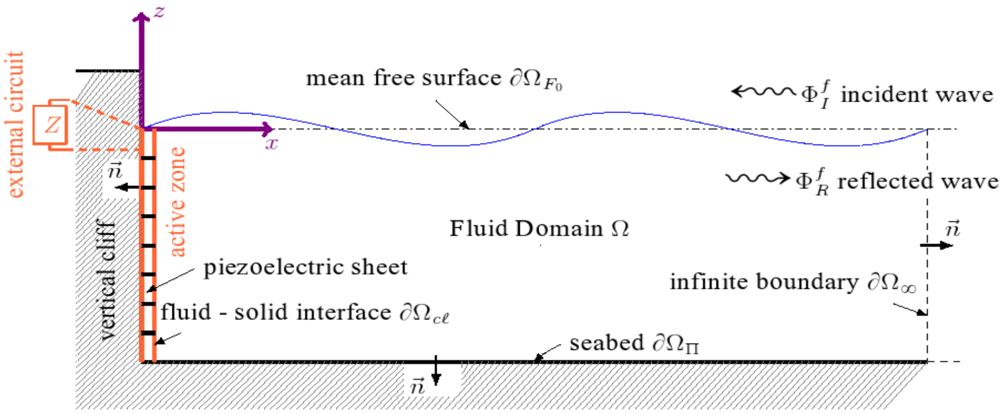


Fig. 1. Geometric configuration of the system

$$\Phi^f(x,z;t) = \mathrm{Re}_j\left\{\Phi^f(x,z;\omega)\exp(j\omega t)\right\} =$$
$$= \mathrm{Re}_j\left\{\left[\Phi_I^f(x,z;\omega)+\Phi_R^f(x,z;\omega)+\Phi_{\ell oc}^f(x,z;\omega)\right]\exp(j\omega t)\right\}, \tag{1a}$$

where

$$\Phi_I^f(x,z;\omega) = \frac{jg}{\omega}\frac{H}{2}\frac{\cosh\left[k_0(h_D+z)\right]}{\cosh(k_0 h_D)}\exp(jk_0 x), \tag{1b}$$

is the incident wave, having amplitude $H/2$,

$$\Phi_R^f(x,z;\omega) = W\frac{jgH}{2\omega}\frac{\cosh\left[k_0(h_D+z)\right]}{\cosh(k_0 h_D)}\exp(-jk_0 x), \tag{1c}$$

is the reflected wave, and $\Phi_{\ell oc}^f(x,z;\omega)$ is a local wave field, vanishing exponentially far from the cliff. (The exact form of $\Phi_{\ell oc}^f(x,z;\omega)$ will be given in Sec.4). In Eqs. (1), $j=\sqrt{-1}$ is the imaginary unit, $g$ is the acceleration due to gravity, $\omega$ is the frequency of the monochromatic incident wave, $k_0$ is the corresponding wave number, $h_D$ is the sea depth in front of the vertical cliff, and $W$ is the reflection coefficient. The latter is, in general, complex valued, $W=\gamma\cdot e^{j\delta}$, $\gamma = |W|$ being the amplitude attenuation factor and $\delta = \mathrm{Arg}(W)$ being the phase shift with respect to the incident wave.

The hydrodynamic pressure field in the fluid, $p(x,z;\omega)$, is given by the linearized Bernoulli's law:

$$p(x,z;\omega) = -j\rho_f\,\omega\,\Phi^f(x,z;\omega), \tag{2}$$

where $\rho_f$ is the mass density of sea water. Note that, when the nonlinear effects are taken into account, the total hydrodynamic pressure induced on the vertical cliff exhibits, in general, larger values than those obtained by means of the linear theory.

### *2.2 The piezoelectric subsystem: energy harvesting elements on the cliff*

Piezoelectricity, initially detected in some crystalline solid materials, is a phenomenon according to which an electric field is developed in the material in response to externally applied mechanical stresses. It is a reversible process; when an external electric field is applied to the piezoelectric material, the latter exhibits deformations. Linear piezoelectricity is quantified macroscopically by means of the piezoelectric constitutive equations, connecting mechanical stress $\{\sigma_i\}_{i=1}^{i=6} = \{\sigma_{11},\sigma_{22},\sigma_{33},\sigma_{23},\sigma_{31},\sigma_{12}\}$ and electric displacement $D_k$ to mechanical strain $\{e_i\}_{i=1}^{i=6} = \{e_{11},e_{22},e_{33},2e_{23},2e_{31},2e_{12}\}$ and electric intensity $E_k$:

$$\sigma_i = c_{ik}^E\,e_k - \epsilon_{ki}\,E_k, \qquad D_i = \epsilon_{ik}\,e_k + \varepsilon_{ij}^S\,E_j \tag{3a,b}$$

where $c_{ik}^{E}$ is the elastic stiffness tensor under constant electric intensity, $\varepsilon_{ij}^{S}$ is the dielectric permittivity tensor under constant strain, and $\epsilon_{ik}$ is the piezoelectric stress tensor. The latter contains null elements since the piezoelectric effect disappears for certain crystallographic and limiting point symmetry groups. (Newnham, 2005, Ch. 12.3).

Piezoelectric materials are available either in small solid pieces or in the form of films or ropes. In this conjunction, and in order to exploit the thickness-mode oscillations, the piezo-elements considered in the present study are assumed to be small plates with transverse dimensions $\ell_1$, $\ell_2$, of order of magnitude of some centimeters, and thickness $h$, of order of magnitude of some millimeters. One of their surfaces $S = \ell_1 \times \ell_2$ is clamped on the vertical cliff and the other is free to oscillate under the influence of the wave impact. Piezoelements are installed contiguously from the sea bottom to the mean free surface and are electrically connected in series, forming a vertical array of $M_1$ piezoelements; see Fig. 1. The repetition of this array for an appreciable length $L_2 = M_2 \ell_2$ in the direction of $y-$ axis (horizontally along the cliff), in conjunction with a parallel electric connection between the vertical arrays, results in a two dimensional active zone of piezoelements, which is also called the piezoelectric sheet; Fig. 2.

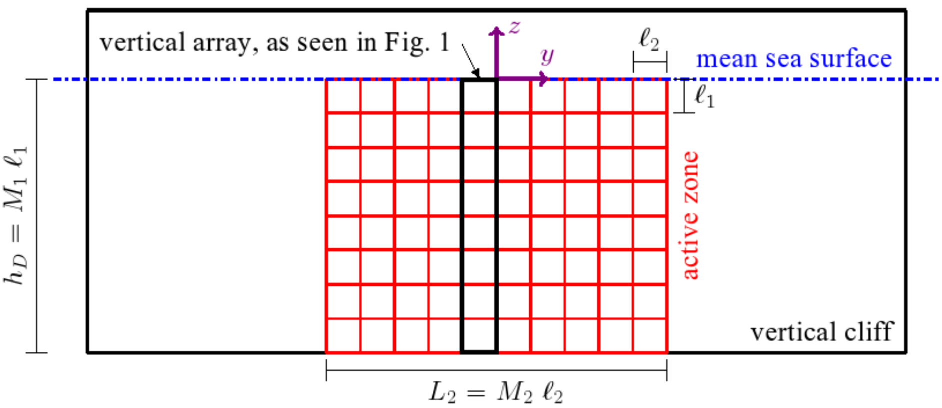


Fig. 2. The two-dimensional, cliff-mounded, piezoelectric active zone

Two basic technical issues, relevant to the formation and the installation of the piezoelectric sheet, are the insulation from the ambient sea water and the fixation on the vertical cliff. Both issues are strongly material dependent and they are out of the scope of the present work, which aims at a feasibility study of the basic concept.

*2.3 The external electrical circuit*

In order to take off power from the impinging waves, the output terminals of the system of piezoelements should be plugged in an external electrical circuit. A typical choice for the latter is the so-called standard energy harvesting (SEH) circuit, including a diode rectifier and a smoothing capacitor; see, e.g., Gyomar *et al* (2005) and Shu & Lien (2006). A simpler choice, which is the usual one in most of the literature emphasizing on the mechanical part of the system, is a standard AC circuit, characterized by its impedance

$$Z(\omega) = R + jX(\omega), \quad (4)$$

where $R$ models the total resistance and $X(\omega)$ models the total reactance. A thorough discussion concerning the effect of the external circuit on the energy flow in piezoelectric harvesters can be found in Liang & Liao (2011).

## 3. THE PIEZOELECTRIC PROBLEM

### *3.1 The piezoelectric problem for a single piezoelement*

For each piezoelement, a local, $(x_1 x_2 x_3)-$ Cartesian coordinate system is introduced, with $x_i$-axis coinciding with the corresponding principal piezoelectric axis; see Fig.3. Each piezo-element is considered geometrically symmetric with respect to the coordinate planes $x_1 = 0$, $x_2 = 0$, $x_3 = 0$. Face $\gamma\delta$ is clamped (on the vertical cliff), while face $\alpha\beta$ is free to oscillate under the influence of incoming sea waves. [Note that in the physical position, faces $\alpha\beta$ and $\gamma\delta$ of each piezoelement are vertical; cf. Fig. 1]. Both faces $\alpha\beta$ and $\gamma\delta$ are electroded.

In this paper the thickness-mode vibration is considered, in which the resulting electric polarization vector has the same direction as the applied stress (thus $i = j = 3$). Thus, the constitutive equations (3a,b) take the form

$$\sigma_3(x_3;t) = c_{33}^E e_3(x_3;t) - \epsilon_{33} E_3(x_3;t), \quad (5a)$$

$$D_3(x_3;t) = \epsilon_{33} e_3(x_3;t) + \varepsilon_3^S E_3(x_3;t), \quad (5b)$$

where $\varepsilon_3^S \equiv \varepsilon_{33}^S$. The external excitation (tensile) stress $\hat{\sigma}_3$, applied to the electroded face $\alpha\beta$, equals to $-p$, where $p$ is the hydrodynamic pressure; the presence of minus sign is due to the fact that $p$ is always compressive. The applied excitation $\hat{\sigma}_3$ gives rise to mechanical displacements $u_3(x_3;t)$ and voltage difference

$$\Delta V(t) \equiv V_1(t) - V_0(t) = \Phi^{el}(h/2;t) - \Phi^{el}(-h/2;t). \quad (6)$$

between the two faces $\alpha\beta$ and $\gamma\delta$, where $\Phi^{el}(x_3;t)$ is the electric potential field developed inside the piezoelement.

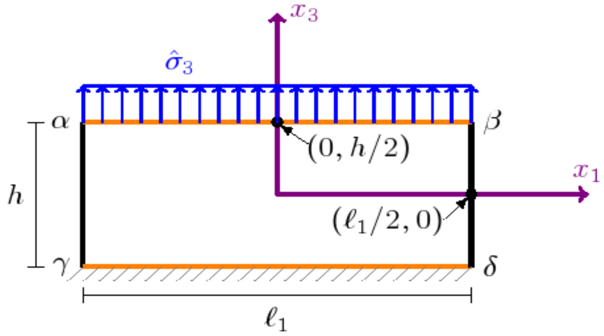


Fig. 3. Mode 3-3 vibration of a single piezoelement with $\alpha\beta$ and $\gamma\delta$ faces electroded

As the physical length (width) of each piezoelement is small in comparison with both the depth $h_D$ of the sea and the wavelength of sea waves, the applied stress $\hat{\sigma}_3$ (due to sea waves) can be considered almost constant on the face $\alpha\beta$ of each piezoelement. Thus, we can consider $\hat{\sigma}_3$ equal to the mean value of the hydrodynamic pressure $-p$ over the face $\alpha\beta$, and simplify the piezoelectric problem assuming that all quantities are dependent only on $x_3$ coordinate. In this way, the piezoelectric phenomenon to be studied becomes essentially one dimensional (1D).

Since the frequency range of sea waves (exciting the piezoelements) is very low in comparison with electromagnetic waves frequencies, the equations governing the piezoelectric phenomenon are the quasi-static ones. See, e.g., Parton & Kudryavtsev (1988), Ch. 1, Bardzokas & Filshtinsky (2006), Ch. 2, Meitzler *et. al.* (1987). Note that mechanical and dielectric dissipative phenomena are ignored in this study. Under the assumption of monochromatic excitation, all quantities can be represented by the corresponding phasors, i.e. $\hat{\sigma}_3(t) = \mathrm{Re}_j\left\{\hat{\sigma}_3(\omega)\exp\left(j\omega t\right)\right\}$. Then, for the present case of 1D linear problem in the frequency domain, the set of equations and boundary conditions governing the piezoelectric phenomenon in a single piezoelement are:

$$c_{33}^E \frac{\partial^2 u_3}{\partial x_3^2}\left(x_3;\omega\right) + \epsilon_{33}\frac{\partial^2 \Phi^{e\ell}}{\partial x_3^2}\left(x_3;\omega\right) = -\rho_b\,\omega^2 u_3\left(x_3;\omega\right), \tag{7}$$

$$\varepsilon_3^S \frac{\partial^2 \Phi^{e\ell}}{\partial x_3^2}\left(x_3;\omega\right) = \epsilon_{33}\frac{\partial^2 u_3}{\partial x_3^2}\left(x_3;\omega\right), \tag{8}$$

$$u_3\left(-h/2;\omega\right) = 0, \tag{9a}$$

$$c_{33}^E \frac{\partial u_3}{\partial x_3}\left(h/2;\omega\right) + \epsilon_{33}\frac{\partial \Phi^{e\ell}}{\partial x_3}\left(h/2;\omega\right) = \hat{\sigma}_3(\omega), \tag{9b}$$

$$\Phi^{e\ell}\left(-h/2;\omega\right) = V_0(\omega), \tag{10a}$$

$$\Phi^{e\ell}\left(h/2;\omega\right) = V_1\left(\omega\right), \tag{10b}$$

where $\rho_b$ is the mass density of the piezoelement.

Eq. (10a) is a gauge condition which sets the level value of the potential. $V_0(\omega)$ is arbitrarily chosen, the quantity having physical meaning being the voltage difference $\Delta V(\omega)$. Eq. (10b) relates the unknown quantity $V_1(\omega) = \Delta V(\omega) + V_0(\omega)$ with the also unknown quantity $\Phi^{e\ell}\left(h/2;\omega\right)$. Accordingly, boundary conditions (10a,b) do not specify boundary data; they just specify relations between unknown quantities. As a consequence, the solution of the boundary problem (7) – (10) is not unique. As we shall see in the sequel, this lack of boundary data will result in an undermined coefficient, the true value of which will be obtained later on by using information from the external electric circuit.

The solution of boundary-value problem (7) – (10), applied at $x_3 = h/2$ is:

$$u_3\left(x_3 = h/2;\omega\right) = \left[\hat{\sigma}_3(\omega) - \epsilon_{33} A(\omega)\right] \frac{h}{c_{33}^D}, \tag{11}$$

$$\Delta V(\omega) = V_1(\omega) - V_0(\omega) = \frac{\left[\epsilon_{33}\sigma_3(\omega) - \epsilon_{33}^2 A(\omega)\right]}{\varepsilon_3^S} \frac{h}{c_{33}^D} + A(\omega)h. \tag{12}$$

The undetermined coefficient $A(\omega)$ will be expressed in terms of the current $I = I(\omega)$ flowing through the piezoelement (and through the external circuit). For a detailed derivation of Eqs. (11) and (12), see Athanassoulis & Mamis ( 2012).

For this, we need some (simplified) electrodynamic equation, not included in the quasi-static problem (7) – (10). (See, e.g., Erturk & Inman, 2011, Sec. 3.1.3, Parton & Kudryavtsev, 1988, Sec. 1.3). Using the definition of displacement current we obtain:

$$\frac{I}{S} = \dot{D}_3, \tag{13}$$

where $S$ is the area of each of the electroded surfaces of the piezoelement. Using constitutive relation (5b), eq. (13) is written as

$$I = j\omega S\left(\epsilon_{33} e_3 + \varepsilon_3^S E_3\right). \tag{14}$$

Recalling that $e_3 = \partial u_3 / \partial x_3$ and $E_3 = -\partial \Phi^{el} / \partial x_3$, we obtain

$$e_3 = \frac{\hat{\sigma}_3(\omega) - \epsilon_{33} A(\omega)}{c_{33}^D}, \quad E_3 = -\frac{\epsilon_{33}\hat{\sigma}_3(\omega) - \epsilon_{33}^2 A(\omega)}{\varepsilon_3^S c_{33}^D} - A(\omega). \tag{15a,b}$$

Substituting Eqs. (15a,b) into Eq. (14) we get

$$I(\omega) = -j\omega\varepsilon_3^S S A(\omega). \tag{16}$$

### *3.2 The system of piezoelements on the vertical cliff in series connection*

We shall now proceed to considering the whole active zone. Various connections are possible between the electrodes of adjacent piezoelements that form each vertical array. In the present work a series connection has been selected, as depicted in Fig. 4.

The results obtained in previous subsection, for a single piezoelement, can be applied to each piezoelement of the group. All quantities associated with the $m-th$ piezoelement, e.g., $u_3\left(h/2,\omega\right)$, $\hat{\sigma}_3(\omega)$, etc., will be now distinguished by a superscript $m$ in parenthesis, e.g. $u_3^{(m)}\left(h/2,\omega\right)$, $\hat{\sigma}_3^{(m)}(\omega)$, etc.. Considering all piezoelements being of the same material and of the same dimensions, we do not use the $m$ superscript for material properties and element dimensions. On the basis of the series connection of adjacent piezoelements, $I^{(m)}(\omega) = I(\omega)$, $V_0^{(m)} = V^{(m-1)}$ and $V_1^{(m)} = V^{(m)}$, $m = 1,2,\ldots,M_1$. Using Eq. (12), the voltages $V^{(m)}(\omega)$ at the output electrode of each piezoelement are given by (see also Fig. 4):

$$V^{(m)}(\omega) - V^{(m-1)}(\omega) = \frac{\left[\epsilon_{33}\,\sigma_3^{(m)}(\omega) - \epsilon_{33}^2\,A^{(m)}(\omega)\right]h}{\varepsilon_3^S\,c_{33}^D} + A^{(m)}(\omega)\,h\,, \tag{17}$$

where $$\hat{\sigma}_3^{(m)}(\omega) = \frac{1}{\ell_1}\int_{\substack{m-th \\ piezoelement}} \hat{\sigma}_3(z;\omega)\,dz\,, \tag{18}$$

and $\hat{\sigma}_3(z;\omega) = -p(z;\omega)$, $-h_D \le z \le 0$, $z$ being the global vertical coordinate; see Fig. 1.

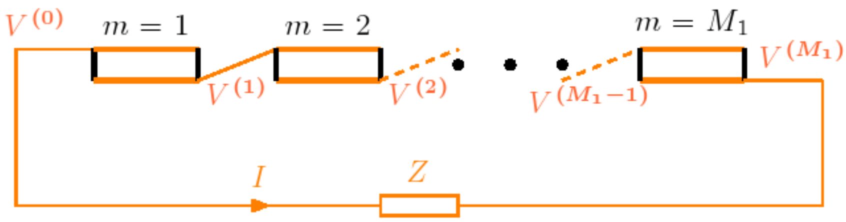


Fig. 4. Series connection of piezoelements forming one vertical array

Setting $V^{(0)} = 0$ on the first electrode of the first piezoelement, and summing up all Eqs. (17), we find the total voltage difference $\Delta V(\omega)$. Then applying Ohm's law to the external circuit, we get the following equation for the current

$$I(\omega) = \frac{\Delta V(\omega)}{Z(\omega)} = \frac{\left(\epsilon_{33}\sum_{m=1}^{M_1}\sigma_3^{(m)}(\omega) - \epsilon_{33}^2\sum_{m=1}^{M_1}A^{(m)}(\omega)\right)h}{\varepsilon_3^S\,c_{33}^D\,Z(\omega)} + \frac{h}{Z(\omega)}\sum_{m=1}^{M_1}A^{(m)}(\omega)\,. \tag{19}$$

As the piezoelements are connected in series, the current $I(\omega)$ is common over the whole circuit. Thus, Eq. (16), applied to each piezoelement, takes the form

$$I(\omega) = -j\,\omega\,\varepsilon_3^S\,S\,A^{(m)}(\omega) \;\Rightarrow\; A^{(1)} = \ldots = A^{(m)} = \ldots = A^{(M_1)} = A\,. \tag{20}$$

By introducing the piezoelectric constants

$$k_t^2 = \frac{\epsilon_{33}^2}{\varepsilon_3^S\,c_{33}^D} \quad \text{and} \quad C_0 = \frac{\varepsilon_3^S\,S}{h} \tag{21a,b}$$

called the coupling factor and clamped capacitance (of each piezoelement) respectively, [for a physical interpretation see, e.g., Lefeuvre *et. al.* (2010), Guygomar *et. al.* (2005)], and setting

$$\mathscr{E}_t(\omega) = \frac{k_t^2}{\left(1-k_t^2\right) + j\,\omega\left(C_0/M_1\right)Z(\omega)}\,, \tag{22}$$

the system of equations (19) – (20) provides the following solution for the common value of $A$'s, $A(\omega)$:

$$A(\omega) = -\frac{\mathscr{E}_t}{\epsilon_{33}} \frac{1}{M_1} \sum_{m=1}^{M_1} \hat{\sigma}_3^{(m)}(\omega). \quad (23)$$

Substituting Eq. (23) into Eq. (19), we obtain the total voltage output:

$$\Delta V(\omega) = I(\omega) Z(\omega) = j\omega C_0 Z(\omega) \frac{\mathscr{E}_t(\omega)}{\epsilon_{33}} h \frac{1}{M_1} \sum_{m=1}^{M_1} \hat{\sigma}_3^{(m)}(\omega). \quad (24)$$

Futhermore, substituting $A(\omega)$ from Eq. (23) into Eq. (11):

$$u_3^{(m)}\left(x_3 = h/2; \omega\right) = \frac{h}{c_{33}^D} \hat{\sigma}_3^{(m)}(\omega) + \frac{h\,\mathscr{E}_t(\omega)}{c_{33}^D} \frac{1}{M_1} \sum_{m=1}^{M_1} \hat{\sigma}_3^{(m)}(\omega), \quad m = 1, \dots, M. \quad (25)$$

Since the vertical width $\ell_1$ of each piezoelement is only a small fraction of the water-wave length, the stress variation over the face $\alpha\beta$ of each piezoelement is negligible, which implies that Eq. (25) can be written in the continuous form as:

$$\hat{u}_3(z;\omega) = \frac{h}{c_{33}^D} \hat{\sigma}_3(z;\omega) + \frac{h}{c_{33}^D} \mathscr{E}_t(\omega)\, \overline{\hat{\sigma}_3(\omega)}, \qquad -h_D \le z \le 0, \quad (26)$$

where $\hat{u}_3(z;\omega) \approx u_3^{(m)}\left(x_3 = h/2; \omega\right)$, for $z$ varying over the face $\alpha\beta$ of the $m-th$ piezo-element and $\overline{\hat{\sigma}_3(\omega)}$ is the mean excitation stress over the whole vertical cliff. The first term in the right-hand side of Eq. (26) is of elastic nature, having a local dependence on the applied pressure and stiffness coefficient $c_{33}^D = c_{33}^E + \epsilon_{33}^2 / \varepsilon_3^S$. The presence of $c_{33}^D$, which is greater than the standard $c_{33}^E$ coefficient appearing in the constitutive equation (5a), models the piezoelectric stiffening phenomenon. (For a general discussion considering piezoelectric stiffening see Auld (1969) and Yang (2006), Sec. 2.2.1.) The second term in the right-hand side of Eq. (26) is of purely piezoelectric nature, it has a global dependence on the applied pressure, and is also dependent on the external electric circuit characteristics through the factor $\mathscr{E}_t(\omega)$.

### *3.4 Power flow relations*

The net (time average) power flowing through the piezoelectric sheet covering an area $h_D \times L_2$ of the cliff (see Fig. 2.) is given by the equation

$$\mathrm{P}_{c\ell}^{\mathrm{piezo}}(\omega) = L_2 \frac{1}{2} \mathrm{Re}_j \left\{ \int_{z=-h_D}^{z=0} j\omega\, \hat{u}_3(z;\omega)\, \hat{\sigma}_3^*(z;\omega)\, dz \right\}, \quad (27)$$

where the asterisk denotes the complex conjugate.

On the other hand, the net electric power $\mathrm{P}_Z(\omega)$ consumed by the external circuit, that is by the impedance $Z(\omega) = R + jX(\omega)$, is calculated in terms of the electric quantities $\Delta V_{\text{tot}}(\omega)$ and $I_{\text{tot}}(\omega)$. Due to the parallel electrical connection between the vertical arrays and the identical electrical quantities of each array, it holds true that $\Delta V_{\text{tot}}(\omega) = \Delta V(\omega)$ and $I_{\text{tot}} = M_2 I = L_2 I / \ell_2$, where $M_2$ is the number of vertical arrays that form the active zone. Thus, $\mathrm{P}_Z(\omega)$ is calculated as

$$\mathrm{P}_Z(\omega) = \frac{1}{2}\,\mathrm{Re}_j\left\{\Delta V_{\text{tot}}(\omega)\, I^*_{\text{tot}}(\omega)\right\} = M_2\,\frac{1}{2}\,\mathrm{Re}_j\left\{\Delta V(\omega)\, I^*(\omega)\right\}. \tag{28}$$

After extended algebraic manipulations, we can show that $\mathrm{P}^{\text{piezo}}_{c\ell}(\omega) = \mathrm{P}_Z(\omega)$. Thus, the whole net power flowing through the piezoelectric sheet is delivered at the external circuit. This is a statement of the conservation of energy, since we have neglected the dissipation within the piezoelements.

## 4. THE HYDRODYNAMIC PROBLEM

### *4.1 Mathematical formulation of the hydrodynamic boundary-value problem*

The 2D liquid domain $\Omega$ extends from the seabed $\partial\Omega_\Pi$ ($z=-h_D$) up to the free surface $\partial\Omega_F$ ($z=\eta(x;t)$), and from the vertical cliff $\partial\Omega_{c\ell}$ up to infinity $\partial\Omega_\infty$; see Fig.1. Two hydrodynamic fields are involved in the problem: the velocity potential field $\Phi^f(x,z;t)$, and the pressure field $p(x,z;t)$. Both fields are independent from the $y$ coordinate, in accordance with the 2D character of the problem, as discussed in subsection 2.2. In the context of linear water-wave theory, the domain of definition of $\Phi^f(x,z;t)$ is restricted to the a half strip $\Omega_0 = \left\{-h_D < z < 0,\ 0 < x < +\infty\right\}$.

Assuming further that the whole (linear) system is excited by a monochromatic incident wave, with angular frequency $\omega$, all fields will be monochromatic with the same frequency and, thus, can be expressed by means of their phasors, as in Eq. (1a). The complete, linearized, boundary-value problem for the total wave potential $\Phi^f(x,z;\omega)$, in the frequency domain, is formulated as follows (see, e.g., Wehausen & Laitone (1960), Sec. 11, Stoker (1957), Sec. 3.1, or Mei *et al.* (2005), Sec. 1.4):

$$\Delta\Phi^f(x,z;\omega) = 0\,, \quad \text{in } \Omega_0\,, \tag{29}$$

$$\frac{\partial\Phi^f}{\partial z}(x,z=0;\omega) - \mu_0\,\Phi^f(x,z=0;\omega) = 0\,, \qquad \mu_0 = \omega^2/g\,, \qquad \text{(on } \partial\Omega_{F_0}) \tag{30}$$

$$\frac{\partial\Phi^f}{\partial z}(x,z=-h_D;\omega) = 0\,, \qquad \text{(on } \partial\Omega_\Pi) \tag{31}$$

$$\frac{\partial\Phi^f}{\partial x}(x=0,z;\omega) = j\omega\,\hat{u}_3(z;\omega)\,, \qquad \text{(on } \partial\Omega_{c\ell}) \tag{32}$$

$$p(x=0,z;\omega) = -\hat{\sigma}_3(z;\omega)\,, \qquad \text{(on } \partial\Omega_{c\ell}) \tag{33}$$

$$\Phi^f \to \Phi^f_I + \Phi^f_R\,, \quad \text{when } x\to+\infty\,, \quad \text{(i.e., at } \partial\Omega_\infty) \tag{34}$$

where $\Phi_I^f = \Phi_I^f(x, z; \omega)$ is the incident wave, and $\Phi_R^f = \Phi_R^f(x, z; \omega)$ is the reflected wave, already prescribed by Eqs (1b) and (1c). The pressure $p(x, z; \omega)$ is given by the linearized Bernoulli's law, Eq. (2).

Conditions (32) and (33) are matching conditions, ensuring the continuity of normal velocity and normal pressure through the fluid-solid interface $\partial\Omega_{c\ell}$, respectively. These conditions match the hydrodynamic quantities $\Phi^f$ and $p$ with the elastodynamic quantities $\hat{u}_3$ and $\hat{\sigma}_3$ and, through them, with the piezoelectric problem.

*4.2 Modal representation of the wave potential*

The solution of the coupled problem is greatly facilitated by means of the following modal representation of the wave potential:

**Modal Representation Theorem of the Wave Potential:** Every function $\Phi^f$ defined in the half strip $\Omega_0$, satisfying the Laplace Eq. (39) therein, the free-surface boundary condition (30) on $\partial\Omega_{F_0}$ $(z=0)$, the seabed boundary condition (31) on $\partial\Omega_\Pi$ $(z=-h_D)$, and the bounded-ness condition $\left|\Phi^f(x, z; \omega)\right| \le \mathrm{M} = const.$, in $\Omega_0$, admits of the following representation

$$\Phi^f(x, z; \omega) = \frac{j g H}{2\omega} Z_0(z) \exp\left(j k_0 x\right) + W \frac{j g H}{2\omega} Z_0(z) \exp\left(-j k_0 x\right) + \Phi_{\ell oc}^f(x, z; \omega), \quad (35)$$

where the first term in the right-hand side of the above equation represents the incident wave, the second term represents the reflected wave, and the third term, $\Phi_{\ell oc}^f(x, z; \omega)$, represents a local wave field, vanishing exponentially far from the cliff, which can be expanded in form of an infinite series of evanescent modes, as follows:

$$\Phi_{\ell oc}^f(x, z; \omega) = \sum_{n=1}^{\infty} \Phi_n^f(x, z; \omega) = \sum_{n=1}^{\infty} C_n Z_n(z) \exp\left(-k_n x\right). \quad (36)$$

$Z_0(z)$, $Z_n(z)$, $n = 1, 2, 3, \ldots$, are the vertical eigenfunctions of the water-wave problem, given by the equations

$$Z_0(z) = \frac{\cosh\left[k_0\left(h_D + z\right)\right]}{\cosh\left(k_0 h_D\right)}, \qquad Z_n(z) = \frac{\cos\left[k_n\left(h_D + z\right)\right]}{\cos\left(k_n h_D\right)}, \quad n = 1, 2, \ldots \quad (37a,b)$$

The constants $k_0$ and $k_n$, $n = 1, 2, 3, \ldots$, appearing in the above equations are the positive roots of the dispersion relation:

$$\frac{\mu_0}{k_0} = \tanh\left(k_0 h_D\right), \qquad \frac{\mu_0}{k_n} = -\tan\left(k_n h_D\right), \quad (38a,b)$$

where $h_D$ is the (constant) sea depth.

The coefficients $W, C_n$, $n = 1, 2, 3, \ldots$, are free; they can be determined by means of the boundary (matching) conditions imposed on the vertical boundary surfaces

$\partial\Omega_{c\ell}, \partial\Omega_{\infty}$. The above representation theorem traces back to Kreisel (1949). It is also discussed by Wehausen & Laitone (1960), Sec. 17 and Mei (2005), Sec. 8.4.1, and it has been extensively used in the study of various water wave problems over a locally varying bathymetry (see, e.g., Bai & Yeung, (1974), Athanassoulis & Belibassakis (1999), Belibassakis & Athanassoulis (2005)).

Using Eqs. (35) and (36) we easily obtain representations of the horizontal velocity $\hat{\Phi}^f_{,x}(z;\omega) = \partial\Phi^f(x=0,z;\omega)/\partial x$ and the pressure $\hat{p}(z;\omega) = p(x=0,z;\omega)$ on the fluid-solid interface $\partial\Omega_{c\ell}$, in terms of the unknown coefficients $W$, $C_n$, $n=1,2,3,\ldots$. These representations will be exploited in the next section in order to solve the coupled problem.

*4.3 Power flow relations*

The net (time average) power flowing towards the cliff through a vertical section at any position $x=a$ within the liquid domain (having horizontal extent $L_2$, normally to the wave front) is given by the equation

$$\mathrm{P}^f_a(\omega) = L_2 \frac{1}{2}\mathrm{Re}_j\left\{ j\omega\rho_f \int_{-h_D}^{0} \Phi^f(x=a,z;\omega)\left(\Phi^f_{,x}(x=a,z;\omega)\right)^* dz \right\}. \tag{39}$$

As expected from energy considerations (and it can be proved by using Green's Theorem) the above quantity is independent from the position $x=a$ of the considered section. Accordingly, the easiest way to calculate $\mathrm{P}^f_a(\omega)$ (in terms of hydrodynamic quantities) is by letting $a\to\infty$ and using Eq. (35) keeping only the first two (non-evanescent) modes. After straightforward calculations, we obtain

$$\mathrm{P}^f_a(\omega) = \frac{1}{8}\rho_f g H^2 L_2 \omega \frac{k_0}{\mu_0}\left(1-|W|^2\right)\|Z_0\|^2. \tag{40}$$

On the other hand, if we apply Eq. (39) to $x=0$, and take into account the matching conditions (32) and (33), we readily see that $\mathrm{P}^f_{a=0}(\omega)$ is exactly the power flowing through the fluid-solid interface $\partial\Omega_{c\ell}$ towards the piezoelements, which, finally, is consumed by the external circuit; see Eq. (28). The equations

$$\mathrm{P}^f_\infty(\omega) = \mathrm{P}^f_a(\omega) = \mathrm{P}^f_{a=0}(\omega) = \mathrm{P}^{\mathrm{piezo}}_{c\ell}(\omega) = \mathrm{P}_Z(\omega) \tag{41}$$

express the conservation of energy under the idealized conditions that the dissipation during the propagation of the sea waves as well as the dissipation in the piezoelements are negligible.

## 5. SOLUTION OF THE COUPLED PROBLEM

The dynamical coupling between the piezoelectric and hydrodynamic problem is realized by means of the matching conditions (32) and (33). Combining these two equations with Eq. (26), we obtain the following condition on the fluid-solid interface $\partial\Omega_{c\ell}$:

$$\hat{\Phi}^{f}_{,x}(z;\omega) + j\omega \frac{h}{c^{D}_{33}} \hat{p}(z;\omega) + j\omega \frac{h}{c^{D}_{33}} \frac{\mathscr{E}_t(\omega)}{h_D} \int_{-h_D}^{0} \hat{p}(z;\omega)\, dz = 0. \tag{42}$$

This is a non-local (because of the last term) condition connecting the hydrodynamic fields $\Phi^{f}(x,z;\omega)$ and $p(x,z;\omega)$ at $x=0$.

The modal expansion, given by Eqs. (35) and (36), permits us to express both quantities, $\hat{\Phi}^{f}_{,x}(z;\omega) = \partial\Phi^{f}(x=0,z;\omega)/\partial x$ and $\hat{p}(z;\omega) = p(x=0,z;\omega)$, in terms of the same set of unknowns, the expansion coefficients $W$, $C_n$, $n=1,2,3,...$. This fact will be now exploited in order to formulate an infinite system for these unknown coefficients. Substituting the modal expansions of the functions $\hat{\Phi}^{f}_{,x}(z;\omega)$ and $\hat{p}(z;\omega)$ into Eq. (42), and performing the appropriate algebraic manipulations, we finally obtain

$$\begin{aligned} &\left\{\alpha_0 Z_0(z) + \beta_0 Z_0(z) + \gamma_0 \mathscr{T}_0\right\} W + \sum_{n=1}^{\infty}\left\{\alpha_n Z_n(z) + \beta Z_n(z) + \gamma \mathscr{T}_n\right\} C_n = \\ &\qquad = \alpha_0 Z_0(z) - \beta_0 Z_0(z) - \gamma_0 \mathscr{T}_0, \quad -h_D \le z \le 0. \end{aligned} \tag{43}$$

where $\alpha_0$, $\alpha_n$, $\beta_0$, $\beta$, $\gamma_0$, $\gamma$ are expressed in terms of various already defined hydrodynamic and piezoelectric properties (see Athanassoulis & Mamis, 2012) and

$$\mathscr{T}_n = \int_{-h_D}^{0} Z_n(z)\, dz. \tag{44}$$

Recall now that the vertical eigenfunctions $Z_0(z), Z_n(z)$, $n=1,2,\ldots$, as defined by Eq. (37a,b), constitute an orthogonal system of functions, complete in the Hilbert space $L^2(-h_D,0)$. Accordingly, by projecting both members of Eq. (43) on each one of the basis functions $Z_0(z)$, $Z_n(z)$, $n=1,2,\ldots$, we obtain the following infinite system of equations with respect to the unknown coefficients $W$, $C_n$, $n=1,2,3,...$:

$$\left\{\mathrm{K}^{+}_{00} + \gamma_0 \Lambda_{00}\right\} W + \sum_{n=1}^{\infty} \gamma \Lambda_{n0} C_n = \mathrm{K}^{-}_{00} - \gamma_0 \Lambda_{00}, \tag{45a}$$

$$\gamma_0 \Lambda_{0m} W + \sum_{n=1}^{\infty}\left\{\mathrm{K}_{nn}\delta_{nm} + \gamma \Lambda_{nm}\right\} C_n = -\gamma_0 \Lambda_{0m}, \quad m=1,2,3,..., \tag{45b}$$

where

$$\mathrm{K}^{+}_{00} = (\alpha_0 + \beta_0)\left\|Z_0\right\|^2, \ \mathrm{K}^{-}_{00} = (\alpha_0 - \beta_0)\left\|Z_0\right\|^2, \ \Lambda_{n0} = \Lambda_{0n} = \mathscr{T}_n \mathscr{T}_0, \tag{46}$$

$$\mathrm{K}_{nn} = (\alpha_n + \beta)\left\|Z_n\right\|^2, \ \Lambda_{nm} = \Lambda_{mn} = \mathscr{T}_n \mathscr{T}_m, \tag{47}$$

and $\left\| Z_n \right\|^2 = \int_{-h_D}^{0} Z_n^2(z)\,dz$ is the square of the norm of $Z_n(z)$ in the space $L^2\left(-h_D, 0\right)$.

From Eq. (45a) we obtain:

$$W = -\sum_{n=1}^{\infty} \frac{\gamma \Lambda_{n0}}{\mathrm{K}_{00}^{+} + \gamma_0 \Lambda_{00}} C_n + \frac{\mathrm{K}_{00}^{-} - \gamma_0 \Lambda_{00}}{\mathrm{K}_{00}^{+} + \gamma_0 \Lambda_{00}}. \tag{48}$$

Since the coefficients multiplying $C_n$ in Eq. (48), are about four orders of magnitude smaller than the $C_n -$independent terms, the effect of the $C_n -$dependent terms on $W$ is small. This has been definitely verified by means of detailed numerical calculations. Thus, it is safe to proceed with our analysis by keeping only the second ($C_n -$independent) term in Eq. (48). This approximation is compatible with the long-wave theory for water waves.

Under the (numerically confirmed) simplification that the $C_n$ coefficients do not practically affect the reflection coefficient $W$, this coefficient can be written in the form

$$W = \frac{1 - j\mathscr{H}\dfrac{h}{c_{33}^{D}}\left(1 + \mathscr{Y}\,\mathscr{E}_t(\omega)\right)}{1 + j\mathscr{H}\dfrac{h}{c_{33}^{D}}\left(1 + \mathscr{Y}\,\mathscr{E}_t(\omega)\right)}, \tag{49}$$

where $\quad \mathscr{H} \equiv \rho_f\, g\left(\mu_0 / k_0\right), \qquad \mathscr{Y} \equiv \left(\mathscr{T}_0 \mathscr{T}_0\right) / \left(h_D \left\| Z_0 \right\|^2\right)$ (50a,b)

are two purely hydrodynamic, real-valued (positive), quantities.

As is seen from Eq. (49), the reflection coefficient $W$ is dependent on the following two (dimensionless) coefficients

$$\varpi \equiv \mathscr{H}\frac{h}{c_{33}^{D}} > 0 \qquad \text{and} \qquad \lambda \equiv \mathscr{Y}\,\mathscr{E}_t(\omega) \in \mathbb{C}, \tag{51a,b}$$

which realize the energetic coupling between the three subsystems (hydrodynamic, piezoelectric and external circuit). We shall call these two coefficients hydro/piezo/electric compliances. From the definition of quantity $\mathscr{E}_t(\omega)$, Eq. (22), compliance $\lambda$ can be written as

$$\lambda \equiv \mathscr{Y}\,\mathscr{E}_t(\omega) = \sigma\frac{\Pi}{\Pi^2 + \chi^2} - j\,\sigma\frac{\chi}{\Pi^2 + \chi^2} = \lambda_R(\chi) + j\lambda_J(\chi), \tag{52}$$

where $\Pi \equiv 1 - k_t^2 - \omega\left(C_0 / M_1\right) X(\omega)$, $\chi \equiv \omega\left(C_0 / M_1\right) R > 0$ and $\sigma \equiv \mathscr{Y}\, k_t^2$.

Using the notation introduced above, we can write $\left| W \right|^2$ in the form

$$|W|^2(\chi) = \frac{1+2\varpi\lambda_J(\chi)+\varpi^2[1+\lambda_R(\chi)]^2+\varpi^2\lambda_J^2(\chi)}{1-2\varpi\lambda_J(\chi)+\varpi^2[1+\lambda_R(\chi)]^2+\varpi^2\lambda_J^2(\chi)} \equiv \frac{F(\varpi,\lambda_R(\chi),\lambda_J(\chi))}{G(\varpi,\lambda_R(\chi),\lambda_J(\chi))}. \tag{53}$$

## 6. OPTIMIZATION AND EFFICIENCY OF THE HYDRO/PIEZO/ELECTRIC HARVESTER

Combining Eqs. (40), (41) with Eq. (53), we readily see that the ratio of the total power taken off the impinging waves over the incident wave power, that is, the efficiency of the hydro/piezo/electric harvester described in Sec. 2, can be expressed as

$$\frac{P^f_{a=0}(\omega,\chi)}{P^f_I(\omega)} = 1-|W(\chi)|^2 \tag{54}$$

where $P^f_I(\omega) = \frac{1}{2}\rho_f g\left(\frac{H}{2}\right)^2 L_2\omega\frac{k_0}{\mu_0}\|Z_0\|^2$ is the incident wave power. Thus, it is clear that the coupling phenomenon between the hydrodynamic wave field, the piezoelectrically vibrating elements and the external electric circuit is solely modeled by $1-|W(\chi)|^2$. Since in the variable $\chi = \omega(C_0/M_1)R$, the easily adjustable ohmic resistance $R$ of the external circuit is involved, it is expedient to maximize $1-|W(\chi)|^2$ (equivalently, the taken-off power) with regard to $\chi$, following the common practice in piezoelectric harvesters (Guygomar *et al.*, 2005, Lefeuvre *et al.*, 2010). Using the first derivative test, we have to solve the equation

$$d\left[1-|W|^2(\chi)\right]/d\chi = 0 \quad \Longleftrightarrow \quad \left\{\frac{dF}{d\chi}G - F\frac{dG}{d\chi}\right\} = 0. \tag{55}$$

After some algebraic manipulations, we find that Eq. (55) reduces to

$$\left(\chi^2+\Pi^2\right)^2\left(\chi^2-\Pi^2\left(1+\mu^2\right)\right) = 0, \tag{56}$$

where $$\mu^2 \equiv \mu^2\left(\frac{\sigma}{\Pi},\varpi\right) = = \frac{\sigma}{\Pi}\left(\frac{\sigma}{\Pi}+2\right)\frac{\varpi^2}{1+\varpi^2}. \tag{57}$$

That is, Eq. (55) has the double negative root $\chi^2_{1,2} = -\Pi^2$, which is of no importance for our purposes, and the positive root $\chi_3^2 = = \Pi^2\left(1+\mu^2\right)$, where

$$\frac{\sigma}{\Pi} = \frac{\mathscr{Y}k_t^2}{1-k_t^2-\omega\left(C_0/M_1\right)X(\omega)} \approx \frac{\mathscr{Y}k_t^2}{1-k_t^2}. \tag{58}$$

[The second (simplified) form of $\sigma/\Pi$ is valid since the reactance $X(\omega)$ is expected to be much smaller than $\left(1-k_t^2\right)/\omega\left(C_0/M_1\right) \sim O\left(10^{12}\,\Omega\right)$.]

Thus, the value $\chi = \chi_{\text{opt}} = \omega \left( C_0 / M_1 \right) R_{\text{opt}} > 0$ which maximizes the taken-off power is given by the formula $\chi_{\text{opt}} = \Pi \sqrt{1+\mu^2}$, which leads to the following optimal external ohmic resistance value $R_{\text{opt}}$

$$R_{\text{opt}} = \left( \frac{1-k_t^2}{\omega \left( C_0 / M_1 \right)} - X(\omega) \right) \sqrt{1+\mu^2} \approx \frac{1-k_t^2}{\omega \left( C_0 / M_1 \right)} \sqrt{1+\mu^2} \,. \tag{59}$$

Introducing $\chi_{\text{opt}}$ in Eq. (53), the following form for the electrically optimized efficiency is obtained

$$1 - \left| W \right|_{\text{opt}}^2 \equiv \mathscr{W}\left( \frac{\sigma}{\Pi}, \varpi \right) =$$

$$= \frac{4\,\varpi \,\frac{\sigma}{\Pi} \sqrt{1+\mu^2\left(\frac{\sigma}{\Pi},\varpi\right)} \Big/ \left( 2+\mu^2\left(\frac{\sigma}{\Pi},\varpi\right) \right)}{1 + 2\,\varpi \,\frac{\sigma}{\Pi} \frac{\sqrt{1+\mu^2\left(\frac{\sigma}{\Pi},\varpi\right)}}{2+\mu^2\left(\frac{\sigma}{\Pi},\varpi\right)} + \frac{\varpi^2}{2+\mu^2\left(\frac{\sigma}{\Pi},\varpi\right)} \left[ 2+\mu^2\left(\frac{\sigma}{\Pi},\varpi\right) + 2\,\frac{\sigma}{\Pi} + \left(\frac{\sigma}{\Pi}\right)^2 \right]} \,. \tag{60}$$

It should be stressed that the optimum value $1 - \left| W \right|_{\text{opt}}^2$ is dependent only on the two dimensionless, positive-valued quantities $\varpi$ and $\sigma / \Pi$, which appropriately combine the hydrodynamic, the piezoelectric and the circuit characteristics affecting the energetic coupling of the system. Furthermore, taking into account the definitions of $\sigma$ and $\Pi$, and the facts that $\mathscr{Y} \in (0,1]$ and (for many interesting materials) $k_t^2 \in (0.01, 0.5)$, we easily find that $\sigma / \Pi$ ranges (for all realistic situations) from 0 to (approximately) 1.0.

The quantity $1 - \left| W \right|_{\text{opt}}^2$ as a function of the two arguments $\varpi$ and $\sigma / \Pi$ is shown in Fig. 5. By observing this figure, it is seen that, for every value of $\sigma / \Pi$, the efficiency of the system is maximized for values of $\varpi \sim O(10^0)$. However, the system absorbs appreciable energy in the range $O(10^{-1}) < \varpi < O(10^1)$. The dependence of the efficiency $1 - \left| W \right|_{\text{opt}}^2$ on $\sigma / \Pi$ is monotonically increasing; the higher the value $\sigma / \Pi$ the better the efficiency is. Since $\varpi \equiv \mathscr{H}\, h / c_{33}^D$ and $\mathscr{H} \sim O\left(10^4 \ \text{Pa/m}\right)$, it is concluded that the piezoelectric material needed for an efficient harvester would be characterized by $h / c_{33}^D \sim O\left(10^{-4} \ \text{m/Pa}\right)$, having also $k_t^2$ as higher as possible in order that the parameter $\sigma / \Pi$ has a relatively high value.

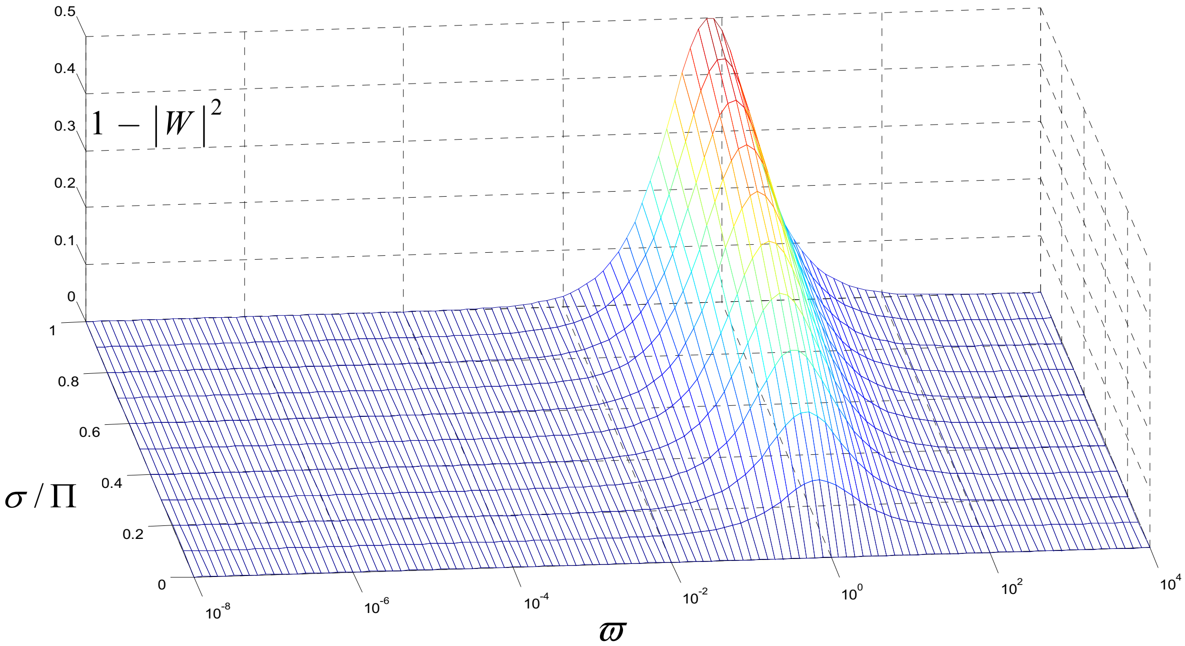


Fig 5. The efficiency $1-|W|^2_{\text{opt}}$ of the hydro/piezo/electric harvester, as a function of the two dimensionless quantities $\varpi$ and $\sigma/\Pi$.

To get a first idea concerning the feasibility of the above requirements in relation with existing materials, we have compiled Table 1, showing the corresponding properties of some common piezoelectric materials. From Table 1 we see that the listed materials do not meet the flexibility requirement for an efficient harvester. It is seen that an improvement of the flexibility coefficient $h/c^D_{33}$ by (approximately) six orders of magnitude is necessary in order that the piezoelectric sheet absorbs enough energy from the impinging waves. Thus, the following two **open questions** arise:

- Is it possible to improve systems' configuration in order to improve the efficiency of the harvester, relaxing the flexibility requirement?

Table 1. Piezoelectric properties of some common materials, assuming $h = 0.1\text{m}$.

| | PZT ceramics | PVDF polymers | 1-3 ceramic(PZT)-polymer composites | Cellular Polypropylenes |
|---|---|---|---|---|
| $h/c^D_{33}$ (m/Pa) | $10^{-13}-10^{-12}$ i) | (1 – 5) $\times 10^{-11}$ ii) | $10^{-12}-10^{-11}$ vi) | $O(5\cdot10^{-8})$ ii) |
| $k_t^2$ | 0.22 – 0.40 i) | 0.012 – 0.023 iv), v) | 0.25 – 0.42 vi) | $O(3.6\cdot10^{-3})$ iv) |

i) Sherman & Butler (2007), Appendix A.5, ii) Bauer & Bauer (2008), Table 6.1, iv) Döring *et al.* (2008), Table 2, v) Splitt (1996), Table 1 vi) Smith & Auld (1991), Figs. 3 and 4.

- Can we expect that new, more flexible piezoelectric materials (e.g., piezo-composites), will be manufactured in the near future ?

Clearly, the latter one should be considered further in collaboration with material scientists.

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