The 2012 World Congress on
Advances in Civil, Environmental, and Materials Research (ACEM' 12)
Seoul, Korea, August 26-30, 2012

Identification of flutter derivatives from the forced and free vibration tests using EEE method

*Y.H. Hong[1)], Y.C. Hwang[2)], H.K. Kim[3)] and H.S. Lee[4)]

[1), 2), 3), 4)] *Department of Civil & Environmental Engineering, Seoul National University, Seoul 110-799, Korea*
[1)] *hyh99@snu.ac.kr*

ABSTRACT

This paper proposes a new approach to identify the flutter derivatives by minimizing an equation error estimator (EEE) which is defined as the least-square errors between structural resistance forces and aeroelastic forces induced by wind. Dissimilar to the other SI algorithms, the proposed method can successfully employed for the identification of flutter derivatives regardless of the experimental procedure and does not require any complicated sensitivity analysis or complex eigen-value analysis to identify the unknown system parameters. The EEE requires complete information on the state variables at all-time steps. In the proposed method, accelerations of a section model are measured with accelerometers in wind tunnel tests while the velocities and displacements corresponding to the measured accelerations are reconstructed by the FEM-based finite impulse response filter (FFIR filter). The validity of the proposed method is demonstrated through both free vibration test and forced vibration test of bridge sectional models in the wind tunnel. It is shown that the flutter derivatives identified by the proposed method agree well with those by previous methods.

1. INTRODUCTION

After the flutter derivative based aeroelastic formula had been by Scanlan, great number of efforts have been made to estimate the flutter derivatives from the test of bridge model in wind tunnel. The flutter derivatives can be identified with different experimental procedures from an idealized 2-DOF section model, i.e. forced-vibration test and free-vibration test. The most widely adopted technique is the free-vibration method. Scanlan and Tomko proposed the extraction scheme for flutter derivatives from 2DOFs coupled motion tests (1971). Sarkar developed the Modified Ibrahim Time Domain (MITD) to estimate cross flutter derivatives along with direct flutter derivatives

[1)] Ph.D.
[2)] Graduate Student
[3), 4)] Professor

(1994). The procedures of these approaches are generally based on the response error estimation, which minimizes the relative error between measured displacement and predicted displacement. Although free-vibration technique requires relatively complicated procedure to identify the flutter derivatives and need additional assumption because of the uncertainty of frequency similarity, but it is appealing for the simple setup and the possibility for the realization of interaction between the structure and the wind.

A more reliable procedure is forced-vibration method [Diana 2004, Falco 1992, Kim 2007 and Matsumoto 1993] in the sense of the law of similarity. As the selection of the experimental procedure, different numerical algorithms are employed for the extraction of the flutter derivatives. Numerous SI algorithms used for the free-vibration technique is inappropriate for the steady state response of the forced-vibration test and vice versa. As far as the author knows of, the general algorithm that can be employed for extraction of flutter derivatives regardless of the testing procedures has not been proposed yet.

This paper proposes a new approach to identify the flutter derivatives by minimizing an equation error estimator (EEE) [Hjelmstad 1995] which is defined as the least-square errors between structural resistance forces and aeroelastic forces induced by wind. Dissimilar to the other SI algorithm, the proposed method can successfully employed for the identification of flutter derivatives regardless of the experimental procedure and do not require any complicated sensitivity analysis or complex eigen-value analysis to identify the unknown system parameters.

The EEE approach requires not only displacement response but also velocity and acceleration history for system identification. In this approach, a displacement and velocity reconstruction scheme [Hong 2010] is used to calculate displacement and velocity history from measured acceleration. Hence, both these reconstructed responses and the measured acceleration is used for EEE method.

2. ESTIMATION OF FLUTTER DERIVATIVES

2.1 Equations of motion for the free vibration test

In this paper, a section model for the wind induced vibration is assumed to have two degrees of freedom in vertical (h) and rotational (α) direction. The equations of motion per unit length for the section model can be expressed by following equations.

$$\mathbf{M}\ddot{\mathbf{U}}(t)+\mathbf{C}\dot{\mathbf{U}}(t)+\mathbf{K}\mathbf{U}(t)=\mathbf{F}_{ae}(t)+\mathbf{F}_{ex}(t) \tag{1}$$

where $\mathbf{M},\mathbf{C},\mathbf{K}$ and $\mathbf{F}_{ae}$ are the mass, damping, stiffness matrix of the structural system and the aeroelastic force vector, respectively, while $\mathbf{U}$, U and **X** are the displacement vector containing *h* and $\alpha$the flow field and the vector of the flutter derivatives which will be defined in next section. The overhead dot denotes differentiation with respect to time.

The aeroelastic force acting on a sinusoidal oscillating section model in a single mode is assumed as a linear function to the motion of the section and its first order derivative [Iwamoto 1995 and Scanlan 1971]:

$$\mathbf{F}_{ae}(t) = \begin{pmatrix} L_{ae}(t) \\ M_{ae}(t) \end{pmatrix} \approx \mathbf{C}_{ae}(\omega)\dot{\mathbf{U}}(t) + \mathbf{K}_{ae}(\omega)\mathbf{U}(t)$$
$$= \begin{bmatrix} H_1(\omega) & H_2(\omega) \\ A_1(\omega) & A_2(\omega) \end{bmatrix} \begin{pmatrix} \dot{h}(t) \\ \dot{\alpha}(t) \end{pmatrix} + \begin{bmatrix} H_4(\omega) & H_3(\omega) \\ A_4(\omega) & A_3(\omega) \end{bmatrix} \begin{pmatrix} h(t) \\ \alpha(t) \end{pmatrix} \tag{2}$$

where L_{ae} and M_{ae} are the aeroelastic lift force and moment, respectively, while is the circular frequency of the oscillation, and H_m and A_m ($m = 1, 2, 3, 4$) are the flutter derivatives. It is customary to use normalized expressions of the flutter derivatives [Scanlan 1971]. For the simplicity of presentation, however, this thesis presents discussions with the un-normalized forms of the flutter derivatives.

2.2 Equation error estimation (EEE)

In case the complete time history of displacement, velocity and acceleration are available, the flutter derivatives are identified by employing the EEE as follows:

$$\underset{\mathbf{X}}{\mathrm{Min}}\,\Pi(\mathbf{X}) = \frac{1}{2}\sum_{i=1}^{nt} \left\| \mathbf{F}_{kn}(t_i) - \mathbf{F}_{un}(\mathbf{X}, t_i) \right\|_2^2$$
$$= \frac{1}{2}\sum_{i=1}^{nt} \left\| \mathbf{F}_{st}(t_i) - \mathbf{F}_{ae}(\mathbf{X}, t_i) \right\|_2^2 \tag{3}$$

here, X is the vector of the flutter derivatives to be identified.

$$\mathbf{X} = (\overline{H}_1 \quad \overline{H}_2 \quad \overline{H}_3 \quad \overline{H}_4 \quad \overline{A}_1 \quad \overline{A}_2 \quad \overline{A}_3 \quad \overline{A}_4)^T \tag{4}$$

$\mathbf{F}_{st}(t_i)$ and $\mathbf{F}_{ae}(\mathbf{X}, t_i)$ are the structural resistance force and the aeroelastic force at time t_i, respectively.

$$\mathbf{F}_{st}(t_i) = \mathbf{M}\ddot{\mathbf{U}}(t_i) + \mathbf{C}\dot{\mathbf{U}}(t_i) + \mathbf{K}\mathbf{U}(t_i) \tag{5}$$

The aeroelastic force given in (4-28) is rewritten in terms of the vector of the flutter derivatives.

$$\mathbf{F}_{ae}(\mathbf{X}, t_i) = \mathbf{s}(t_i)\mathbf{X}$$
$$= \begin{bmatrix} \dot{h}(t_i) & \dot{\alpha}(t_i) & \alpha(t_i) & h(t_i) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dot{h}(t_i) & \dot{\alpha}(t_i) & \alpha(t_i) & h(t_i) \end{bmatrix} \mathbf{X} \tag{6}$$

Following the solution procedure of the EEE method in Eq. (3) ~ Eq. (6), a unique solution is always determined by Eq. (6) as long as a sufficient amount of measured dynamic responses of a section model are provided.

Since the unknown force in Eq. (6) is linear with respect to the system parameter, $\mathbf{X}$, the minimization problem in Eq. (3) forms a quadratic problem with respect to the system parameter. Hence, the solution of Eq. (3) is simply obtained by solving the first-order necessary condition for the quadratic problem, which is linear algebraic equation.

$$\frac{\partial \Pi(\mathbf{X})}{\partial \mathbf{X}} = \mathbf{SX} - \mathbf{G} = 0 \rightarrow \mathbf{X} = \mathbf{S}^{-1}\mathbf{G} \tag{7}$$

where $\mathbf{S}$ and $\mathbf{G}$ are global sensitivity matrix and gradient vector expressed as following equation, respectively.

$$\mathbf{S} = \sum_{i=1}^{nt} \mathbf{s}^{\mathrm{T}}(t_i)\mathbf{s}(t_i) \ , \quad \mathbf{G} = \sum_{i=1}^{nt} \mathbf{s}^{\mathrm{T}}(t_i)\mathbf{F}_{kn}(t_i) \tag{8}$$

3. RECONSTRUCTION OF DISPLACEMENT AND VELOCITY HISTORY FROM MEASURED ACCELERATION

Hong et al. have proposed a new class of FEM-based finite impulse response (FFIR) filter to reconstruct displacement and velocity simultaneously. In their approach, the displacement is reconstructed by solving the following minimization problem defined in a time interval, $T_1 < t < T_2$.

$$\underset{u}{\mathrm{Min}}\, \Pi(u) = \frac{1}{2}\int_{T_1}^{T_2} \left(\frac{d^2 u}{dt^2} - \bar{a}\right)^2 dt + \frac{\beta^2}{2}\int_{T_1}^{T_2} u^2 dt \tag{9}$$

where u and $\bar{a}$ are the displacement and the measured acceleration, respectively. The direct discretization of the variation statement of Eq. (6) with the finite element method using $2k$-th elements leads to a FFIR filter.

$$\delta\Pi(u) = \sum_{e=1}^{2k} \int_{\Delta t} \frac{d^2 \delta u^e}{dt^2}\left(\frac{d^2 u^e}{dt^2} - \bar{a}^e\right) dt + \beta^2 \sum_{e=1}^{2k} \int_{\Delta t} \delta u^e u^e dt = 0 \tag{10}$$

Here, u^e and $\bar{a}^e$ denote the displacement and acceleration in element, e, respectively. The standard FEM formulation for a beam on an elastic foundation is adopted to derive the following matrix expression of Eq. (10).

$$(\mathbf{K} + \beta^2 (\Delta t)^4 \mathbf{M})\mathbf{u} = (\Delta t)^2 \mathbf{Q}\bar{\mathbf{a}} \tag{11}$$

where **u** and $\bar{\mathbf{a}}$ denote the nodal unknown vector and the measured acceleration vector associated with all sampling points of measurement. The nodal unknown vector consists of the nodal displacements and the nodal velocities. The matrixes in Eq. (11) are defined as

$$\mathbf{K}=\sum_{e}\int_{0}^{1}\frac{d^{2}\mathbf{N}_{H}^{T}}{d\xi^{2}}\frac{d^{2}\mathbf{N}_{H}}{d\xi^{2}}d\xi\ ,\ \mathbf{M}=\sum_{e}\int_{0}^{1}\mathbf{N}_{H}^{T}\mathbf{N}_{H}dt\ ,\ \mathbf{Q}=\sum_{e}\int_{0}^{1}\frac{d^{2}\mathbf{N}_{H}^{T}}{d\xi^{2}}\mathbf{N}_{L}d\xi \tag{12}$$

where $\sum_{e}$ is the assembly operator of the FEM, and ξ is the natural coordinate for the time variable ranging from 0 to 1, $\mathbf{N}_H$ and $\mathbf{N}_L$ are Hermitian shape function and the linear shape function, respectively. The nodal unknown vector is obtained by solving Eq. (11).

$$\mathbf{u}=(\Delta t)^{2}(\mathbf{K}+\beta^{2}(\Delta t)^{4}\mathbf{M})^{-1}\mathbf{Q}\overline{\mathbf{a}}=(\Delta t)^{2}\mathbf{C}\overline{\mathbf{a}} \tag{13}$$

where **C** is the coefficient matrix of order $2(2k+1)\times(2k+1)$.
Assuming the time step at the center of a time window represents time *t*, the reconstructed displacement and velocity are expressed as following equations

$$u(t)=u_{k+1}=(\Delta t)^{2}\sum_{p=1}^{2k+1}C_{2k+1,p}\overline{a}_{p}=(\Delta t)^{2}\sum_{p=-k}^{k}c_{p+k+1}\overline{a}(t+p\Delta t) \tag{14}$$

$$v(t)=v_{k+1}=\sum_{p=1}^{2k+1}C_{2k+2,p}\overline{a}_{p}=\Delta t\sum_{p=-k}^{k}\hat{c}_{p+k+1}\overline{a}(t+p\Delta t) \tag{15}$$

4. EXPERIMENTAL VERIFICATION

For the verification of the proposed method for the free-vibration test, the flutter derivatives are identified using measurements taken from a series of free-oscillation tests for a thin rectangular plate with the width-to-depth (B/D) ratio of 20. The flutter derivatives are identified by the proposed method using the measured accelerations and by the MITD method using the measured displacements for comparison. The identified results from both of the methods presented here are the representative values for multiple measurements. The flutter derivatives identified for each measurement are averaged for the MITD method, while multiple measurements are considered together in optimization for the proposed method. Same B/D20 deck section also tested by the forced vibration test. In this testing procedure, the sisnusoidal fitting method is adopted for the comparision.

Fig. 1 and Fig. 2 show the flutter derivatives for the free vibration test and the forced vibration test, respectively. The proposed method yields well-matched results compared to the MITD method and the sinusoidal fitting method for all eight of the flutter derivatives in an overall sense.

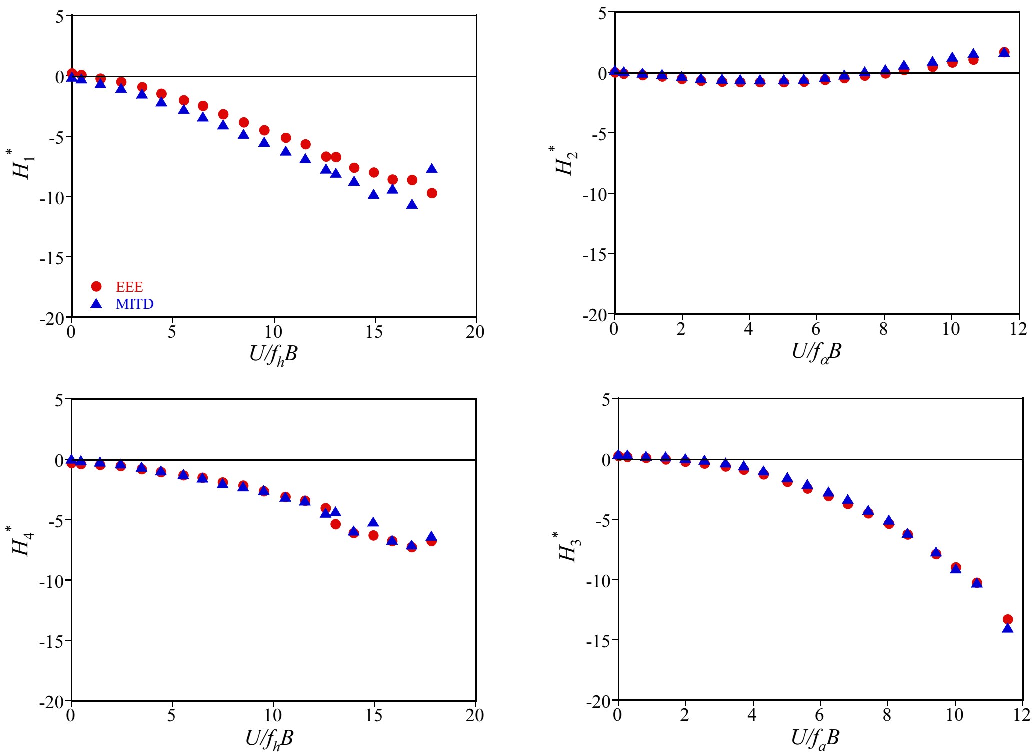

Fig. 1 Identified flutter derivatives from the free vibration test for the B/D20 section - H^* components: (a) H_1^*, (b) H_2^*, (c) H_4^* and (d) H_3^*

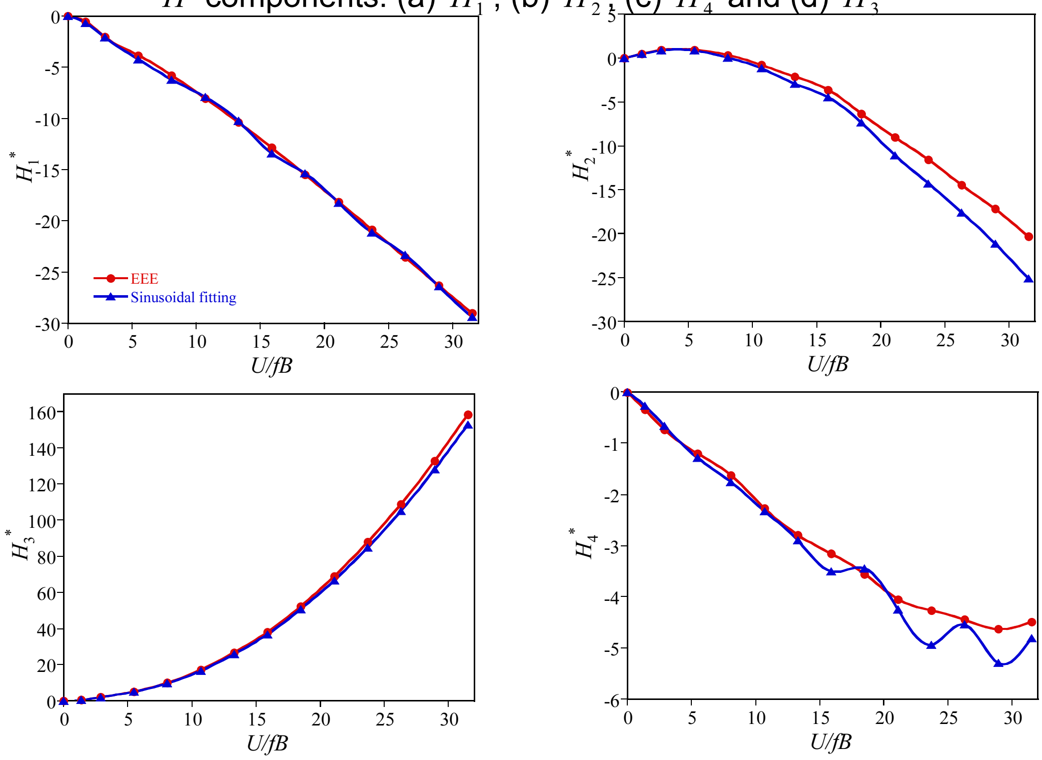

Fig. 2 Identified flutter derivatives from the forced vibration test for the B/D20 section - H^* components: (a) H_1^*, (b) H_2^*, (c) H_4^* and (d) H_3^*

CONCLUSION

This paper proposes a new approach to identify the flutter derivatives by minimizing an equation error estimator (EEE) which is defined as the least-square errors between structural resistance forces and aeroelastic forces induced by wind. The results of EEE method have a good agreement with the results of the conventional methods. Dissimilar to the other SI algorithms, the proposed method can successfully employed for the identification of flutter derivatives regardless of the experimental procedure and does not require any complicated sensitivity analysis or complex eigen-value analysis to identify the unknown system parameters

ACKNOWLEDGEMENT

This research was supported by the grant (09CCTI-A052531-02-000000) from the Ministry of Land, Transport and Maritime of Korean government through the Core Research Institute at Seoul National University for Core Engineering Technology Development of Super Long Span Bridge R&D Center.

REFERENCES

Hjelmstad K.D., Banan M.R. and Banan M.R. (1995), "Time-domain parameter estimation algorithm for structures. I: computational aspects", Journal of Engineering Mechanics, vol. **121** (3), pp. 424–434.

Hong Y.H., Kim H.K. and Lee H.S., "Reconstruction of dynamic displacement and velocity from measured accelerations using the variational statement of an inverse problem", *JSV*, vol. **329**, pp. 4980–5003.

Iwamoto M. & Fujino Y. 1995, Identificatio of flutter derivatives of bridge deck from free vibration data. Journal of Wind Engneering and Industrial Aerodynamics, vol. **54**, pp. 55-63

Lee H.S., Hong Y.H. and Park H.W. (2010), "Design of an FIR Filter for the Displacement Reconstruction Using Measured Acceleration in Low-frequency Dominant Structures", *IJNME*, vol. **82**, pp. 403-434.

Sarkar P. P., Jonse P. N. & Scanlan H. R. (1994), Identification of Aeroelastic Parameters of Flexible Bridges. Journal of Engineering Mechanics, ASCE, vol. **120**(8), pp. 1718-1742.

Scanlan H. R., & Tomko J.J. (1971), "Airfoil and bridge deck flutter derivatives" Journal of the Engineering Mechanics Division, ASCE, vol. **97**(6), pp. 1717-1737.