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Probabilistic numerical modeling of a RCC gravity dam's construction

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ABSTRACT

In this paper, a thermo-chemo-mechanical model is applied to the simulation of the construction of a roller-compacted concrete (RCC) dam. The model accounts for the hydration degree evolution and its influence on the mechanical properties. The simulation is performed using a 2D plane-strain finite element analysis. The model is applied in both a deterministic and a probabilistic manner. A probabilistic numerical model of the RCC gravity dam is proposed and performed by giving a random character to some model parameters. Two probabilistic tools are used: the First Order Second Moment (FOSM) method and the Random Balanced Design FAST (RBD-FAST) method. A global sensitivity analysis is performed using the RBD-FAST method.

1. INTRODUCTION

Risk Management and Risk Analysis tools applied to dams' safety control have undergone significant developments and gained increased application over the past two decades. However, even if it is known that the security of dams is a concept of probabilistic nature, its evaluation has been done in a deterministic manner over the years, based on the concept of the global security coefficient. Therefore, that evaluation should take into account uncertain parameters by means of probabilistic tools as a complement to the deterministic classical tools based on an empiric global security coefficient. With the arrival of the Eurocodes, the limit state and partial factors of safety concepts were introduced and a semi-probabilistic safety assessment of dams has then begun to be implemented in European countries.

The work presented in this paper intends to be a contribution to the use of probabilistic tools on the risk assessment of dams.

In the following the thermo-chemo-mechanical behaviour of concrete will be discussed as well as the probabilistic tools used in this work. Then, some results concerning an application to a gravity RCC dam are presented.

2. PROBLEM DESCRIPTION

Concrete is a composite construction material often described by being the mixed paste of the following three parts: cement, water and aggregates. Due to the exothermic and thermally activated chemical reaction that occurs between water and cement, a thermo-chemo-mechanical model is needed in order to simulate the behaviour of concrete at early ages.

Several authors (Debruyne 1997, Cervera et al. 2002, De Schutter 2004, Lackner and Mang 2004, Buffo-Lacarrière 2007,) developed different models to simulate this coupled behaviour of concrete. In this paper, a model based on the one described in Cervera et al. 2002 is adopted. In the following, the hydration model will be presented, followed by a brief description of the thermo-chemo-mechanical coupling.

2.1. Hydration model

The general heat transfer problem, assuming an isotropic medium, is described by Eq.1 and 2, where ρ [kg/m^3] is the volumetric weight, c [J/(kg ºC)] is the specific heat, T [ºC] is the temperature ($\dot{T}$ being its first derivate with respect to time), $\underline{q}$ is the heat flow vector described by the Fourier law, k [W/(m ºC)] being the thermal conductivity, and $\dot{Q}$ [W/m^3] is the rate of heat generation per unit volume.

$$\rho \cdot c \cdot \dot{T} = -div(\underline{q}) + \dot{Q} \quad (1)$$

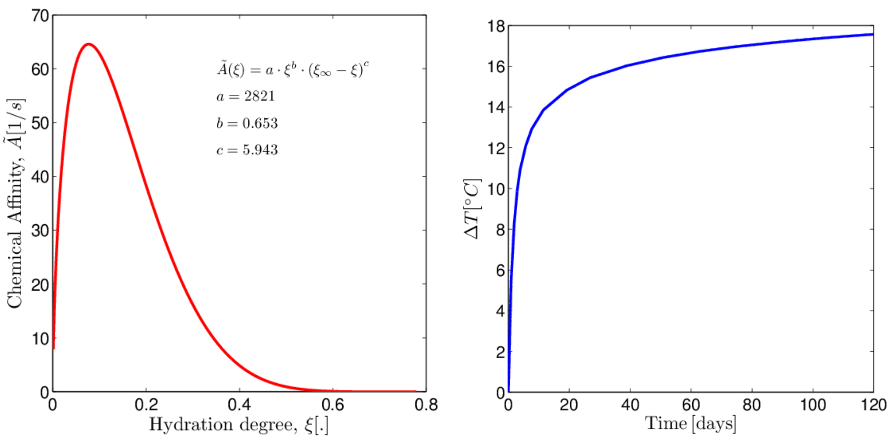
$$\underline{q} = -\underline{\underline{k}} \cdot \underline{grad(T)} \tag{2}$$

The rate of heat generation per unit volume will result in an evolution of temperature in the concrete mass because of the hydration reaction between cement and water. The rate of heat generation is generally described by an equation such as Eq.3, where l_ξ [kJ/m^3] is the (constant) heat of hydration per unit volume and $\dot{\xi}$ the hydration degree rate. The hydration degree rate may be described by means of an Arrhenius-type formula, such as the one presented in Eq.4, where E_a [J/mol] is the apparent activation energy, R [J/(mol ºC)] is the universal gas constant and $\tilde{A}$ [1/s] is the chemical affinity.

$$\dot{Q} = l_\xi \cdot \dot{\xi} \tag{3}$$

$$\dot{\xi} = \tilde{A}(\xi) \cdot \exp\left(-\frac{E_a}{R \cdot T}\right) \tag{4}$$

In this work, the chemical affinity evolution $\tilde{A}(\xi)$ is given by Fig.1a) and the respective increase of temperature ΔT under adiabatic conditions by Fig.1b).



a) Chemical affinity evolution b) Temperature evolution

Fig.1 Hydration model

2.2. *Thermo-chemo-mechanical coupling*

The mechanical properties of concrete result from its hardening, which is a consequence of the formation of hydration products. Therefore, there is an evident coupling between the exothermic hydration reaction and the evolution of the mechanical properties of the skeleton. The thermo-chemo-mechanical coupling used in this work is based on the one in Cervera et al. (2002). In their work, Cervera and his team introduce an aging parameter, which traduces the concrete stiffening during time, right after it has begun to cure. That aging degree (κ) is a function of both the temperature and the hydration degree, as described in Eq.5. The beginning of the curing phase is ruled by the parameter ξ_{set} , known as the mechanical percolation threshold. Thus, since this parameter enters in the definition of λ_{f_c} , it will also rule the beginning of the compressive strength evolution, which is in agreement with the definition given before: ξ_{set} gives the moment the concrete may have turned into a solid and begun stiffening (Cervera et al., 1999). In Eq.6, T_{ref} is the reference temperature for the determination of $f_{c,\infty}$, T_T is the maximum temperature at which concrete hardening may occur and n_T is a rate parameter. In Eq.7, A_f and B_f are constants.

$$\dot{\kappa} = \lambda_T(T) \cdot \lambda_{f_c}(\xi) \cdot \dot{\xi} \geq 0 \tag{5}$$

$$\lambda_T(T) = \left(\frac{T_T - T}{T_T - T_{ref}} \right)^{n_T} \tag{6}$$

$$\lambda_{f_c}(\xi) = A_f \cdot \xi + B_f \quad , \ \xi \geq \xi_{set} \tag{7}$$

Concluding, the mechanical properties of concrete such as the compressive strength (f_c), tensile strength (f_t) and elasticity modulus (E), will be functions of the aging degree, such as in Eq.8, Eq.9 and Eq.10, where the index ∞ designates the long-term values. In Fig.2 are plotted the results of three different isothermal tests at three different initial temperatures. As it can be seen, by using the aging degree concept the model is able to reproduce the mechanical properties dependency on temperature (i.e.,

f_c in this case). In this work, the material behaviour is assumed to be isotropic linear elastic, neglecting creep effects and orthotropy.

$$f_c(\kappa) = \kappa \cdot f_{c,\infty} \qquad (8)$$

$$f_t(\kappa) = \kappa^{2/3} \cdot f_{c,\infty} \qquad (9)$$

$$E(\kappa) = \kappa^{1/2} \cdot E_\infty \qquad (10)$$

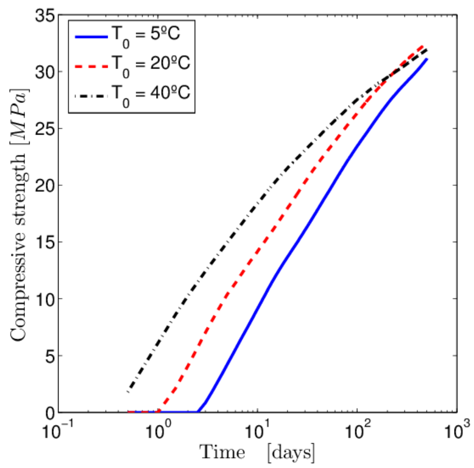


Fig.2 Compressive strength evolution, isothermal tests

3. PROBABILISTIC FRAMEWORK

In this work, two different probabilistic tools were used to propagate uncertainties in the model: the Random Balanced Design Fourier Amplitude Sensitivity Test (RBD-FAST) and the First Order Second Moment method (FOSM). In this section, these methods will be briefly presented to allow a better understanding of the results presented in the next section.

According to Christian (2004), reliability analysis tools may be divided in three categories: direct reliability analysis, event trees/fault trees and other statistical methods. Direct reliability analysis is characterized by the propagation of uncertainties in properties, geometries, loads, etc. through an analytical model of the system to be analyzed, giving as a result a probabilistic description of the behaviour of that system (Christian, 2004). Methods such as FOSM, FORM (First Order Reliability Method), LHS (Latin Hypercube Sampling method)and MC (Monte Carlo) are included in this category.

The FOSM method is a level II reliability method (in a scale of IV) which is of much more simple application than LHS or MC (level III) (Madsen et al., 1986). FOSM is based on the Taylor series development and is simplified by the use of the finite differences approach to solve the sequence of derivatives. Even if this method is of easy implementation and only requires $2 \times n + 1$ calculations, n being the number of random variables, it only gives information about the mean and standard deviation of the output of the model. This is because, in fact, each random variable is not described by its probability density function (pdf) but only by its mean and standard deviation values.

The RBD-FAST (Xu and Gertner, 2008; Mara, 2009) is a sensitivity test derived from the Fourier Amplitude Sensitivity Test (FAST) and is based on the variance decomposition, used here to compute the first order global sensitivity indices. The essence of this method relies on the fact that each random variable is sampled from a periodic function and therefore, the output of the model becomes also a periodic function. Then, a Fourier analysis can be performed over that output and the Fourier coefficients of the Fourier spectrum are determined. Those coefficients will allow the evaluation of the partial and total variances of the model output and the sensitivity index S_i (which is the ratio between the partial and total variances) assigned to each random variable may finally be computed. The differences between RBD-FAST and FAST arise at the very beginning of the process on the sampling of the random variables: in the RBD-FAST method a single integer frequency is assigned to all the random variables; then, the order of those sample values is randomly switched and then combined to be the input of the model; at the end of the process those random vectors are used to re-arrange the order of the input random variables and therefore assign the sensitivity indices of each one. Please refer to Xu and Gertner (2008) and Mara (2009) for more details. This procedure allows a faster convergence of the method when compared to FAST. Moreover, the interest of the RBD-FAST method is to perform a sensitivity analysis, which allows the understanding of the influence of each random variable in the model output.

4. APPLICATION TO A RCC DAM

The roller-compacted concrete, commonly called RCC, is a concrete which is compacted by vibratory roller devices and classified by the American Concrete Institute (ACI) as probably being «*the most important development in concrete dam technology in the past quarter century*» (ACI, 2007). One of the main advantages of this material is the rapid construction and consequent reduced cost.

In the following will be first presented the results of a deterministic construction simulation of a RCC dam. Secondly, the construction of the same dam is simulated taking into account some uncertain parameters by means of the probabilistic tools described previously in this work.

4.1. Deterministic approach

The simulation of the construction of the RCC dam was performed using the model previously described. For now, only the thermal transfer phenomena are simulated, that is, the mechanical behaviour of the material is not explicitly taken into account in the simulation. However, we are able to calculate the elastic modulus, compressive and tensile strengths by means of the aging (κ) parameter. The rate of construction is of 1 layer per day (without stops) and each layer has 0.6m height. In Fig.3 is represented the dam model. The numbers inside the RCC material correspond to the points were the results are further analyzed.

The initial conditions of this model are described by Eq.11. The boundary conditions at the dam faces and the foundation surface are described by Eq.12 , where h[W/(m^2 ºC)] is the convection coefficient, equal to 4 for RCC and 20 for the foundation. The bottom and lateral faces of the foundation are insulated (as indicated in Fig.3) and the correspondent boundary condition is given by Eq.13.

$$T(\underline{x}, t=0) = T_0 \tag{11}$$

$$\frac{\partial T}{\partial n} = \frac{h}{k}\left(T_{ext}(t) - T\right) \tag{12}$$

$$\frac{\partial T}{\partial n} = 0 \tag{13}$$

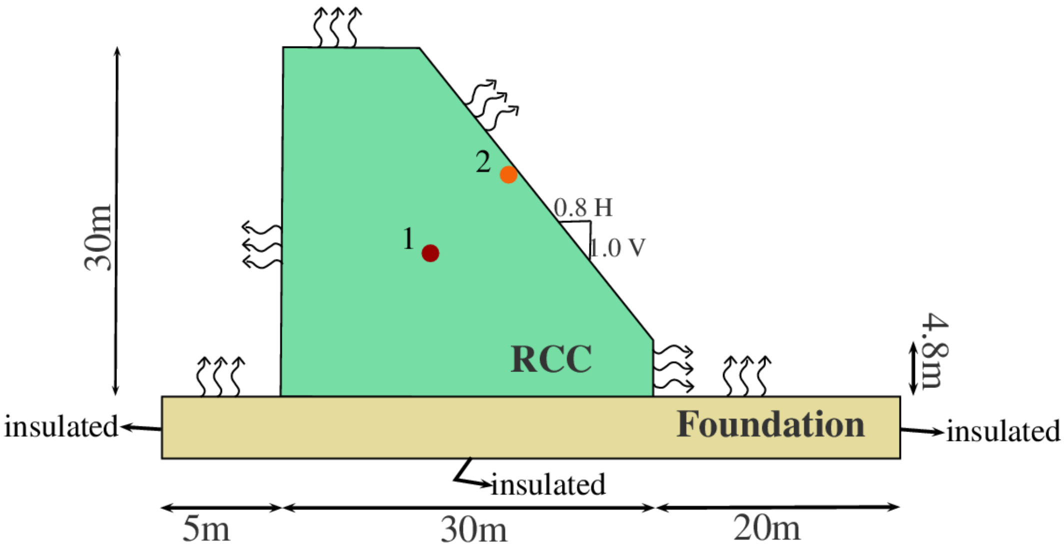


Fig.3 RCC dam model

Table 1 Model parameters

	ρ [kg/m^3]	c[J/(kg°C)]	k [W/(m°C)]	T_0[°C]	h[W/(m^2 °C)]	
RCC	2400	921	2.3	21	4	
Foundation	2600	720	2.9	15	20	
	l_ξ [J/m^3]	E_a / R [K]	ξ_∞	ξ_{set}	A_f	B_f
RCC	6.24E7	5000	0.779	0.2	3.0	0.23
	$f_{c,\infty}$ [MPa]	E_∞ [GPa]	T_{ref} [°C]	T_T [°C]	n_T	
RCC	34.6	29.6	20	100	0.42	

In this study, four case scenarios concerning the ambient temperature (T_{ext}) were studied: three constant daily temperatures of 10, 20 and 30ºC and a sinusoidal temperature evolution varying daily from T_{min} = 10 to T_{max} = 30ºC (described by Eq.12, with a frequency $f = 1/\text{day}$ and a phase $\phi = 0.5$).

$$T_{ext} = \frac{T_{\max} + T_{\min}}{2} + \frac{T_{\max} - T_{\min}}{2} \cdot \sin\left(2\pi \cdot f \cdot t + \phi\right) \quad (14)$$

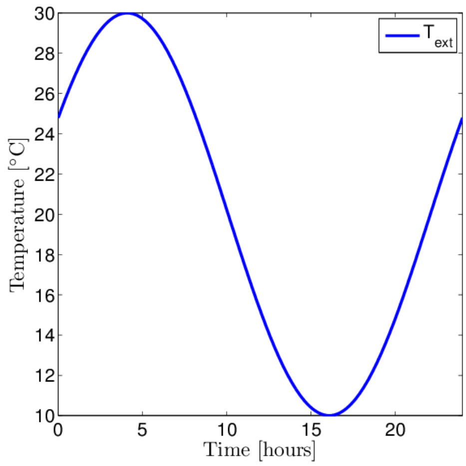
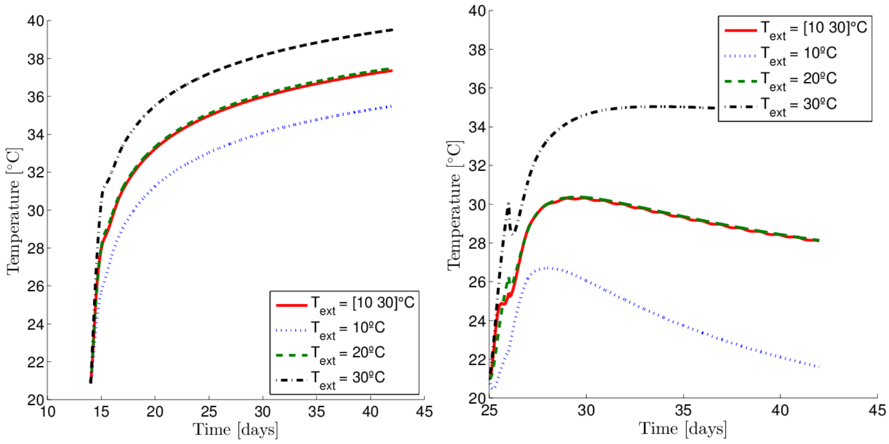


Fig.4 Daily ambient temperature variation

In Fig.5 are plotted the temperature evolutions in the two points depicted in Fig.3, for all of the four case scenarios. The results clearly show the influence of the ambient temperature. As expected, in general lines, the higher the T_{ext}, the higher the obtained temperature at each point. In the case of the sinusoidal temperature variation, its influence is more evident in the point located on the upstream face of the dam (red bold curve that stars at 25 days).



a) Point 1, at the center of the dam b) Point 2, near the upstream face

Fig.5 Temperature evolution

In Fig.6 are plotted the temperature contours on 3 different construction stages. The heat transfer with the environment is well reproduced at each stage as it can be seen by lower temperature near the upstream, downstream and upper surfaces. On the bottom interface between the foundation and the dam body can be noticed the influence of the foundation temperature (15ºC) on the temperature of the first RCC layers.

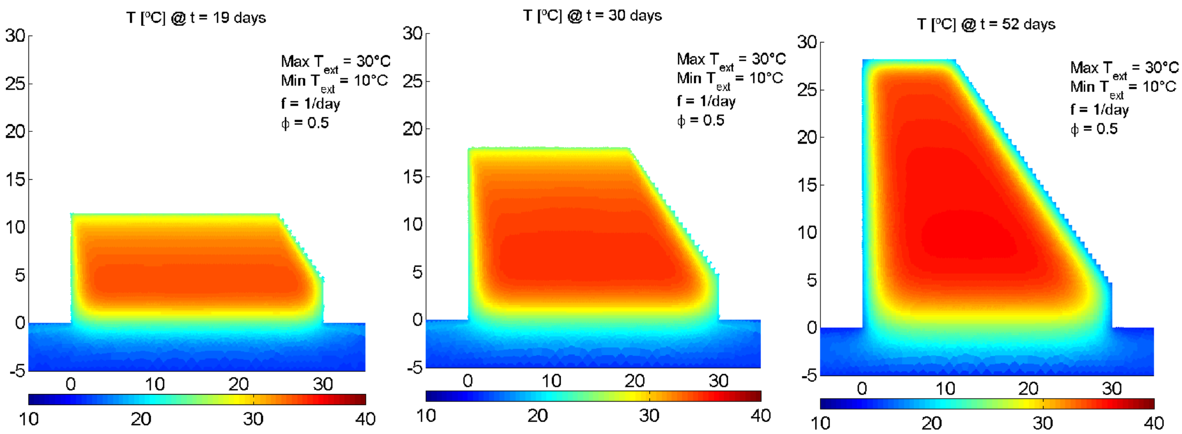


Fig.6 Temperature contours evolution during construction

In Fig.7 are presented the contours of the compressive strength, tensile strength and Young's modulus respectively, at the end of the simulation. These results are coherent since the effect of the layered construction and boundary conditions is well represented, as it can be noticed that the maturation of the layers decreases with the increase of the dam height.

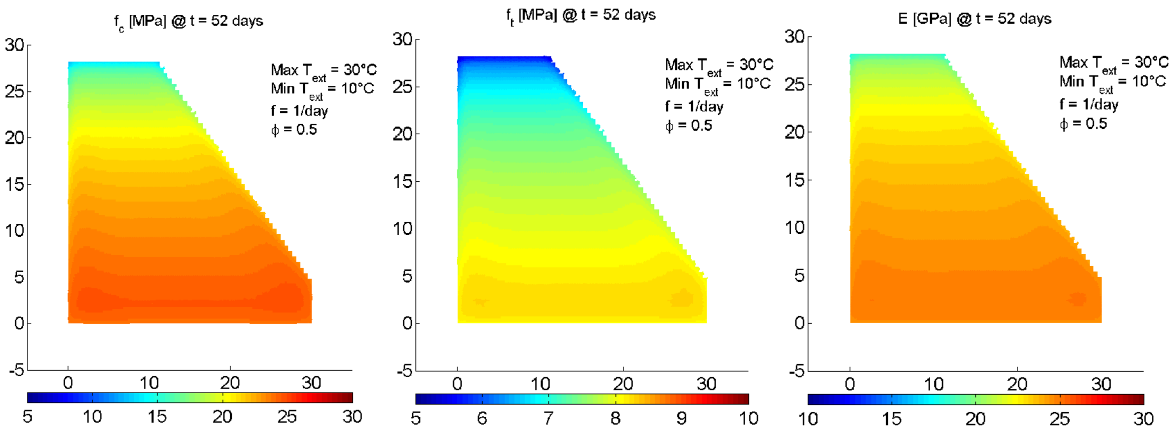


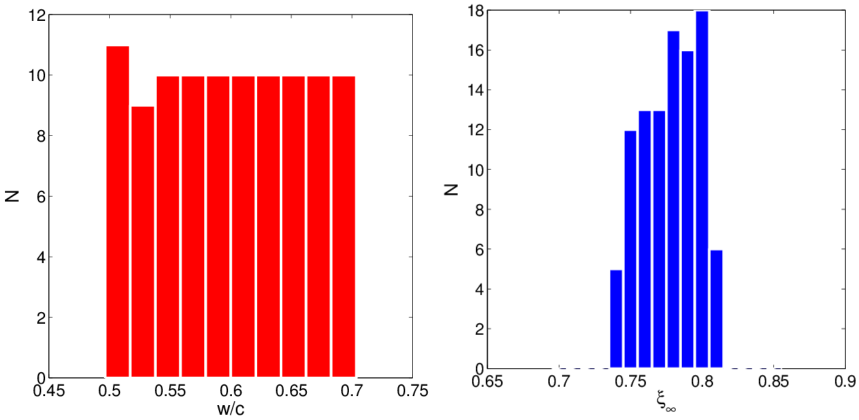
Fig.7 Properties' contours at the end of construction

4.2. Probabilistic approach

As already mentioned before, uncertainties are propagated in the model by means of FOSM and RBD-FAST methods. The results obtained by using those two methods will be presented in this section.

In this work, three uncertain parameters were considered: the water-to-cement ratio the cement content and the thermal conductivity. The water-to-cement ratio (w/c) influences some of the model parameters, such as the final hydration degree ξ_∞ (Eq.13 from Pantazopoulo (1995)) that will in turn influence the hydration model. In Fig.8b is plotted the obtained distribution for ξ_∞ given the uniform distribution of w/c. The cement content will in turn influence the latent heat per cubic meter of concrete (l_ξ). All the three random variables are described by a uniform distribution (for the RBD-FAST method) with mean and covariation coefficient (CV) described in table 2. For the FOSM mehod, the same mean and CV values are applied to each random variable.

$$\xi_\infty = \frac{1.031 \cdot w/c}{0.194 + w/c} \tag{13}$$



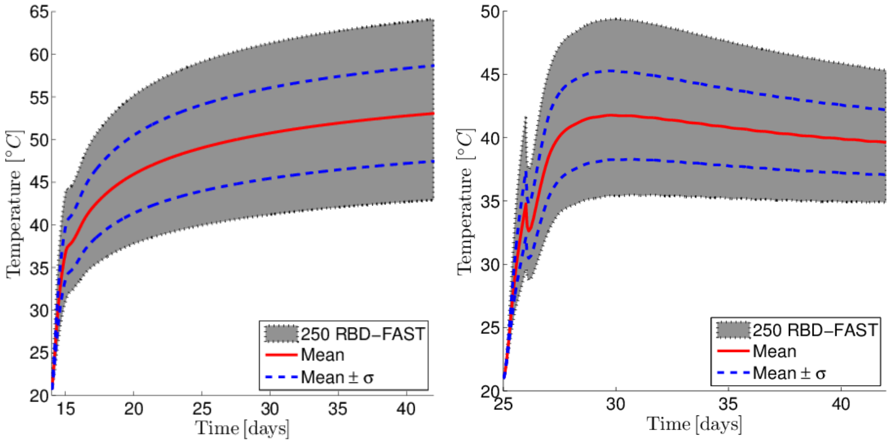
a) Uniform distribution for w/c   b) Obtained distribution for ξ_∞

Fig.8 Histogram of r.v.

For the RBD-FAST, 250 computations were performed and the results of the temperature evolution for both points 1 and 2 are plotted in Fig.9. For these computations, the external loading case consists on a sinusoidal varying ambient temperature evolution with an increasing daily mean temperature.

Table 2 Input uncertain parameters used in FOSM and RBD-FAST

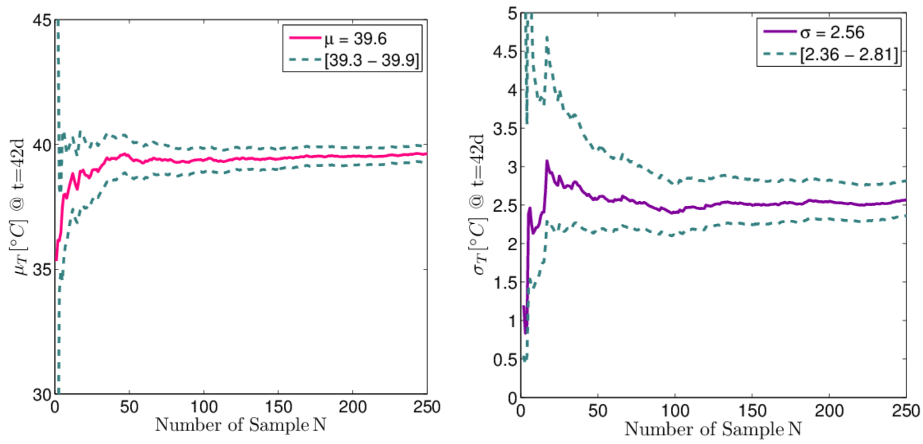
	μ	CV	σ
w/c [.]	0.6	0.1	0.06
c [kg/m^3]	220	0.15	33
k [W/mºC]	2.96	0.25	0.74



a) Point 1, in the center of the dam b) Point 2, near the upstream face

Fig.9 Temperature evolution, 250 RBD-FAST

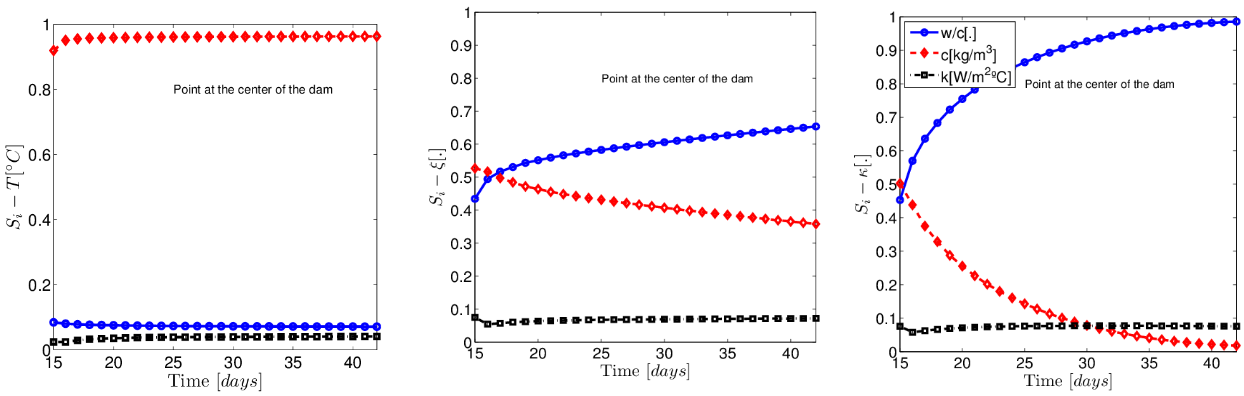
The statistical convergence of the 250 RBD-FAST computations were verified and is here plotted in Fig.10. It can be concluded that the mean and standard deviations of temperature at the end of construction for the point in the surface of the dam are converging.



a) μ_T [ºC], point 2 b) σ_T [ºC], point 2

Fig.10 Confidence intervals for the mean and standard deviation estimators of temperature at the end of construction

As mentioned before, the interest of the RBD-FAST simulations is to perform a global sensitivity analysis. The first order sensitivity indices for the three random variables studied here are plotted in Fig.11. The sensitivity indices described here concern the response of the point 1 at the center of the dam. The results for the point 2 are not presented here but their evolution were very close to the ones found in point 1.

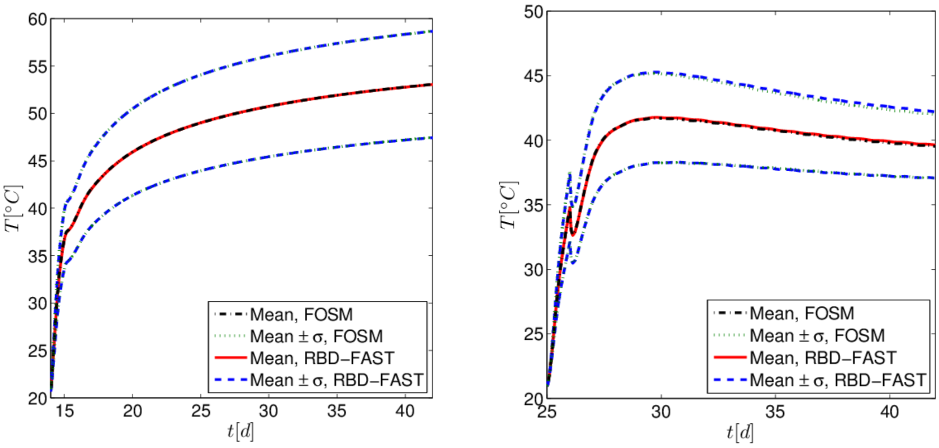


a) S_i for $T\left[{}^{\circ}C\right]$, point 1 b) S_i for $\xi\left[.\right]$, point 1 c) S_i for $\kappa\left[.\right]$, point 1

Fig.11 First-order sensitivity indices

From the results of the sensitivity analysis it may be concluded that the thermal conductivity has no influence on the evolution of temperature, hydration degree and aging degree. The water-to-cement ratio has little influence on the temperature evolution, the cement content being the most influent parameter on that output. In what concerns the hydration and aging degrees, they are both sensitive to the water-to-cement ratio and cement content at the beginning of the analysis. The sensitivity index grows during the construction for the water-to-cement ratio while it decreases for the cement content.

Finally, a comparison between the FOSM and RBD-FAST output results (mean and standard deviation) are plotted in Fig.12. As mentioned before, the FOSM method is a simple method that allows the evaluation of the mean and standard deviations of the model output by propagating uncertainties described by only their mean and CV. This analysis shows that the results are the same, which enhances the power of the FOSM as a useful tool to predict the output of the model. However, FOSM only gives information about the two first moments (mean and standard deviation) of the model output, which can only be considered as an approximation of the response.



a) Point 1, in the center of the dam b) Point 2, near the upstream face

Fig.12 Temperature evolution, FOSM vs. RBD-FAST

CONCLUSION

In this work a thermal analysis of the construction of a RCC dam was performed. A thermo-chemo-mechanical model based on the one presented by Cervera et al. (2002) is used.

The model of the dam's construction was ran in both deterministic and probabilistic manners. Several external loading cases were applied concerning the ambient temperature variation: a constant temperature during all the construction procedure and a daily varying temperature with a constant or an increasing daily mean temperature. The temperature evolution for each of the loading cases was analyzed in two different points within the dam: one at the center of the dam, which presents an adiabatic behaviour, and another one near the upstream face of the dam which is more influenced by the external ambient temperature evolution.

Two different probabilistic tools were used in this work: the First Order Second Moment (FOSM) method and the Random Balanced Design FAST (RBD-FAST) method. From the probabilistic approaches used in this studied the following main conclusions may be drawn: the thermal conductivity has very little influence on the temperature, hydration degree and aging degree evolutions; even if it is a level II reliability method and only gives information about the mean and standard deviation of the model output, the FOSM method is very useful to have an idea of the model output before running the much heavier level III reliability methods, such as LHS or RBD-FAST.

In further works, the authors expect to develop the mechanical part of the model, accounting creep effects. Concerning the probabilistic approach, random fields for the input random variables will be considered.

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