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Fabric of Granular Materials at the Critical State

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ABSTRACT

A series of numerical tests on idealized two-dimensional granular assemblages having different initial fabrics but identical density is modeled by the discrete element method. The specimen is confined by two-pairs of rigid walls which simulates a biaxial compression. The aims of the investigation are twofold: (1) to study the effect of fabric of an assemblage on its global stress-strain response; (2) to address the fabric of an assemblage at its critical state. The geometrical arrangement of the particles and associated void spaces as well as the distribution of normal contact force is examined to shed light on the evolution of the assemblage at the particulate level. At the end of the loading, particles and void spaces are found aligning with their major axis perpendicular and parallel, respectively, to the loading direction. When the shear strain of the granular system becomes significantly large, the specimens appear to achieve a unique stress-volume-fabric state regardless of the initial fabric.

1. INTRODUCTION

The concept of critical state has an utmost importance to soil mechanics. It is defined as a unique state where the particulate materials deform continuously at constant stress and volume when subject to shear loading. Despite its phenomenological nature, a large number of constitutive models have been developed since the pioneer works by Roscoe and Schofield (1963) and Roscoe and Burland (1968) under such framework – the existence of a unique critical state. On the one hand, these models successfully capture the key mechanical behaviors of many geomaterials subject to simple loading condition. On the other hand, while more experimental evidence became available (Vaid et al. 1990; Negussey and Islam 1994; Mooney et al. 1998; Finno and Rechenmacher 2003; etc), the uniqueness of critical state has been greatly challenged. Material anisotropy has been referred to the main reason for the observation of non-unique critical state. Li and Dafalias (2002) proposed a constitutive model for sands

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from the perspective of fabric dependent critical state lines in the *e*-*p* space, where *e* is the void ratio and *p* is the mean effective stress. Their model successfully simulates the effect of principal stress direction and intermediate principal stress on the sand behavior. However, as also pointed out in their paper, the evolution of fabric and stress-induced anisotropy has not been included in their model formulation. More recently, fabric at the critical state has been revisited from a more fundamental approach – thermodynamics perspective (Li and Dafalias 2012). Based on Gibbs stability requirement, the uniqueness of a critical state line has been proved. Moreover, there exists a unique fabric at the critical state and such fabric depends on the loading direction.

This study explores the evolution of the fabric of a two-dimensional granular assemblage subject to numerical biaxial shearing via the discrete element method. Particular attention is paid on the assemblage fabric after a prolonged period of shearing with fixed loading direction. It aims to address the uniqueness of fabric at the critical state from a particulate perspective.

2. NUMERICAL BIAXIAL TEST

Two dimensional, non-crushable, elongated rigid particles with length-to-width (aspect) ratio of two are generated with the built-in clump function in PFC2D (Itasca 2008). The linear contact model between particles is adopted for simplicity. The elongated particles are then rained into a model container where an assemblage with preferential particle orientations – inherent anisotropy, should be obtained. Isotropic stress is applied at the boundary of the assemblage and equilibrium is reached. Square specimens making different angles with the particle deposition direction are extracted from the container. Each specimen, bounded by two pairs of rigid walls, contains approximately 3,000 particles (i.e., 9,000 elementary discs). To mimic a "drained" biaxial compression test, the top and bottom boundary walls are moved simultaneously inwards while the horizontal positions of the left and right boundary walls are continuously adjusted by a servo-controlled mechanism which maintains the lateral confining stress at its initial confining value. The wall velocity is essentially slow such that a quasi-static equilibrium is always maintained throughout the analysis. Attempt has been made to shear the specimen to 80% axial (Hencky) strain. $\beta = 0°$ corresponds to a specimen with majority of particles aligning parallel to the major principal stress plane prior to shear, while $\beta = 90°$ represents a specimen with particles mainly align perpendicular to the major principal stress plane. As a result, multiple specimens with essentially the same density but different fabrics are obtained. A specimen is labeled by its initial confining stress and the angle β. For instance, I100D60 denotes a specimen formed by $\beta = 60°$ with an isotropic confining stress of 100 kPa prior to shearing. Fig. 1 summarizes the preparation of the assemblage and the testing of the specimen. Details should be referred to Yan and Zhang (2012).

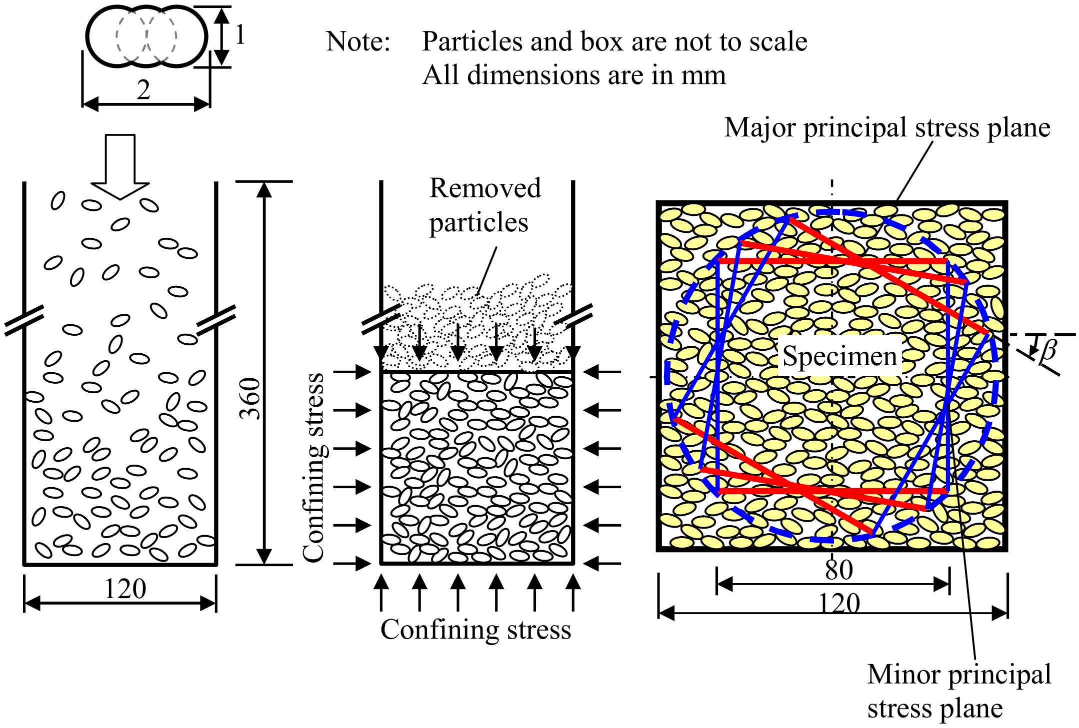

Fig. 1 Preparation of the numerical specimens.

3. FABRIC

Following the discussion of Oda et al. (1985), three sources of fabric are considered in herein: (1) particle orientation, (2) contact normals, (3) void distribution. Fig. 2 denotes the set of vectorial parameters that are used to describe the fabric of an assemblage. In Fig. 2(a), $\mathbf{p}_{\mathrm{A}}$ and $-\mathbf{p}_{\mathrm{A}}$ are a pair of vectors describing both the length and orientation of particle A with respect to the reference coordinate system. $\mathbf{f}_{\mathrm{AB}}^{\mathrm{n}}$ and $\mathbf{f}_{\mathrm{AB}}^{\mathrm{s}}$ denote respectively the normal and shear components of the contact force between particle A and B. Fig. 2(b) shows the void cell system as proposed by Li and Li (2009). $\mathbf{v}(\mathbf{n})$ describes a vector from the centroid of a void space to the inter-particle contacts of the void space. Fabric of an assemblage is described by statistical collection of the tensorial components in a rose diagram (see also the discussion in Oda et al. 1985 and Yan and Zhang 2012). Rothenburg (1981) proposed the use of two scalar quantities with clear physical meaning, a and θ_a, to approximate the distribution density represented in the rose diagram (see Fig. 3).

$$f(\mathbf{n}) \simeq \frac{1}{2\pi}\left[1 + a\cos 2(\theta - \theta_a)\right] \tag{1}$$

where $\oint f(\mathbf{n})\mathrm{d}\mathbf{n} = 1$, a is a positive number called the coefficient of anisotropy that describes the degree of anisotropy, $0° \le \theta_a < 180°$ denotes the preferred direction of such anisotropy (in this study, $0°$ is the plane parallel to the major principal stress plane). Clearly the average degree of anisotropy increases as a increases. More details discussion of the formulation of the fabric descriptors should be referred to Yan and Zhang (2012).

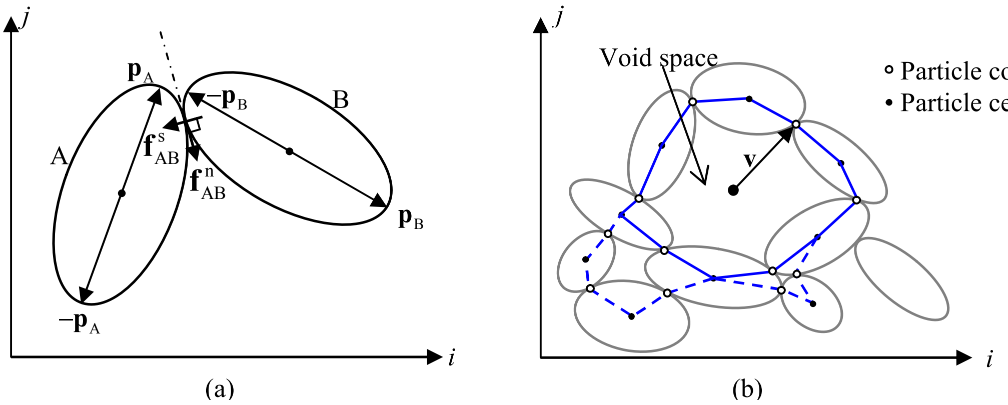

Fig. 2 Definition of the vectorial parameters.

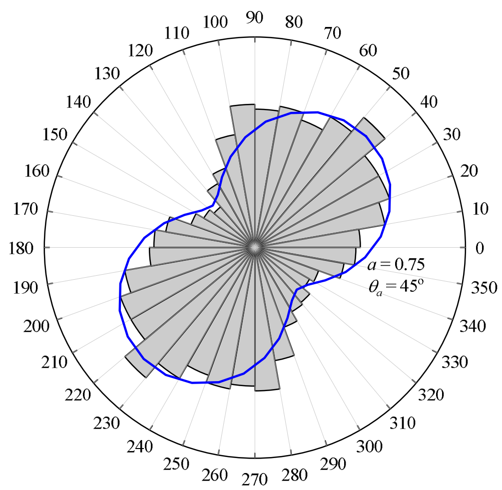

Fig. 3 A typical rose diagram with the distribution density represented by a and θ_a.

4. MACROSCOPIC STRSS-STRAIN RESPONSES

The stress state of a specimen is represented by two stress variables, $s = (\sigma_1 + \sigma_3)/2$ and $t = (\sigma_1 - \sigma_3)/2$ where σ_1 and σ_3 is the major and minor principal stress, respectively. The influence of fabric (specimens with different β) on the stress-strain response is clearly shown in Fig. 4(a) and (b). Key observations include: (1) assemblage stiffness (beyond 1% strain) decreases with increasing β; (2) dilatancy reduces with increasing β; (3) essentially the same stress ratio is mobilized at a very large strain; (4) the dilatancy only becomes negligibly small at 60% strain and beyond; and (5) essentially the same overall volumetric strain is induced at a very large strain. In other words, the specimens shear at a constant volume at the same stress state when the strain level becomes high, which means the phenomenological critical state is achieved. Since the specimens have identical void ratio prior to shear, the ultimate void ratio at the end of the shearing is also the same. It implies that a unique stress-volume relation at the critical state is reached.

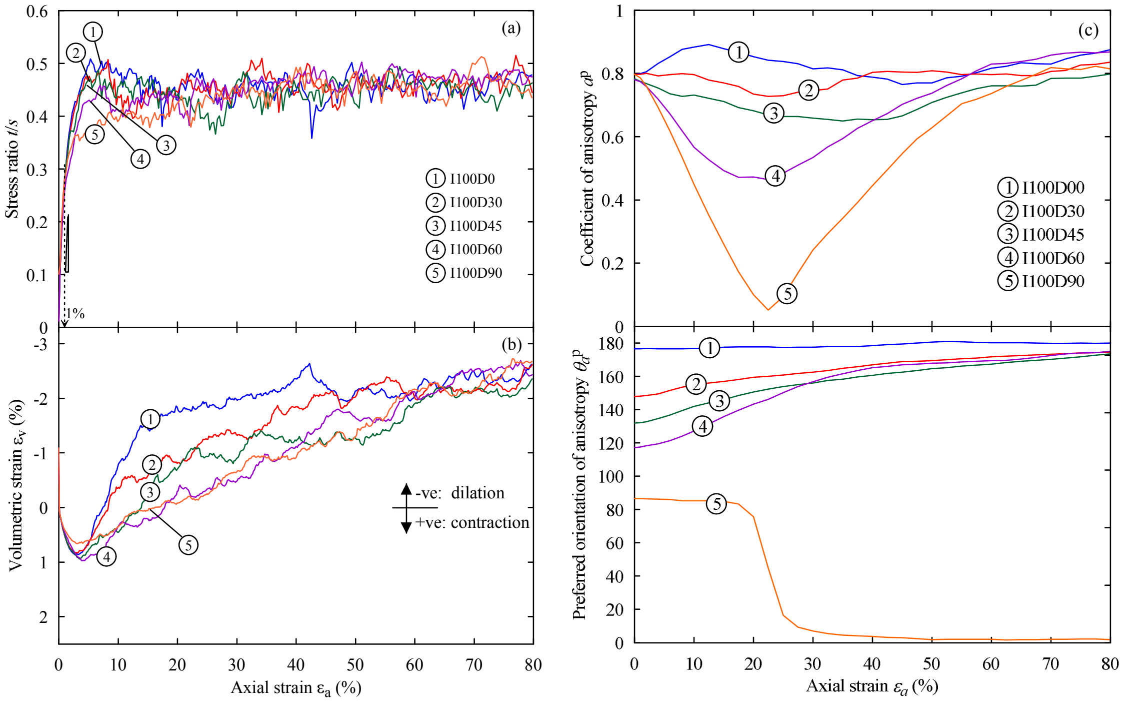

Fig. 4 The influence of initial fabric on the behavior of the specimens: (a) stress-strain response; (b) dilatancy; (c) evolution of particle arrangement

5. EVOLUTION OF FABRIC

In this section, the evolution of the fabric (including particle geometric arrangement, distribution of normal contact forces, and distribution of void shape) is presented. Fig. 4(c) shows the distribution of particle arrangement evolving with shear. It is found that except I100D60 and I100D90, specimens exhibit a small change in the degree of anisotropy (varies between 0.65 and 0.90) during the biaxial shear. Yet, I100D90 shows an abrupt decrease in a^{p} (below about 22% strain) and then a substantial regain (beyond 22% strain), and a noticeable change in θ_a^{p} at around 20% strain. It suggests that a significant re-orientation of the particles occurs in this specimen. The particles align from mainly parallel to the loading direction ($\theta_a^{\mathrm{p}} = 90°$) before shear to mainly perpendicular to it ($\theta_a^{\mathrm{p}} = 0°$) at large strain. Other specimens show a gradual change in the preferential particle orientation during shear. More importantly, the results show a unique fabric (anisotropy of particle arrangement) at large strain. At the end of the shearing, the particles orientate perpendicular to the loading direction (i.e., $\theta_a^{\mathrm{p}} = 0$ or $180°$). Furthermore, plotting the fabric of the normal contact force also demonstrates a unique force fabric at large strain. The plot is not shown here for brevity. The contact force chains align parallel to the loading direction, as expected.

The evolution of the average void shape of the assemblage, as described by the void cell system proposed by Li and Li (2009), is shown in Fig. 5. As the specimens are prepared by rotating the same assemblage, the initial coefficient of anisotropy for the void space a^{v} is the same for different specimens. Yet, their initial θ_a^{v} are different as they should be in accordance with β (i.e., $\theta_a^{\mathrm{v}} \approx 180° - \beta$). The figure shows that though the degree of anisotropy of the void space generally decreases with shearing, a noticeable anisotropy is still observed at large strain. The void aligns with its long axis parallel to the loading direction ($\theta_a^{\mathrm{v}} \approx 90°$). It is important to note that such findings echoes the proposal of Li and Dafalias (2012) that the void fabric evolves with its major direction aligning towards that of the loading.

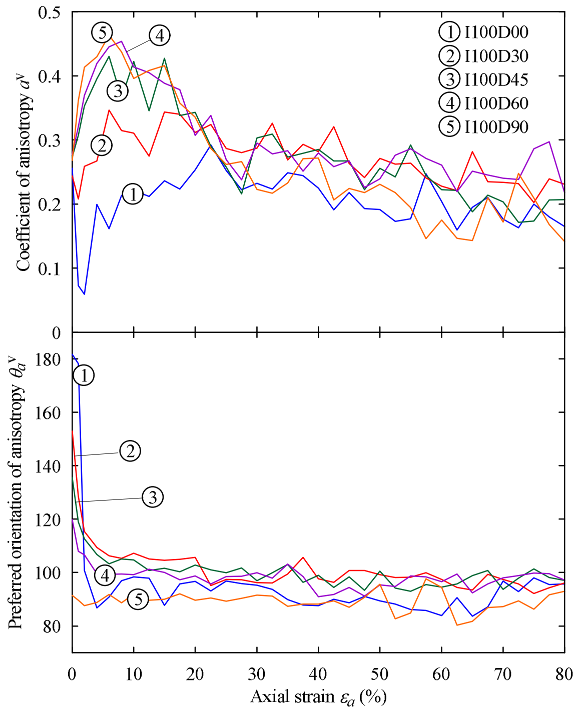

Fig. 5 Evolution of the void fabric during the biaxial shearing

The above results are also found on specimens under different initial confining stresses. In summary, a unique critical state can be achieved at very large strain. At such state, a unique stress-volume-fabric relation can be observed.

CONCLUSIONS

A series of numerical biaxial shear tests is performed with the aid of the discrete element method. Specimens prepared by idealized two-dimensional elongated particles, having different initial fabric are sheared to very large strain. The evolution of fabric, including particle and void orientation, and normal contact force, is explored with respect to the statistical representation of the directional vectorial quantities. The results show that a unique fabric is finally reached regardless of their initial fabric. The particles and void space align with their major axis perpendicular and parallel to the loading direction, respectively. A unique stress-volume-fabric state appears to occur at the critical state.

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