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Modelling of the transport and dispersion of oil spill in the sea by particles

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ABSTRACT

The hydrodynamic Navier-Stokes equations are solved by a finite-difference scheme to describe the behaviour of seawater bodies under the influence of wind stress, atmospheric pressure gradients and tidal forces. This numerical model outputs the three-dimensional field of the coastal currents. Here the scheme is applied to describe the motion of an oil spot in the sea around Giglio Island situated in the Tyrrhenian Sea (Italy). In order to account for the surface tension of the oil spot an additional friction term proportional to the velocity and to the number of particles for area unity is added, thus simulating the presence of an attractive interaction between particles with low computational cost. This term counteracts the continuous spreading of particles in presence of an inhomogeneous current field in the coastal area. The magnitude of the additional term can be set to match the actual magnitude of the surface tension of oil spot, a factor not taken into consideration in the most widely used simulation of oil spills. In conclusion the present model provides as output the time evolution of the spreading of the oil spill including the possible fragmentation of the plot.

1. INTRODUCTION

The Department of Physics of the University of Genoa operates on environmental subjects in order to favour and promote projects and initiatives involving numerical models and experimental measurements of sea water circulation and coastal pollution. A three-dimensional model is able to determine ocean current fields in a generic basin with changing wind, atmospheric pressure and tidal forces (Bazzurro 2002). One of the problems in formulating the "3D" model is to specify the shearing stresses in terms of velocity components. In order to calculate the vertical profile of velocity from the equations of motion, it is necessary to relate the shearing stress components to the velocity components. In specifying the stress of the wind, we assumed it to act in the direction of the wind relative to the sea surface and its magnitude to be proportional to the square of the wind speed relative to the sea surface. Thus $\tau_s = C_D \rho_a W^2$ where W is the wind speed measured at 10 m above the sea surface, ρ_a is the density of the air and C_D is a standard drag coefficient (Gill 1982). An eddy viscosity term has been used, allowing the eddy coefficient N_z to vary with depth following the modelling technique described by Ramming and Kowalik (Ramming 1980). A non-linear model has been developed to study the interaction between the forces responsible of sea level oscillations and the meteorological forcing (Papa 1980). Right-handed rectangular coordinates have been taken with the x and y axes horizontal and the z axis vertically downward. A finite-differences scheme has been used to solve the equations of motion in the x, y, and z directions with Coriolis terms and the equation of continuity of volume (Gill 1982) using a numerical technique which is an extension of the two-dimensional method of Courant, Isaacson and Rees (Courant 1952) to solve the quasi-linear hyperbolic systems of first-order partial differential equations. The vertical and horizontal discretization of the model is very accurate in describing both the coastal boundaries, especially along irregular coasts, and the sea bottom topography. In fact the surface elevations z and the components (u,v,w) of the current field are computed at each node of the mesh. It is also possible to compute the surface Stokes drift current produced by the sea waves: $U_d = 3.2\ H_s^2/T_m^3$, where H_s (in m) is the significant wave height and T_m (in s) is the mean period of the waves. This term was added to the wind model to simulate ocean currents.

In our calculations a grid of square meshes with size of 250 m for a limited area of about 14 Km^2 of the Tyrrhenian Sea around the Giglio island was employed to describe the bottom and the lateral boundaries of the model. The time step was set to 1 s to satisfy the Courant –Friedrichs –Lewy criterion of numerical stability (Courant 1952).

2. METHOD

In order to model the motion of an oil spot we model it here as a set of particles. As long as the motion of the spot is merely determined by the velocity field, i.e. as long as no friction force or no interaction between different parts of the spot is present, the mass is not needed to solve the equation of motion. To this purpose starting from the "3D" numerical model we used here a code able to calculate numerically the trajectories of a set of particles released in the sea in a given surface two-dimensional field (u,v) of velocities. The particles leave from a randomly generated initial position within a pre-defined size. It is also possible to define multiple sets of particles to be released at different times to simulate a continuous source, as required in a realistic model of oil spilling (Reed 1999, Elliott 2000, Guo 2009, Korotenko 2010, Coppini 2011).

The subsequent motion is determined by numerically integrating the trajectories. The value of the velocity at an arbitrary point is obtained by bilinear interpolation from the values at the four nodes of the square including the point. The coastal morphology and depths are given as input to the code, which can thus be easily used in different seas provided that the appropriate matrices identifying the coastline and the bottom topograhy are known. Their trajectories are determined by numerically integrating the equation of motion with the following scheme (Eq. (1)): (Danish Hydraulic Institute 1991, Papa 2010, Papa 2011):

$$X(t+\Delta t) = x(t) + u\,\Delta t + \Delta L\, u/|V| - \Delta T\, v/|V| + \Delta D_x + \varepsilon\, w_x \Delta t$$

$$Y(t+\Delta t) = y(t) + v\,\Delta t + \Delta L\, v/|V| + \Delta T\, u/|V| + \Delta D_y + \varepsilon\, w_y\, \Delta t \qquad (1)$$

with (u,v) the components of velocity of modulus V. The terms u Δt and v Δt are the deterministic changes of the position due to the action of the velocity field. ΔL and ΔT are stochastic terms accounting for longitudinal and transverse active dispersion respectively. Their amplitude, whose maximal value is fixed by setting the parameters α_L and α_T , depends on the local velocity and thus it changes with position according to:

$$\Delta L = 2\,(6\,\alpha_L\, v\, \Delta t\,)^{1/2}\ (ran(n_L) - 0.5)$$
$$\Delta T = 2\,(6\,\alpha_T\, v\, \Delta t\,)^{1/2}\ (ran(n_T) - 0.5) \qquad (2)$$

where $ran(n_L)$ and $ran(n_T)$ are independent random numbers in the interval [0,1].

The effect of longitudinal active dispersion is to produce at each time step a further change in position in the direction of the velocity, while transverse dispersion produces a further change in position in direction normal to the velocity.

ΔD_x and ΔD_y are also stochastic terms but they are independent of the local velocity and take neutral dispersion into account (Danish Hydraulic Institute 1991).

$$\Delta D_x = 2\,(6\, d_0\ \Delta t)^{1/2}\ (ran(n_x) - 0.5)$$
$$\Delta D_y = 2\,(6\, d_0\ \Delta t)^{1/2}\ (ran(n_y) - 0.5) \qquad (3)$$

here $ran(n_x)$ and $ran(n_y)$ are independent random numbers in the interval [0,1].

Their magnitude is determined by d_0 which describes for pure diffusion; as apparent from the above relationships α_L and α_T have the dimension of length while ([L]) d_0 has the dimension of squared length divided by time ($[L^2T^{-1}]$). With an appropriate choice of the parameters α_L, α_T and d_0 it is thus possible to determine the motion under conditions when both deterministic (velocity field) and random terms (neutral and/or active dispersion) are present.

In order to consider the additional effect of wind, a further term is added. If w_x, and w_y are the components of the wind field, it induces an additional drift of the form $\varepsilon\,(w_x, w_y)\,\Delta t$, where $\varepsilon \sim 0.03$, according to literature.

In absence of friction and of interaction between particles, the deterministic part of the trajectory is entirely determined by the velocity field and by the wind field.The addition of a random term, both diffusive and advective, as done in (Papa 2010, Papa 2011), may mimic effectively the presence of significant wave motion, influencing significantly the actual trajectory and thus the spreading of the spot which is being modelled.

This is indeed the common procedure followed in modelling the motion of an oil spot on the sea.

A qualitative step forward is possible only by adding further terms into the equation of motion. The need for this arises from the fact that surface tension causes a force opposing to the split of a spot into smaller ones. In order to model, at least qualitatively, such an effect we need to add either a direct interaction between different parts of the spot or some “mean field” term.

The former choice would have a more immediate physical meaning but is more demanding from the computational point of view; the latter choice is on the contrary less time consuming, although at the price of a less straightforward physical meaning. In both cases we have to face however the problem of defining the “particle” which experiences the forces and to assign it a proper mass to proceed with the solution of the equation of motion.

The simplest approach is to define a particle as a fraction of the whole spot having an area A_{part} and to assign it a mass $m = \rho A_{part} \Delta z$, where ρ is the density of oil (~ 881 kg /m^3) and Δz is the thickness of the layer (typically ~ 100 μm).

A_{part} is somewhat arbitrary in the sense that the same amount of oil can be described by a certain number of particles having an area A_{part} each or by a double number of particles with halved area.

In general A_{part} should be such that the velocity field can be considered constant over this area for any position of the spot in the velocity field. In our simulations we assumed A_{part} = 100 m^2 so that the mass m of such an equivalent particle is 8,81 Kg. The velocity field should not change appreciably on a length scale $L \sim (A_{part})^{½} \sim 10$ m. More formally the condition (Eq. (4)):

$$\nabla.\mathbf{v}\ L/v \ll 1 \qquad (4)$$

should be satisfied at any place in the velocity field.

A friction term is then added to the equation of motion providing a force of modulus:

$$F = -\beta\ (N(i,j)/N_{max})\ v\ A_{part} \qquad (5)$$

$N_{max} = \Delta x \Delta y / A_{part}$ being the maximum number of particles which can be found in a grid of area Δx*Δy and v the velocity at the position of the particle, while N(i,j) is the number of particles in cell (i,j). The acceleration of such a particle reads then:

$$a = F/m = -\ \beta\ (N(i,j)/N_{max})\ v\ A_{part} / (\rho\ {*}\Delta z\ A_{part}) = -\ w\ v\ N(i,j)/N_{max} \qquad (6)$$

with $w = \beta/(\rho\ {*}\Delta z)$. The motion of the particle is thus decelerated proportionally to the local speed and to the relative density of particles with respect to the maximal one. When $N(i,j) \sim N_{max}$ the deceleration is maximal and the model describes an area entirely covered with oil. In this limit β is the force for unity of area slowing down an oil spot when the speed of the current is 1 m/s. This definition is independent of the choice of A_{part} : if indeed A_{part} is doubled, N_{max} is halved. In practice the choice of A_{part} (and consequently of N_{max}) determines the “granularity” of the model, i.e. the ultimate spatial resolution of the model.

The integration of the equation of the motion requires moreover an appropriate choice of the integration time step Δt, since the change in position due to the friction term reads ½ a(Δt)^2.This is not trivial: if Δt is too small, the computational effort is abnormally increased without any real gain in the final accuracy while if Δt is not small

enough the integration of the trajectory comes out to be inaccurate and unreliable. To asses this point we set the condition that the change in position due to friction alone cannot exceed ½ the change in position due to the velocity field alone, i.e.
$\frac{1}{2} a (\Delta t)^2 < \frac{1}{2} v\Delta t$ leading finally to: $\Delta t < 1/w$.

3. RESULTS

We applied our model to a limited area around Giglio Island, the choice being motivated by the accident recently occurred to the ship Concordia on January 13th, 2012 with the possible release of oil and other materials into the sea. Fortunately the former scenario has been avoided by removing oil form the tanks of the ship, however the model can be applied in general to any regional area potentially involved in spilling of oil as well as of any liquid having density lower than the one of water. The model requires knowledge of the map contour of the coast, the bottom topography and the current field at the sea surface.

In the present case the velocity field (u,v) has been obtained as described in the Introduction. The velocity field in presence of wind blowing from the North West has been computed at the points of a square grid for cells having size $\Delta x=\Delta y=250$ m for an area of about 14 Km^2 around the Giglio island.

Fig. 1 shows the snapshots of a square spot of 1 Km side, immediately after a release (t=0) close to the north-eastern coast of the Giglio Island and after 3 hours, for different values of β and α, where α denotes the set (α_L,α_T). $\beta=0$ indicates the absence of any friction force while $\alpha=0$ indicates the absence of any random (both diffusive and advective) motion.

It is apparent that:

a) without diffusion and advection, i.e. for pure deterministic motion, the shape of the spots after 3 hours are similar, except for a slight delay for a slight delay in presence of quite relevant friction ($\beta=8$)
b) In presence of diffusion and advection, i.e. with significant random part as required to model the motion in presence of significant wave motion, the evolution gives rise to a more irregular shape with significant spreading of the particles.

The chosen values of α and β correspond to quite relevant random and friction effects: $\alpha=2.5$ corresponds indeed to extremely large wave motion, living rise to a change in position of the order of tens of meters for second; $\beta=8$ requires a time step of the order of 0.07 s and in that time interval the deceleration is such that the motion of the particle can be delayed up to 0.12 m for each second, giving a visible effect already after 3 hours.

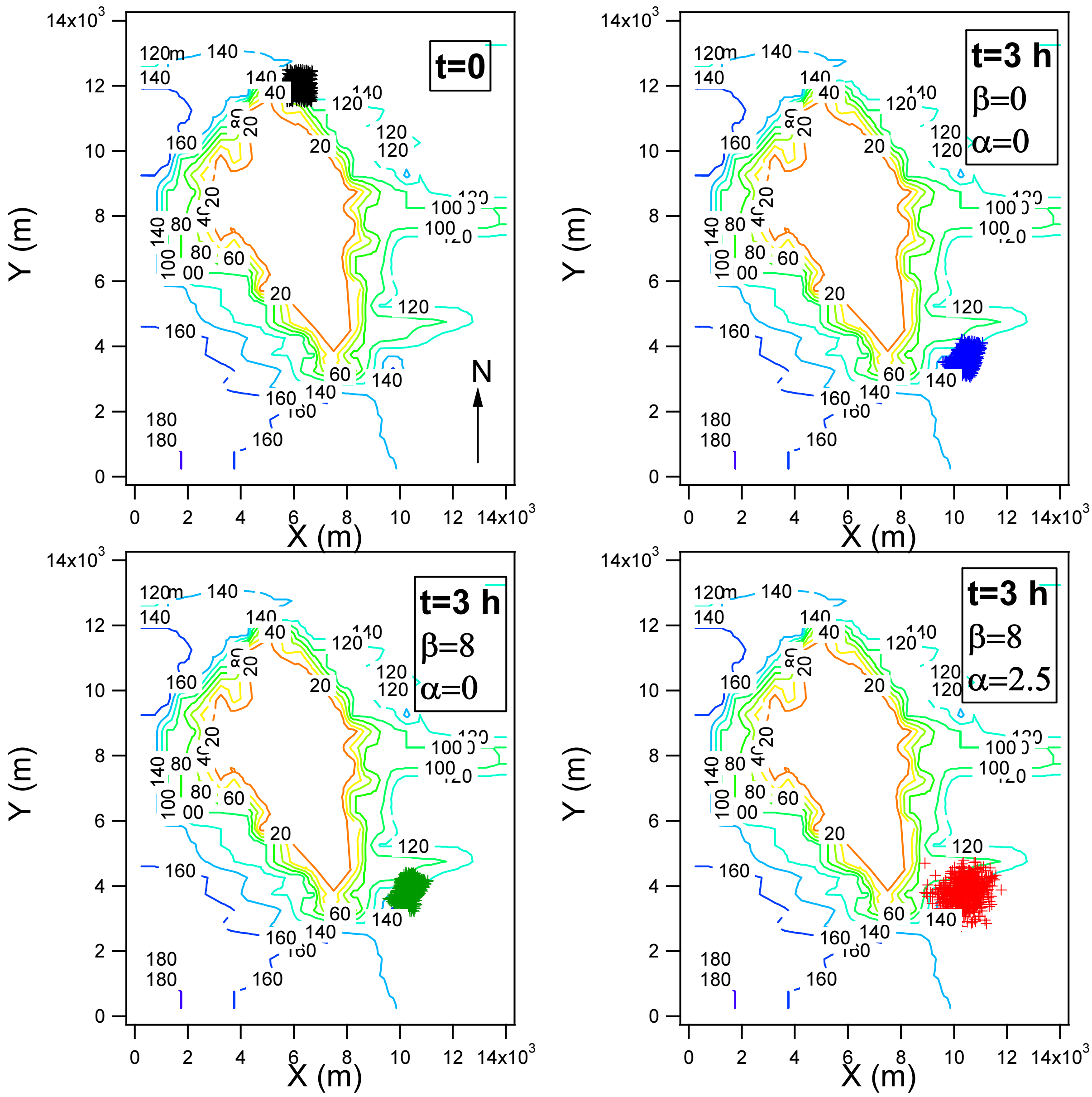

Fig. 1. Snapshots of initial (t=0) and final positions (t=3 h) of a set of particles having initially 1 Km size, for different values of α and β. The lines indicate constant depth values of the sea around Giglio Island. The wind blows from the North West with constant speed of 2.8 m/s.

Point a) indicates that the presence of friction is relevant for precise timing of the arrival of an oil spot at a desired position but does not affect significantly the final shape, at least in presence of a constant current field.

Point b) indicates that wave motion is a key issue to predict the actual evolution of an oil spot: although our choice of α corresponds to an extremely high wave motion, this result indicates that even much smaller wave motion can produce measurable effects on a longer time scale.

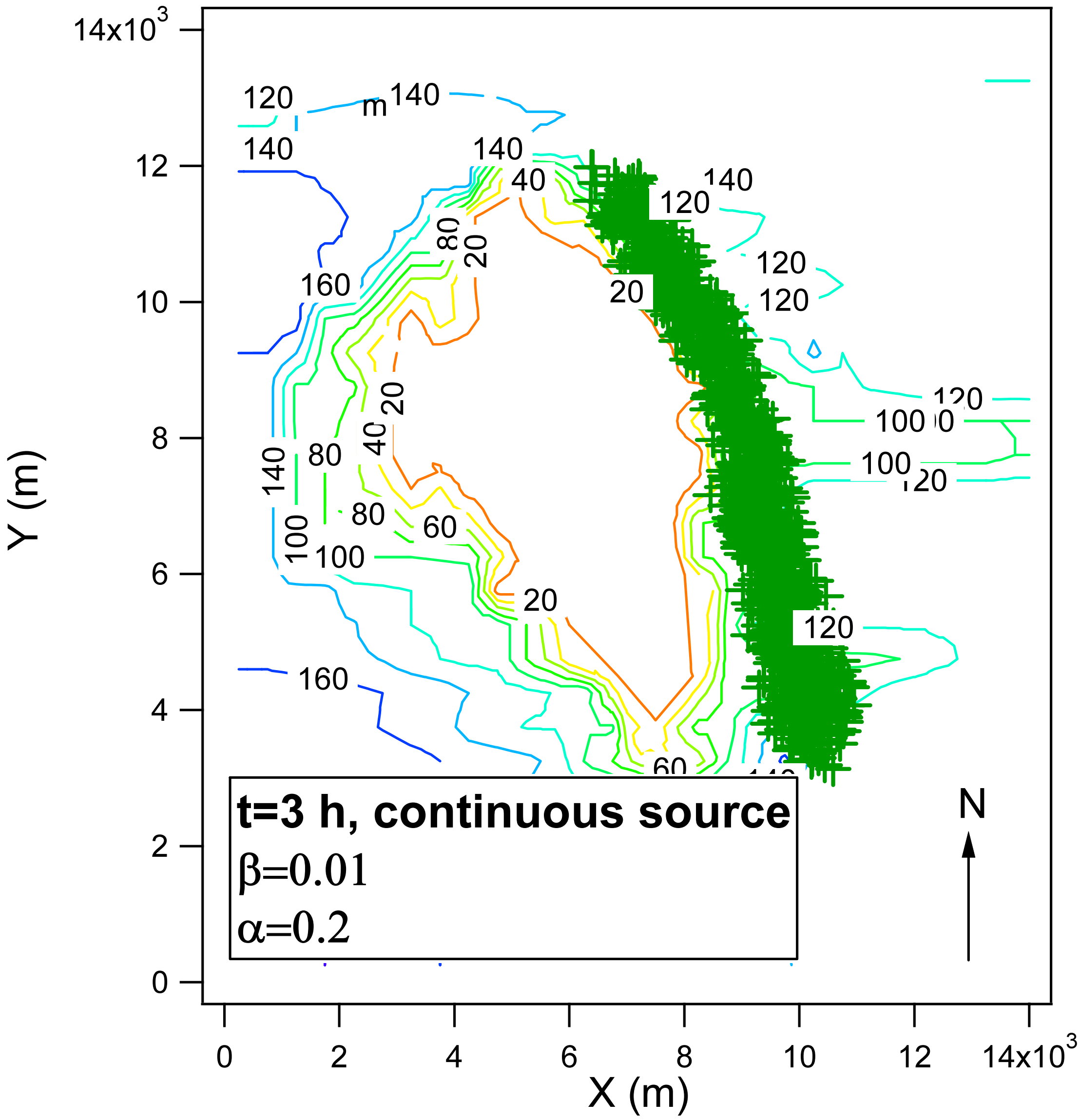

Fig. 2. Snapshot after 3 h for a continuous source with the same initial position as in Fig. 1. The continuous source is modelled as subsets of 10 particles each released every 100 s for a total of 1000 particles.

In presence of a time varying current and wind, which is the truly realistic case, both effects may be relevant on the intermediate time scale (ranging from a few to several hours).

In order to perform a realistic calculation it is necessary to select a physically sound value of the parameters. The set of parameters α measures the magnitude of the random and diffusive terms while β determines the magnitude of the friction term.

The surface tension γ of oil reads about 20 dyne/cm (Darwish 1995). The separation of two particles of area A_{part} creates edges for an additional length of $(A_{part})^{½}$ requiring a force $\propto \gamma\ (A_{part})^{1/2}$. This force must equal the force acting on A_{part} which reads $\beta\ v\ A_{part}$ leading to the relationship (Eq. (7)):

$$\beta \sim \gamma/(v\ (A_{part})^{1/2}) \tag{7}$$

The resulting value of β for oil is definitely small, suggesting that in short time scales this effect is not dominant and thus justifying the fact that it was neglected in most previous studies of oil spilling. It can however play a role on longer times due to the delay effect mentioned above and when the surface tension of the released fluid is significantly higher than for oil.

Fig. 2 shows the evolution of a continuous source for a more realistic choice of the parameters. In presence of an actual spilling, the results might be compared with the images of the spot recorded at subsequent times, thus providing a more reliable set of parametric values to be used in future forecasting of the evolution of the spot.

4. CONCLUSIONS

We have shown here that it is possible to model the spilling of a liquid substance into the sea taking into account not only the velocity field and the transport due to wind, as routinely done in similar simulations, but also the random part due to wave motion and a friction term, accounting of the surface tension of the liquid.

The latter factor comes out to play a minor role in case of oil and for short time scales, thus justifying its absence in previous studies. The present model is however apt to include it, without significant computational cost, for situations (higher surface tension and longer time scales) for which it may not be negligible.

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