

## **Air damping induced from the interaction between a vibrating cantilever and its surroundings**

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### **ABSTRACT**

To develop a mechanical spectrometer to measure linear viscoelastic properties of solid materials, it is essential to eliminate parasitic damping sources in the apparatus. On the experimental front, we develop a thin-film beam shaker apparatus consisting of a bimorph piezo actuator and two fiber-optics probes for displacement measurements on the piezo and sample, respectively. The cantilever sample is sinusoidally loaded at its fixed end. From the shape of experimentally measured resonant peaks, the damping of the vibration system can be determined either by the full-width-at-half-maximum method or lorentzian curve fit. In this work, numerical coupled fluid-structure interaction is simulated by the finite element method to detect the effects of air damping from the vibrating cantilever. Both base excitation and tip loading are considered, and driving frequency is set to be 1 Hz. It is found that at this low frequency, the base excitation makes the sample behave as a rigid body. For the tip loading case, significant deformation on cantilever is generated, and the surrounding flow velocity pattern is quite different from that of the base excitation case.

### **1. INTRODUCTION**

Loss mechanisms are important in experimental measurements of viscoelastic properties. The overall tangent delta is equal to the sum of loss tangent contributed from various mechanisms [1].

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$$\tan \delta_{\text{total}} = \sum_i \tan \delta_i \quad (1)$$

Here the subscript  $i$  denotes different mechanisms. The  $\tan \delta$  can be defined as follows.

$$\tan \delta = \frac{\Delta W}{2\pi W} = \frac{1}{Q} \quad (2)$$

where  $W$  is the stored energy and  $\Delta W$  the energy loss per cycle.  $Q$  is the quality factor, often used in the vibration community. Hence, the total damping can be subtracted from measured total damping, the inverse of quality factor  $Q$ , since  $Q^{-1} = Q_1^{-1} + Q_2^{-1} + \dots$ , where  $Q^{-1}$  is the total damping and the inverse of  $Q$ 's with subscripts represents damping from different mechanisms. Various loss mechanisms to the vibrating cantilever beam are known as follows [2].

Loss tangent due to the interaction between the cantilever beam and the surrounding air can be estimated by

$$\tan \delta = \alpha \sqrt{\frac{12}{\rho E}} \left( \frac{L}{h} \right)^2 \quad (3)$$

Here  $\alpha$  is a constant related to vibration modes, surrounding air and pressure,  $L$  the length of the beam,  $h$  the thickness of the beam,  $E$  the Young's modulus and  $\rho$  the density. It can be seen that increasing length increases the loss tangent. In the dilute limit, damping due to surrounding air can be estimated by the following equation [3].

$$\tan \delta_{\text{air}} = \frac{2P}{\pi \rho c} \sqrt{\frac{C_p}{C_v} \frac{\mu}{RT}} \quad (4)$$

$C_p$  and  $C_v$  are heat capacity under constant pressure and volume, respectively.  $\mu$  is the density of surrounding fluid, and  $\rho$  is the density of the solid. The speed of sound in solid is denoted as  $c$ . The symbols  $P$ ,  $R$  and  $T$  represent the surrounding pressure, gas constant (8.315 J/K/mol) and temperature in Kelvin, respectively. This expression is independent on sample geometry.

A vibrating cantilever beam loses its energy through the fixed end. This phenomenon is called the support loss, and is significant when  $L/h < 100$ . Loss tangent can be estimated by

$$\tan \delta \approx 3 \left( \frac{h}{L} \right)^3 \quad (5)$$

It can be seen from the equation that longer beams have smaller loss tangent contributed from the support loss.

The thermoelastic loss is due to the heat conduction in the cantilever beam, causing transverse heat flow while vibrating. This phenomenon is frequency ( $f$  in Hz) dependent.

$$\tan \delta = \beta f h^2 \quad (6)$$

The  $\beta$  depends on the thermomechanical properties of the material. The thermoelastic loss does not depend on beam length.

When the cantilever is extremely thin, surface-to-volume ratio increases dramatically. Hence, the surface loss, caused by surface stress due to absorbents on the surface or surface defects, is not negligible. For rectangular cantilever beams,

$$\tan \delta = \gamma \frac{\Delta(3w + h)}{wh} \quad (7)$$

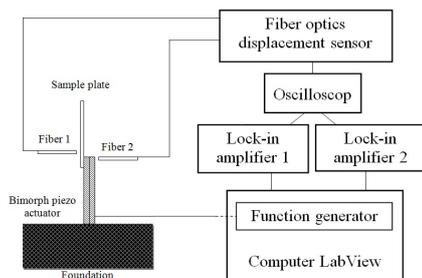
Here  $w$  is the width of the beam and  $\Delta$  the surface layer thickness. It can be seen that surface loss does not depend on beam length.

Among the many mechanisms mentioned above, we perform numerical study to model the interaction between vibrating cantilever and its surrounding air. The purpose of this work is to provide guidelines for thin-film-shaker experiment, which measures  $\tan \delta$  and elastic modulus of thin film materials. Only air damping is studied in the present work.

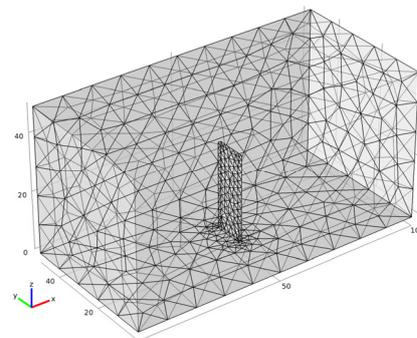
## 2. EXPERIMENT

The schematic of our experimental setup is shown in Figure 1 (a). A bimorph piezoelectric plate-like actuator with the dimensions on the millimeter scales. The bimorph actuator was clamped into the foundation with proper electrical connections, and driven by a function generator. The free length of the piezo actuator was 10mm. The specimen was mounted onto the top of the piezo actuator with superglue. The deflection of the specimen and aluminum clamp was monitored by an MTI fiber-optic measurement system with the probes labeled Fiber 1 or Fiber 2, respectively, in the figure. Lock-in amplifiers (Stanford Research Systems, Inc., Sunnyvale, CA, U.S.A.) were connected with the fiber-optic probes to reduce noise. Vacuum system is to be completed in the near future. Detailed description of the apparatus can be found in [4]. Figure 1 (b) shows the finite element model to calculate the interaction between the vibrating beam and its surrounding air.

It has been shown that experimentally measured loss tangent is on the order of  $10^{-2}$ , For low damping materials, such as metal, which has loss tangent on the order of  $10^{-5}$  or smaller, the TFS device cannot provide damping formation due to parasitic damping sources. One of the goals of the present study is to identify parasitic damping due to air damping, and with Eq. (1), accurate material damping may be obtained.



(a)



(b)

Figure 1. (a) Schematic of the thin film shaker (TFS), and (b) the finite element model to study the interaction between the sample and it surrounding air.

### 3. FINITE ELEMENT CALCULATION

The coupled fluid-solid problem, that consists of a vibrating cantilever beam in air is solved with COMSOL Multiphysics [5]. From the Navier-Stokes equations, Eq. (1) and (2), the fluid motion is modeled.

$$\mu \frac{\partial \mathbf{v}}{\partial t} - \nabla \cdot \left[ -p\mathbf{I} + \eta(\nabla \mathbf{v} + \nabla \mathbf{v}^T) \right] + \mu((\mathbf{v} - \mathbf{v}_m) \cdot \nabla) \mathbf{v} = \mathbf{F} \quad (8)$$

$$\nabla \cdot \mathbf{v} = 0 \quad (9)$$

Here,  $\mathbf{v}$  is the velocity field,  $p$  pressure,  $\eta$  viscosity, and  $\mu$  the density of the fluid. The volume force of the fluid  $\mathbf{F}$  is equal to zero in the present analysis. The traction force that is exerted by the fluid to the solid can be calculated as follows.

$$\mathbf{F}_T = -\mathbf{n} \cdot \left[ -p\mathbf{I} + \eta(\nabla \mathbf{v} + \nabla \mathbf{v}^T) \right] \quad (10)$$

Here, the normal vector to the boundary of the solid is denoted by  $\mathbf{n}$ . The subscript T indicates the force that is be transmitted from fluid to solid, and superscript T the matrix transpose operator. The deformation of the cantilever beam is modeled by Navier's equation with displacement field  $\mathbf{u}$  and body force  $\mathbf{b}$ . The density of the solid is denoted as  $\rho$ .

$$\mu \nabla^2 \mathbf{u} + (\lambda + \mu) \nabla(\nabla \cdot \mathbf{u}) + \rho \mathbf{b} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} \quad (11)$$

The motion of the deformed mesh is modeled using Winslow smoothing and arbitrary euler lagrangian (ALE) mesh is adopted. The boundary conditions control the displacement of the moving mesh with respect to the initial geometry. At the boundaries of the obstacle, this displacement is the same as the structural deformation. On the exterior boundaries of the air domain, the deformation is set to zero in all directions.

### 4. RESULTS AND DISCUSSION

Both base excitation and tip loading are considered in the coupled fluid-structure interaction with the finite element method. All simulations are under displacement control at 1 Hz. Figures 2 (a) and (b) are the displacement and pressure vs. time for the base excitation case. For the tip loading case, the displacement and pressure at various locations are shown in Figures 2 (c) and (d). Due to the rigidity of the cantilever beam, the base excitation induces rigid body motion of the cantilever, and hence all displacements are identify in Figure 2 (a). Under the tip loading, all rigid body motion is

constrained, and hence deformation is discernable. Pressure on the surface at various locations show that it is proportional to the deformation of the cantilever, as expected.

The air flow around the cantilever is significantly affected by the motion of the sample vibration, as shown in Figure 3. Since the base excitation at low frequency only creates rigid body motion, the air flow around is symmetric (Figure 3 a). On the contrary, the tip loading deforms the cantilever, which creates asymmetric motion in the air flow, as shown in Figure 3 (b).

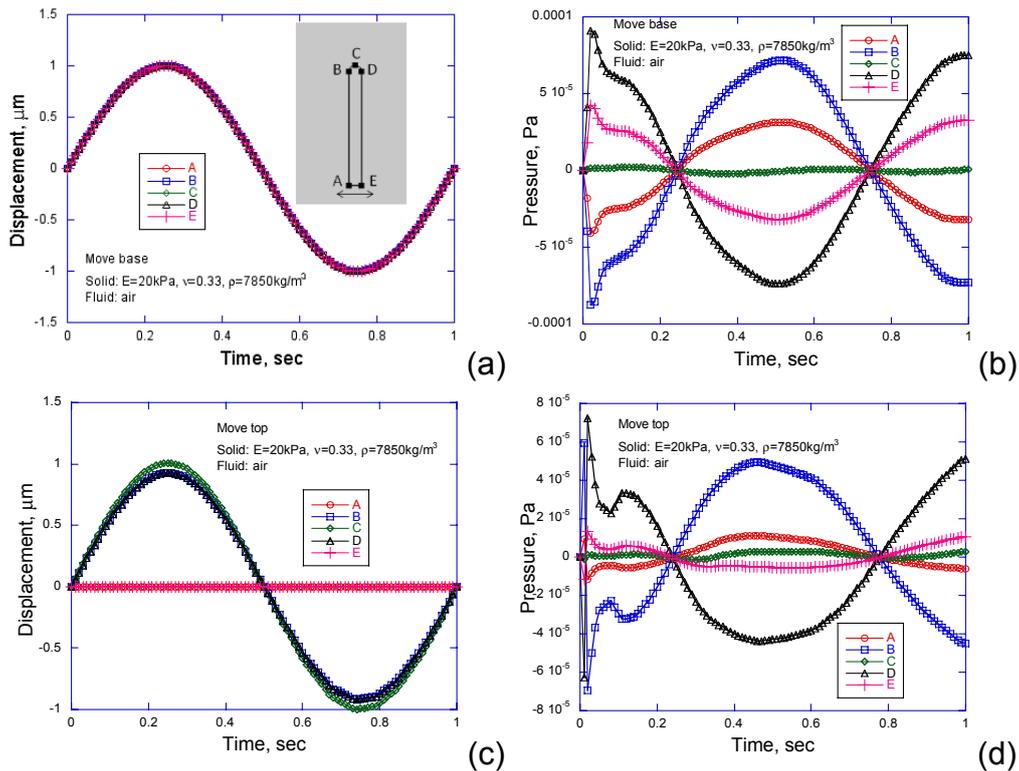


Figure 2. For the base excitation case, (a) displacement vs. time, and (b) pressure on the cantilever at various locations vs. time. For the tip loading, (c) displacement vs. time, and (d) pressure at various locations vs. time.

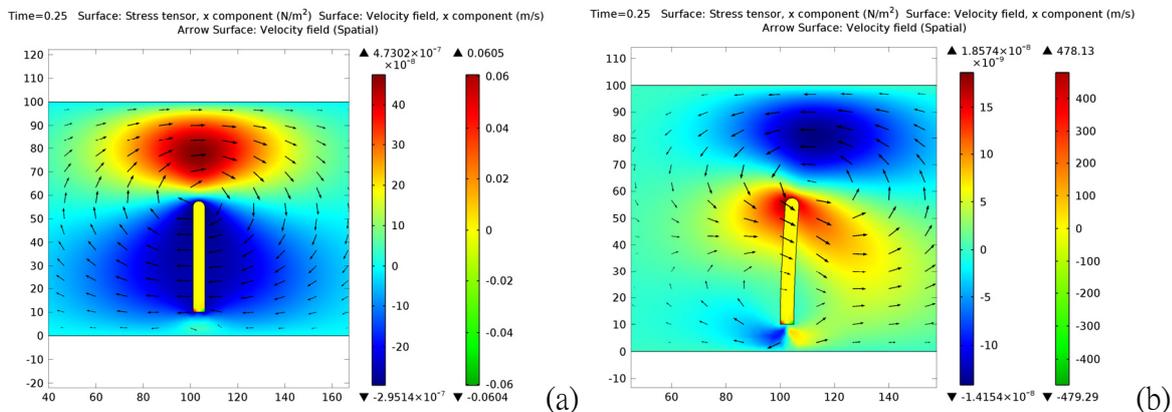


Figure 3. Stress (first scale bar on the right) and flow velocity (second scale bar on the right) in the solid for (a) the base excitation case, and (b) the tip loading case.

## 5. CONCLUSION

Structure-fluid interaction between the cantilever beam and its surrounding air is modeled by coupled finite element analysis. Our simulation shows the effects of surrounding air are able to induce air damping in the TFS measurements. In low frequency, the base excitation causes rigid body motion, and the tip loading deforms sample significantly. The two different deformation modes cause different air flow patterns around the sample, and hence different contribution to air damping. At high frequency, base excitation may induce deformation on the sample due to inertia effects, which are currently under investigation.

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