The 2012 World Congress on
Advances in Civil, Environmental, and Materials Research (ACEM' 12)
Seoul, Korea, August 26-30, 2012

Minimizing the scattered field of layered acoustic cloak through optimization

*Chung-Ning Weng1), Wen-Hsiao Chung2), Tungyang Chen3)

1), 2), 3) *Department of Civil Engineering, National Cheng-Kung University, Tainan, Taiwan*
3)tchen@mail.ncku.edu.tw

ABSTRACT

In this paper, we present a quasi-Newton method together with a genetic algorithm as the optimization procedure in the design of multilayered acoustic cloaks. Examples on refining the material parameters of anisotropic layers as well as thickness distributions are demonstrate. Numerical results show that the proposed optimization can efficiently reduce the scattering fields.

1. INTRODUCTION

Recently, the use of coordinate transformations to design materials specifications that control the propagation of electromagnetic waves as desire has been extensively discussed. This starts with the pioneering study of invisibility cloaks by Pendry (2006) and Leonhardt (2006). They proposed that one can enclose a region by a properly engineered material, whose refractive index are tailored according to the result of geometric transformation, while light pass through, the fields would be bent around the region without penetrating and finally return to their original trajectories. The idea is soon confirmed by a few works of theoretical models (Ruan 2007, Chen 2007), numerical ray tracing (Schurig 2006), full wave simulation (Cummer 2006) and experiment in microwave regime (Schurig 2006). Similar concept can be extended to the design of acoustic cloak (Milton 2006, Cummer 2007, Chen 2007). Experiment for protecting underwater structure from sonar detection (Zhang 2011) is performed as well. Unfortunately, such media usually appears strong anisotropy and nonhomogeneity which is not easy to fabricate even using metamaterials. In practice, a possible way to yield this property is the use of anisotropic multilayered structures (Schurig 2006, Cai 2007) or bi-layer isotropic media (Huang *et al*. 2007, Torrent 2008, Qiu 2009, Chen and Liu, 2009). However, the unavoidable scattering will occur due to impedance mismatch. In electromagnetics, some researches devoted to the reduction of scattering of

---

1) Postdoctoral research fellow
2) Graduate Student
3) Professor, corresponding author

multilayered cloak through optimizations (Popa 2009, Xi 2009). In this paper, we extent the concept in the design of acoustic cloaks and two optimization procedures respectively correspond to the refinement of materials and thicknesses are introduced via two cylindrical case studies.

2. Optimization of material parameters

According to the transformation techniques, the ideal parameters of a cylindrical acoustic cloak can be written as

$$\frac{\kappa}{\kappa_0}=\left(\frac{b-a}{b}\right)^2\frac{r}{r-a},\quad \frac{\rho_r}{\rho_0}=\frac{r}{r-a},\quad \frac{\rho_\theta}{\rho_0}=\frac{r-a}{r}, \tag{1}$$

in which *a* and *b* are the inner and outer radius of the cloak, respectively. In practice, such graded anisotropic material can be approached by using discrete cylindrical shells as shown in Fig. 1. The inner and outer radiuses of the discrete system are now denoted by r_0 and r_M, respectively and *M* is the total number of layers. Each layer is characterized by a constant bulk modulus and density tensor as follow:

$$\begin{cases}\dfrac{\kappa_m}{\kappa_0}=\left(\dfrac{b-a}{b}\right)^2\dfrac{\frac{r_m+r_{m-1}}{2}}{\frac{r_m+r_{m-1}}{2}-a},\\[2ex] \dfrac{\rho_{mr}}{\rho_0}=\dfrac{\rho_r}{\rho_0}=\dfrac{\frac{r_m+r_{m-1}}{2}}{\frac{r_m+r_{m-1}}{2}-a}, \qquad \text{for } m=1-M,\\[2ex] \dfrac{\rho_{m\theta}}{\rho_0}=\dfrac{\frac{r_m+r_{m-1}}{2}-a}{\frac{r_m+r_{m-1}}{2}},\end{cases} \tag{2}$$

where κ_m, ρ_{mr} *and* ρ_m denotes the bulk modulus, radial and tangential components of density of layer *m*. As we will show next, the scattering fields are inevitable for such discrete layered structures.

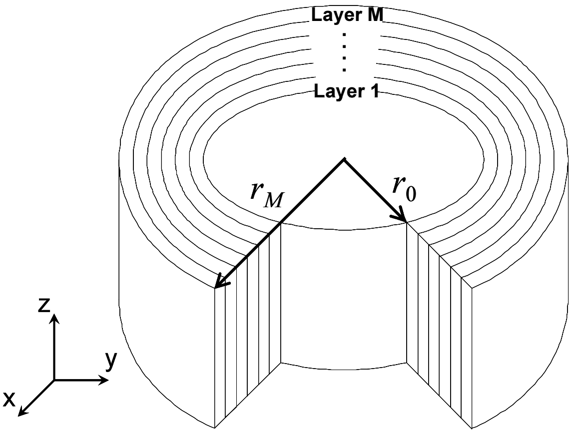

Fig.1 A schematic representation of a multilayered cylindrical acoustic cloak.

Consider the time harmonic problem, the pressure fields *p* for each layer satisfies the following Helmholtz equation:

$$r^2 \frac{\partial^2 p_m}{\partial r^2} + r\frac{\partial p_m}{\partial r} + \frac{\rho_{mr}}{\rho_{m\theta}}\left(\frac{\partial^2 p}{\partial \theta^2}\right) + k_m^2 r^2 p_m = 0, \tag{3}$$

Where $k_m = \omega\sqrt{\rho_{mr}/\kappa_m}$. Based on the Mie theory, a uniform incident field propagating along *x* direction upon the layered structure can be expressed as

$$p_{inc} = \sum_n i^n J_n(k_0 r)e^{in\theta} \tag{4}$$

and the corresponding scattered field can be written as

$$p_{sc} = \sum_n A_n H_n^{(2)}(k_0 r)e^{in\theta} \tag{5}$$

in which A_n is the unknown scattering coefficient, $k_0 = \omega\sqrt{\rho_0/\kappa_0}$ is the wave number in background material. J_n and $H_n^{(2)}$ denote the *n*th order Bessel functions and Hankel functions of the second kind. Analyzing Eq. (3) by separation of variables, the field in layer *m* are given by

$$p_m = \sum_n \left[C_{mn} J_{\nu_{mn}}(k_m r) + D_{mn} Y_{\nu_{mn}}(k_m r) \right] e^{in\theta}, \text{ for } m = 1 - M \tag{6}$$

where

$$\nu_{mn} = n\sqrt{\frac{\rho_{mr}}{\rho_{m\theta}}}. \tag{7}$$

The coefficients A_n, C_{mn} and D_{mn} can be determined by matching the continuity conditions of pressure and velocities at boundaries

$$\begin{aligned} &p_m\big|_{r=r_m} = p_{m+1}\big|_{r=r_m}, \\ &\frac{1}{\rho_m}\frac{\partial p_m}{\partial r}\bigg|_{r=r_m} = \frac{1}{\rho_{m+1}}\frac{\partial p_{m+1}}{\partial r}\bigg|_{r=r_m+1} \quad \text{for } m = 1 - M. \end{aligned} \tag{8}$$

At the innermost surface (r=r_0), sound hard condition is assumed. Working out the algebra, the non-vanishing scattering coefficient A_n is obtained for any finite *M* layers and thus cloaking effect is therefore broken. To further exam the induced scattering, we discuss about the scattering width, a quantitative parameter in scattering studies, which is the measure of power scattered in a given direction when an object is illuminated by an incident wave. For the proposed model, the angle distribution of scattering cross section can be expressed as

$$\sigma = \lim_{r\to\infty}\left[2\pi r\frac{\left|p_{sc}\right|^2}{\left|p_{inc}\right|^2}\right]. \tag{9}$$

In far-fields, Eq. (9) can be expressed through asymptotic approximation and yields

$$\sigma = \frac{4}{k_0}\left|\sum_n i^n A_n e^{in\theta}\right|^2 . \tag{10}$$

If the incident frequency and observation angle are prescribed, Eq. (10) is only in terms of material parameters with $3M+1$ unknowns. For simplicity, we only focus on the bistatic scattering width in the forward direction ($\theta = 0$) since the strongest scattering occurs in this direction generally. For better cloaking efficiency, the smaller value of σ is demanded and consequently the problem can be treated as an optimization design, aiming to the reduction of scattering width, via control variable, ρ_{mr}, $\rho_{m\theta}$ and κ_m.

Table I The material parameters for the three-layer cloak

	Initial guess			Optimized parameters		
Layer	κ_m/κ_0	ρ_r/ρ_0	ρ_t/ρ_0	κ_m/κ_0	ρ_r/ρ_0	ρ_t/ρ_0
1	0.698	41.00	0.024	0.433	23.67	0.031
2	0.244	14.33	0.070	0.242	7.690	0.100
3	0.153	9.00	0.111	0.172	7.358	0.147

To begin with, we illustrate a simple design of a three layered thin-cloak as an example with respect to sound frequency 9875Hz. The geometric parameters are selected as r_0=0.1m and $r_M = r_3$= 0.115m, and the thickness of each layer is equal. Background material is water with density ρ_0=998kg/m^3 and bulk modulus κ_0=1.9Gpa. Then we employ the Quasi-Newton method which is also called Broyden-Fletcher-Goldfarb-Shanno (BFGS) method in optimization processes. This algorithm can numerically solve unconstrained nonlinear problems, and the solution converges well locally if we set a suitable initial guess. Table I shows the initial guess chosen from Eq. (2) and optimized material parameters. It is interesting to see that the optimized parameters are different from initial parameters. The maximum difference is about 38% in bulk modulus and 46% in the component of densities. We also plot the angle distributions of scattering width normalized to wavelength in Fig. 2. The black curve represents the normalized scattering efficiency of a sound rigid cylinder without any cloak. The blue and red curves represent the cases of layered cloak with initial and optimized parameters, respectively. It can be observed that the initial layered cloak only reduce 15dB lower in maximum, compared with the scattering of uncloaked example. However, the reduction of scattering in each angle is more than 25dB when the optimized cloak is chosen. This remarkable improvement indicates that more scattering fields can be cancelled by the three optimized layers.

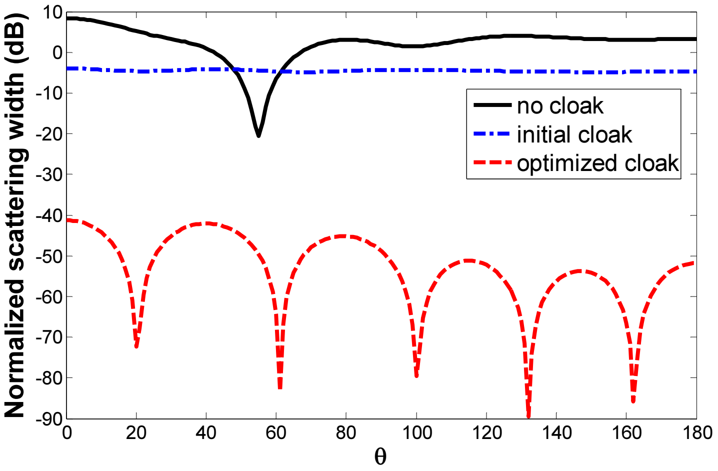

Fig. 2 Normalized scattering efficiency for a sound rigid sphere without cloak (black), three-layer cloak with initial parameters (blue) and three-layer cloak with optimized parameters (red).

3. Optimization of thicknesses

However, the efficiency of the propose optimization appears size dependent. The above example is designed for protecting an object with 1.2λ in diameters and the total thickness of cloaking shell is 0.1λ. According to our numerical results, three-layer approximation is sufficient for this size. When the objects become larger, increasing the number of layer as well as total thickness is required so as to preserve satisfactory performance. For example, if the inner radius increases from 0.1m to 0.2m, the required numbers of layers becomes six. To get over it, we propose an alternative optimization to minimize scattering widths for thick cloaks by altering the thickness of each layer while the total number of layers could be decreased. Consider a cloak with inner radius 0.2m and outer radius 0.23m. For this size, the required number of layers increases to six since the shell is 2 times thicker than the previous case. Here we assume the shell is divided into four layers only, and each thickness $r_m - r_{m-1}$ (m=0–4) needless to be identical. The objective function σ can be expressed in terms of radius r_i (i =1–3) form Eq. (2). The question now is to find a fitting set of thickness distributions that σ is minimized. In this case, we cannot assign a proper initial guess of r_i so that the BFGS method might be helpless. We utilize the generic algorithm (GA) to search for global optimum. The bounds of each radius [0.2, 0.23] together with local constrains $r_i < r_{i+1}$ are considered in the calculation.

Fig. 3 shows the comparisons of material parameters verse radius of the proposed multilayered cloak. The blue, black, and red-dashed curves represent the cases of ideal distribution, six layers of equal thickness, and four-layer optimized distributions of material parameters, respectively. Note that the parameters for each layer follow the results of Eq. (2), each radius is obtained from GA processes. A truncation at $\rho = 0.01\lambda$ is chosen in order to avoid the singularity of ideal parameter at inner boundary. It can be found that each thickness of optimized four layers is not the same. Each thickness can be interpreted as the weighting factor of the corresponding materials for that layer. Next we plot the normalized scattering efficiency of the same cloak with six layers of equal thickness and that with four layers of optimized thickness in Fig. 4. It can be observed that the notable reduction occurs between $\theta = 100^\circ$ and $\theta = 160^\circ$. The scattering fields in the vicinities of forward and backward direction appear a little enhancement. This example validates the possibility that a comparable cloaking performance can be yielded by a multilayered structure with fewer layers after optimization.

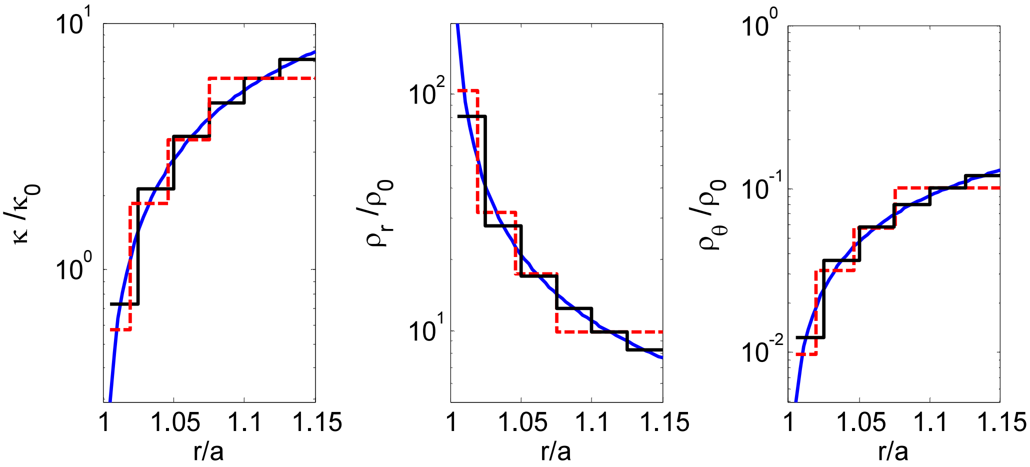

Fig. 3 The comparison of material distributions within the cloak.

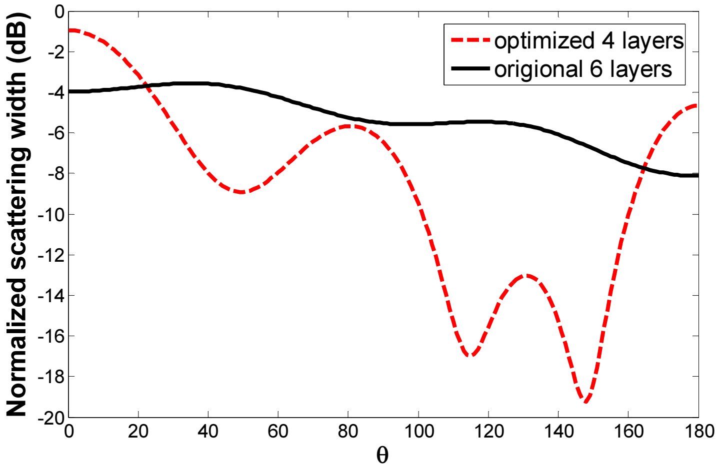

Fig. 4 Normalized scattering width for a six-layer cloak of equal thickness (solid) and four-layer cloak with optimized thickness distribution (dashed).

In addition, the GA processes and the BFGS method can be applied together in the design. We can run the GA search to find appropriate thickness distributions and obtain the corresponding material parameters. Then, we take these material parameters as the initial guess in the calculation of BFGS method and thus approach the optimized values. The advantage of running GA before BFGS optimization is that the required layers as well as material parameters of multilayered structure can be decreased. We again proposed the same thick cloak and follow the two-step procedure. Note that the radius for each layer is identical to that in Fig. 3 while the corresponding bulk modulus and densities are distinct due to BFGS optimization. Fig. 5 shows the comparison of simulation results of pressure fields. The left panel shows the pressure field with a sound hard scatterer, in which a strong scattering is induced. In contrast, when a multilayered cloaking shell is placed around the same scatterer, the incident wave is smoothly directed around the inner surface with remarkable reduced scattering shown as the right panel of Fig. 5 and thus anything inside the cloak will become undetectable.

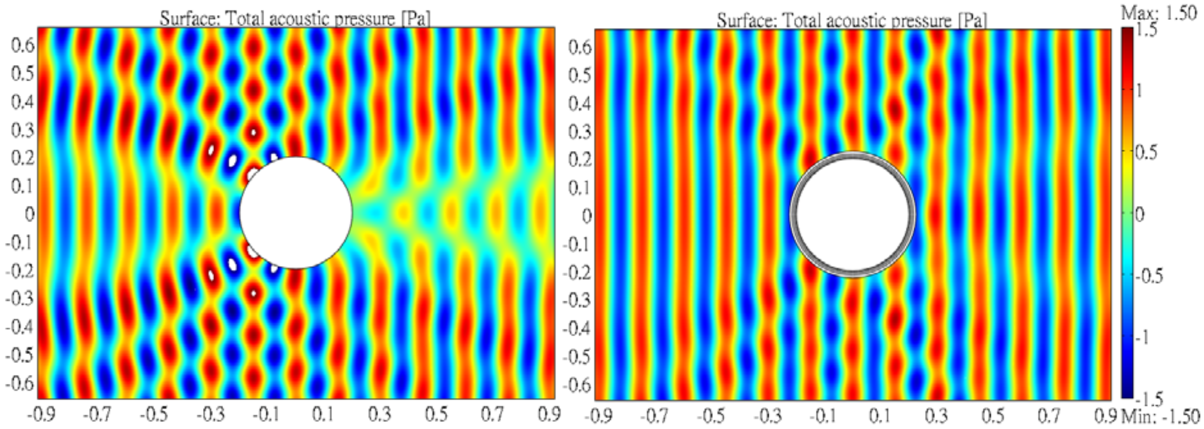

Fig. 5 The pressure field distribution for the two-step optimized four-layer cloak.

CONCLUSION

In conclusion, we introduced two different optimization algorithms in the design of multilayered cylindrical cloaks. The utilization of quasi-Newton method can locally adjust the material properties which are suitable in the design of thin cloak, while the genetic algorithm provides a global search for thick cloak with appropriate thickness distributions, and thus together they can efficiently improve the cloaking performance. Numerical demonstrations confirm that the proposed procedure can indeed reduce the scattering fields. For further study, both algorithms can be extended to three dimensional cases or applied to the design of other structures with complex materials.

REFERENCES

Cai, W., Chettiar, U. K., Kildishev, A. V. and Shalaev, V. M. (2007), “Optical cloaking with metamaterials”*, Nat. Photon.* **1**, 224-227.

Chen, H. and Chan, C. T. (2007), “Acoustic cloaking in three dimensions using acoustic metamaterials”, *Appl. Phys. Lett.* **91**, 183518.

Chen, H., Wu, B. I., Zhang, B., and Kong, J. A. (2007), “Electromagnetic wave interactions with a metamaterial cloak”, *Phys. Rev. Lett.* **99**, 063903.

Cheng, Y., Yang, F., Xu, J. Y. and Liu, X. J. (2008), “A multilayer structured acoustic cloak with homogeneous isotropic materials", *Appl. Phys. Lett.*, **92**, 151913.

Cummer, S. A., Popa, B.-I., Schurig, D. and Smith, D. R. (2006), “*Full-wave simulations* of electromagnetic cloaking structures,” *Phys. Rev. E* **74**, 036621.

Cummer, S.A., and Schurig, D. (2007), “One path to acoustic cloaking”, *New J. Phys.* **9**, 45.

Huang, Y., Feng, Y. and Jiang, T. (2007), “*Electromagnetic cloaking by layered structure of homogeneous isotropic materials*”, *Opt. Express* **15**, 11133.

Leonhardt, U. (2006), “Optical conformal mapping”, *Science* **312**, 1777–80.

Milton, G. W., Briane, M. and Willis, J. R. (2006), “On cloaking for elasticity and physical equations with a transformation invariant form”, *New J. Phys.* **8**, 248.

Pendry, J. B., Schurig, D. and Smith, D. R. (2006), “Controlling electromagnetic fields”, *Science* **312**, 1780-2.

Popa, B-I. and Cummer, S. A. (2009), “Cloaking with optimized anisotropic layers”, *Phys. Rev. A,* **79**, 023806.

Qiu, C. W., Hu, L. Xu, X. and Feng, Y. (2009), “*Spherical cloaking with homogeneous isotropic multilayered structures*”, *Phys. Rev. E* **79**, 047602.

Ruan, Z., Yan, M., Neff, C. W. and Qiu, M. (2007), “Ideal cylindrical cloak: perfect but sensitive to tiny perturbations”, *Phys. Revs. Lett.,* **99**, 113903.

Schurig, D., Mock, J. J., Justice, B. J., Cummer, S. A., Pendry, J. B., Starr, A. F. and Smith, D. R. (2006), “Metamaterial electromagnetic cloak at microwave frequencies”, *Science* **314**, 977.

Torrent, D. and J. Sánchez-Dehesa (2008), "*Acoustic cloaking in two dimensions* a feasible approach", *New J. Phys.*, **10**, 063015.

Xi, S., Chen, H., Zhang, B., Wu, B-I. and Kong, J. A. (2009), “Route to low-scattering cylindrical cloaks with finite permittivity and permeability”, *Phys. Rev. B,* **79**, 155122.

Zhang, S., Xia, C. and Fang, N. (2011), “Broadband Acoustic Cloak For Ultrasound Waves”, *Phys. Rev. Lett.*, **106**, 024301.