The 2012 World Congress on
Advances in Civil, Environmental, and Materials Research (ACEM' 12)
Seoul, Korea, August 26-30, 2012

Reliability based Stabilization of Failed Slopes

Yinggang Zhang [1)], Jie Zhang [2)] and Hongwei Huang [3)]

[1), 2), 3)] *Department of Geotechnical Engineering, Tongji University, Siping Road 1239, Shanghai, China*
[1)] *10zyg@tongji.edu.cn*

ABSTRACT

Stabilization of failed slopes is often complicated by the presence of uncertainties of parameters of the slope. In this study, the slope failure event is viewed as a field test performed directly on the slope, which is then used to back analyze the slope stability model parameters. The output from the back analysis is the probabilistic distribution of the uncertain slope stability parameters. The back analyzed distributions of the soil parameters are then used to analyze the failure probability of the slope when various stabilization parameters are adopted. With the method illustrated in this paper, appropriate stabilization measures can be chosen to meet the target reliability level. By comparing slope stabilization with and without considering the slope failure information, the value and importance of considering the slope failure information is also highlighted.

1. INTRODUCTION

Landslides occur frequently in China. Among them large-scale landslides are dominant and extremely important (Huang 2007). Though slope failing may involve many factors, heavy rainfall always play an essential role in the process, especially in the South China due to subtropical monsoon climate.

Slope stability assessment is often associated with a considerable amount of uncertainties, including both soil strength parameters and pore water pressure parameters. Some recent researches highlight the primary sources of geotechnical uncertainties (Phoon 1999). Reliability analysis is a probabilistic based approach to account for variability and has been applied in geotechnical engineering over the years (Phoon 2008). In order to reduce the variability of parameters and obtain a more realistic reliability analysis of slope stability, more knowledge about the slope should be collected. In addition to the data from geotechnical tests, the past performance of the slope, i.e., survive or fail at a certain state, may provide valuable information in safety assessment (Zhang 2011).

The objective of this paper is to illustrate how a large-scale slope can be repaired considering its past failure information through probabilistic methods with explicit consideration of uncertainties from various sources. The structure of this paper is as

[1)] Graduate Student
[2)] Lecturer
[3)] Professor

follows. First, the background of the slope under investigation is briefly introduced. Then, detailed procedure used to back analyze the slope failure is described. Finally, the slope is repaired with and without considering its past failure information, where the value of back analysis is highlighted. The case study reported in this paper can provide valuable reference to the design of similar slopes.

2. EXAMPLE PROBLEM AND SITE DESCRIPTION

The 6# landslide of Shang-Shan Highway lies in Zhejiang Province, China. From May to June in 2000, the slope deformation accelerated due to continuous rainfall. Finally, obvious overall landslide occurred on June 10th, 2000. The typical geological cross section A2-A2’ of 6# landslide is adopted for the reliability analysis in this paper, as shown in Fig. 1.

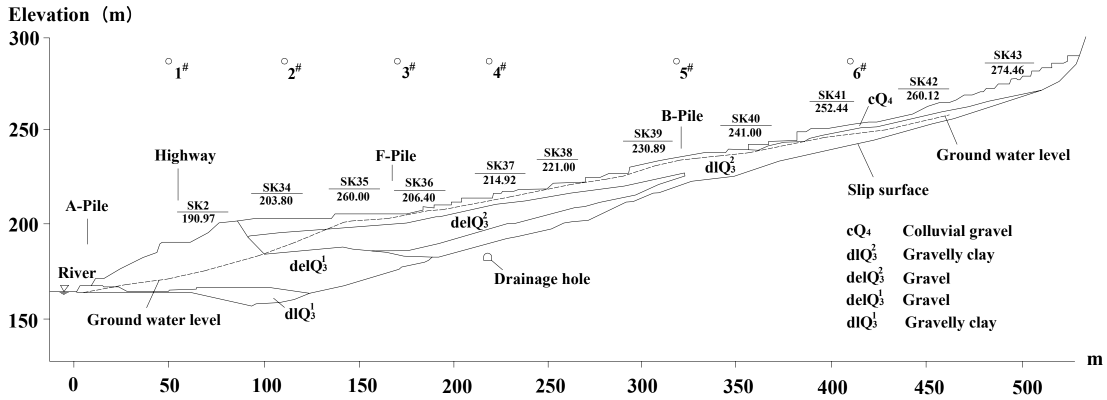

Fig. 1 Cross section A2-A2’ of 6# landslide

The postfailure investigation indicates that the slip surface of the overall landslide is formed along the contact surface between the overlying soil and the bedrock as marked in Fig. 1. The soil is mainly composed of gravel and gravelly clay, of which the average unit weight is 19.3kN/m^3. After the failure event, a serial of engineering measures have been adopted including drainage holes and stabilizing piles to prevent the sliding and reinforce the slope in December, 2001.

3. RESPONSE SURFACE METHOD

The Limit Equilibrium Method for analyzing slope stability has been proposed by many researchers in the past decades for its simplicity in practice and clear mechanical concept. In this study the factor of safety (F_s) is calculated using Morgenstern-Price method (1965) which is implemented by the computer program SLOPE/W (Geo-slope Ltd 2004).

Let $g(\theta)$ denotes the slope stability model, i.e., the factor of safety calculated by Morgenstern-Price method, where θ denotes uncertain variables including pore pressure ratio r_u and soil strength parameters c and ϕ in this study. Accounting for errors denoted by ε in model prediction, the actual factor of safety of the slope can be written as:

$$F_s = g(\boldsymbol{\theta}) + \varepsilon \tag{1}$$

To provide an explicit expression for analysis, a second-order polynomial function is adopted as suggested by Xu (2006) for relatively linear slope stability model:

$$g(\boldsymbol{\theta}) \approx a_0 + \sum_{i=1}^{3} b_i \theta_i + \sum_{j=1}^{3} c_j \theta_j^2 \tag{2}$$

The F_s of the slope is first estimated by Morgenstern-Price method to get the unknown coefficients in Eq. (2) at the following points: $\{\mu_{r_u}, \mu_c, \mu_\phi\}, \{\mu_{r_u} \pm \sigma_{r_u}, \mu_c, \mu_\phi\}, \{\mu_{r_u}, \mu_c \pm \sigma_c, \mu_\phi\}$, and $\{\mu_{r_u}, \mu_c, \mu_\phi \pm \sigma_\phi\}$.

Let $\mathbf{x} = \{\theta, \varepsilon\}$, the limit state surface which separates the safe and failure domains can be represented as:

$$G(\mathbf{x}) = F_s - 1 = 0 \tag{3}$$

Advanced first-order second-moment (AFOSM) method is used to calculate the reliability index β in this paper. Failure probability (p_f) can then be calculated as follows when $\mathbf{x}$ follows a multivariate normal distribution:

$$\beta = \min_{G(x)=0} \sqrt{y^T R_y^{-1} y} \tag{4}$$

$$p_f = 1 - \Phi(\beta) \tag{5}$$

where $\mathbf{y}$ = reduced variable of $\mathbf{x}$; $\mathbf{R_y}$ = correlation matrix of $\mathbf{y}$; and Φ = cumulative distribution function (**CDF**) of a standard normal variable.

The minimization when determining β is implemented automatically in a spreadsheet which is suggested by Low (1997). The process is to find a point which is often called the *design point* in limit state surface, wherein the shortest distance from the reduced variables to the origin of the standard normal space is obtained as the reliability index β.

4. QUANTIFICATION OF UNCERTAIN PARAMETERS

The determination of the pore pressure ratio r_u at the moment of slope failure seems difficult because the ground water level was not measured then. Based on the monitoring data of the ground water level after the slope failure, the prior mean value of r_u is assumed to be 0.35.

Large numbers of soil samples from different positions are collected for laboratory tests. The prior mean value of c and ϕ in reliability analysis is the standard value of the saturated shear strength of the test results considering that the slip surface lies below the water table.

We assume that θ follows an uncorrelated multivariate normal distribution. Therefore, the prior distribution of θ can be presented with a mean of μ_θ = {0.35, 21.6, 7.3} and an assumed standard deviation σ_θ = {0.1, 9, 3}.

The model correction factor ε for Morgenstern-Price method is assumed to be a normal variable with a mean of 0.02 and a standard deviation of 0.07 in this paper according to the study of Zhang (2009).

5. BACK-ANALYSIS CONSIDERING FAILURE INFORMATION

In the back-analysis based on the failure information, i.e., the factor of safety of the slope at the moment of slope failure is unity, the deterministic method seems hard to be implemented because only one piece of information is not adequate to determine all the uncertain variables. Thus, an efficient probabilistic back-analysis method (Zhang 2010) on the basis of Bayes' theory is adopted.

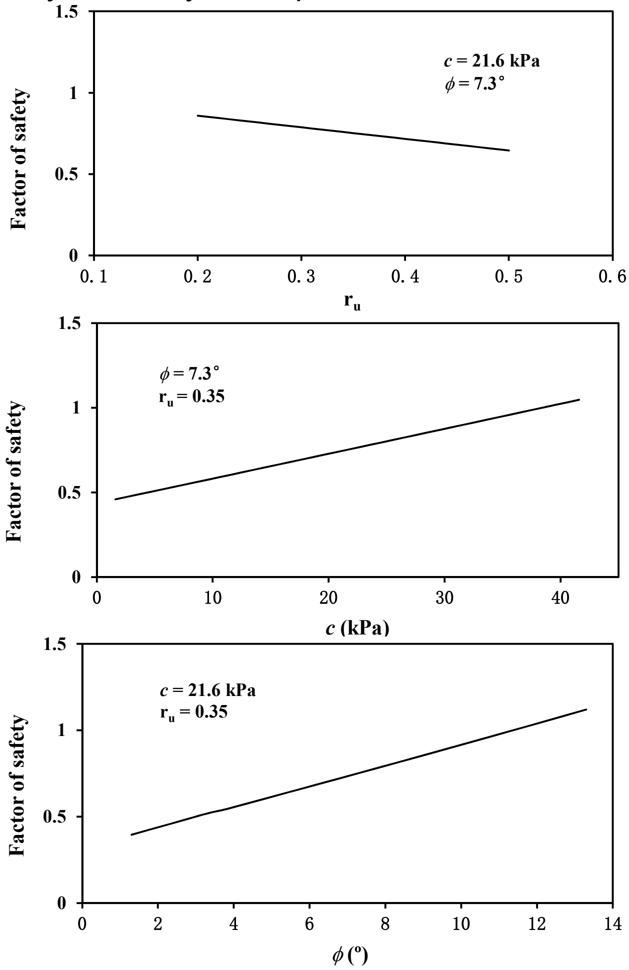

Fig. 2 Relationships between factor of safety and variables

Let $\mu_{\theta|d}$ and $C_{\theta|d}$ denote the improved mean and covariance matrix of θ, respectively. For approximately linear slope stability model, they can be obtained as follows:

$$\mu_{\theta|d} = \mu_\theta + C_\theta H^T (H C_\theta H^T + \sigma_\varepsilon^2)^{-1} [1 - g(\mu_\theta) - \mu_\varepsilon] \quad (6)$$

$$\mathbf{C}_{\theta|\mathbf{d}} = \left(\frac{H^T H}{\sigma_\varepsilon^2} + \mathbf{C}_\theta^{-1} \right)^{-1} \tag{7}$$

$$H = \left. \frac{\partial g(\theta)}{\partial \theta} \right|_{\theta=\mu_\theta} \tag{8}$$

Where $\mathbf{H}$ = row vector representing the sensitivity of $g(\theta)$ with respect to θ at $\boldsymbol{\mu}_\theta$.

The relationships between factor of safety calculated by Morgenstern-Price method and variables are shown in Fig. 2, which seem to be rather linear. Hence, Eq. (2) is applicable to obtain the response surface.

Based on the probabilistic back-analysis method mentioned about, a spreadsheet is specially designed to calculate $\boldsymbol{\mu}_{\theta|\mathbf{d}}$ and $\mathbf{C}_{\theta|\mathbf{d}}$. The layout of the spreadsheet is shown in Fig.3. Since the improved θ also follows a multivariate normal distribution, the posterior distribution of each parameter can be obtained based on the results of back-analysis. The prior and posterior probability density for r_u, c and ϕ are plotted in Fig. 4.

	A	B	C	D	E	F	G	H	I	G	K
1		μ_θ	C_θ					r_u	c	ϕ	
2	r_u	0.35	0.01	0	0		H	-0.712	0.0147	0.06	
3	c	21.6	0	81	0		$g(\mu_\theta)$	0.752			
4	ϕ	7.3	0	0	9		d_{obs}	1			
5											
6		μ_ε	σ_ε				$HC_\theta H^T$				
7		0.02	0.07				0.055084				
8											
9		$H^T H$					$\mu_{\theta\|d}$	$C_{\theta\|d}$			
10		0.5072	-0.01	-0.043			0.322929	0.0092	0.1414	0.064	
11		-0.01	0.0002	9E-04			26.12589	0.1414	57.364	-10.74	
12		-0.043	0.0009	0.004			9.35598	0.0642	-10.74	4.122	
13											

Fig. 3 Spreadsheet of probabilistic back-analysis of slope failure

As shown in Fig. 4, the improved θ is correlated normally distributed with a mean of $\boldsymbol{\mu}_\theta$ = {0.323, 26.126, 9.356} and a standard deviation $\boldsymbol{\sigma}_\theta$ = {0.096, 7.574, 2.03}. The results also reveal the dependence information between c and ϕ which are assumed to be statistically independent at the beginning, while $\mathbf{C}_{\theta|\mathbf{d}}$ indicates that the value of $\rho_{c,\phi}$ changes from zero to -0.698.

6. DESIGN OF STABILIZING PILES

The reliability analysis is first carried out in this section on the slope stabilized with the designed piles shown in Fig. 1. A comparison is made to observe the results with and without considering the slope failure information. Then a reliability based design of piles is suggested to meet a target reliability level.

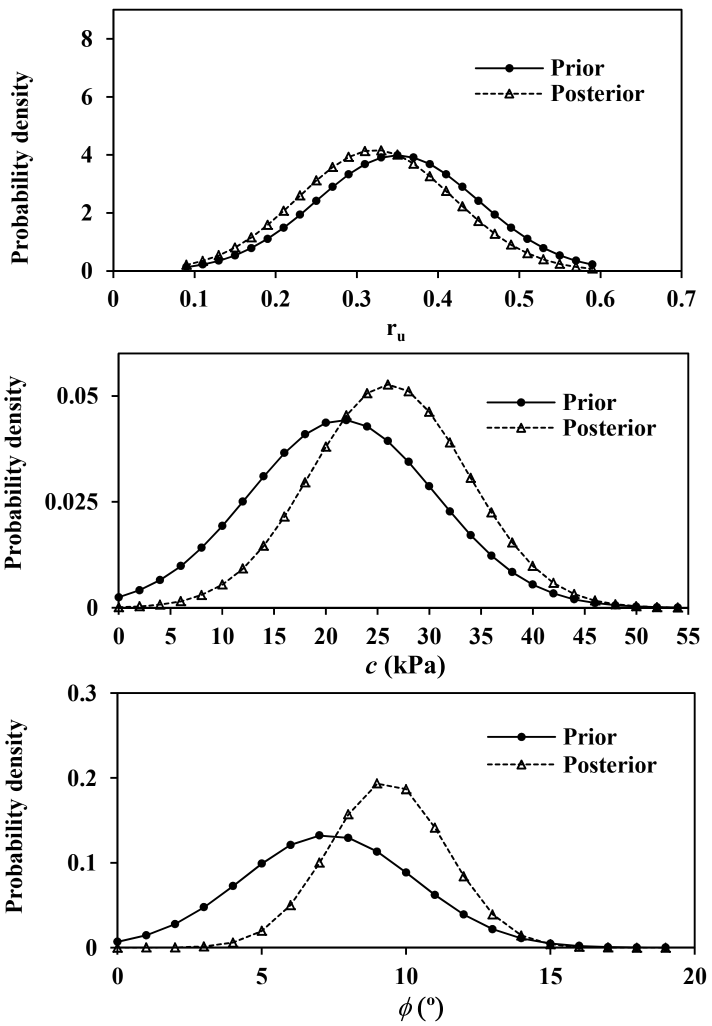

Fig. 4 Prior and posterior probability density of $\mathbf{r}_u$, c and ϕ

When limit equilibrium methods like Morgenstern-Price method is implemented to account for the contribution of piles to the stability of slope, the piles are assumed to only provide a reinforcing resistance (Poulos 1995). The piles in Fig. 1 are designed based on the Chinese *Specifications for Design of Highway Subgrades* (JTGD30—2004), the target factor of safety is set to be 1.3 considering the grade of Shang-Shan Highway. The adopted values of c and ϕ in the design are 26 kPa and 9.1°, respectively. As r_u is not reported, 0.35 is assumed considering that the ground water level does not change significantly. A lateral resisting force of 14334 kN which stands for the effect of piles is calculated out after several trials to meet the target F_s of 1.3.

To analyze the reliability of the slope stabilized with piles, the stability model is rebuilt to take into account the added lateral resisting force. Parametric study shows that the relationships between factor of safety calculated by Morgenstern-Price method and variables remain linear when the slope is stabilized. Therefore, a response surface can be obtained in the form of Eq. (2).

	A	B	C	D	E	F	G	H	I	G	K	L
1	**AFORM**		The reliability index of the slope is calculated by the optimization tool: Solver. The setting in Solver is "minimize C11									
2			by changing C7, D7, E7 and F7, subjected to B11=0" in the first reliability analysis. It is similar in the second one.									
3	**Reliability analysis without considering the failure information**											
4			r_u	c	ϕ	ε		**R**				
5		μ_x	0.35	21.6	7.3	0.02		1	0	0	0	
6		σ_x	0.1	9	3	0.07		0	1	0	0	
7		y	0.1667995	-0.3134763	-0.4297242	-0.1660049		0	0	1	0	
8		x	0.3666799	18.778713	6.0108275	0.0083797		0	0	0	1	
9												
10		H	-0.74	0.0144556	0.0584	0.05	6.173E-06	0.0001667				
11		a	0.6255533									
12												
13		G(x)	β	pro. of F								
14		5.613E-08	0.5816442	28.04%		$g(\theta)=-0.74r_u+0.01446c+0.0584\phi+0.05Ru^2+6.17\times10^{-6}c^2+0.000167\phi^2+0.625553$						
15												
16	**Reliability analysis considering the failure information**											
17			r_u	c	ϕ	ε		**R**				
18		μ_x	0.323	26.126	9.356	0.02		1	0.1950906	0.3305926	0	
19		σ_x	0.096	7.574	2.03	0.07		0.1950906	1	-0.698213	0	
20		y	0.8068164	-0.1938563	-0.7367441	-2.4648258		0.3305926	-0.698213	1	0	
21		x	0.4004544	24.657732	7.8604095	-0.1525378		0	0	0	1	
22												
23		H	-0.9465061	0.014266	0.056982	0.0542535	8.716E-06	0.000364				
24		a	0.6954131									
25												
26		G(x)	β	pro. of F								
27		8.774E-07	3.549879	0.02%		$g(\theta)=-0.947Ru+0.01427c+0.057\phi+0.0543Ru^2+8.716\times10^{-6}c^2+0.000364\phi^2+0.695413$						
28												

Fig. 5 Spreadsheet of probabilistic back-analysis of slope failure

As shown in Fig. 5, the AFOSM for slope reliability analysis is implemented in a spreadsheet. The reliability index is improved from 0.582 to 3.550 when the variables are updated based on the failure information. Considering the reliability request in design, the improved β seems to be more reasonable. On the contrary, the reliability assessment without site performance information underestimates the stabilization measures because the calculated failure probability of about 28% is unacceptable. Consequently, a more conservative design will be proposed to lower the failure probability which will increase the cost of the stabilization project.

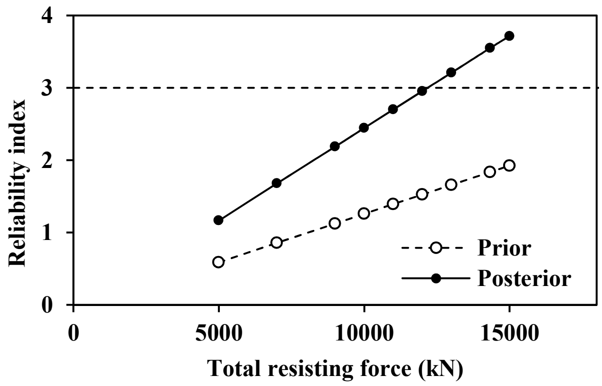

Fig. 6 Reliability index at various lateral resisting forces

On the basis of AFOSM, the reliability index β or failure probability of the slope when various lateral resisting forces are provided by the piles can also be calculated. The relationship between the lateral resisting force and reliability index is plotted in Fig. 6.

As would be expected, the reliability index increases when more resisting force is provided by piles. To meet a target reliability index of 3.0, the lateral resisting force should be about 12500 kN.

SUMMARY AND CONCLUSIONS

The research work reported in this study can be summarized as follows:

(1)For the landslide assessed in this paper with the well- developed slip surface, the relationships between factor of safety and model input parameters, i.e., r_u, c and ϕ, are quite linear. A second-order polynomial function can be adopted as the response surface.

(2) Based on the failure information of 6# landslide of Shang-Shan Highway, the variables in the slope stability model are updated through probabilistic back-analysis which is implemented in a convenient spreadsheet.

(3) More reliable and reasonable assessment of the stabilization measures is obtained with the improved parameters. Then appropriate stabilization measures can be chosen to meet the target reliability level in design.

REFERENCES

Geo-slope Ltd (2004), "Slope/W for Slope Stability Analysis: User's Guide", Geo-Slope Ltd, Calgary, Canada.

Huang, R. Q. (2007), "Large-scale landslides and their sliding mechanisms in china since the 20th century", *Chinese Journal of Rock Mechanics and Engineering*, **26**(3), 433-454. (In Chinese)

Low, B. K., Tang, W. H. (1997), "Efficient reliability evaluation using spreadsheet", *Journal of Engineering Mechanics*, **123**(7), 749–752.

Morgenstern, N. R., Price, V. E. (1965), "The analysis of the stability of general slip surfaces", *Geotechnique*, **15**(1), 79–93.

Phoon, K. K., Kulhawy, F.H., (1999), "Characterization of geotechnical variability", *Canadian Geotechnical Journal*, **36**, 612–624.

Phoon, K. K. (2008), *Reliability-Based Design in Geotechnical Engineering*, Taylor & Francis, New York.

Poulos, H. G. (1995), "Design of reinforcing piles to increase slope stability", *Canadian Geotechnical Journal*, **32**, 808-818.

Xu, B., Low, B. K. (2006), "Probabilistic stability analysis of embankment based on finite-element analysis", *Journal of Geotechnical and Geoenvironmental Engineering*, **132**(11), 1444–1454.

Zhang, J. (2009), "Characterizing geotechnical model uncertainty", Ph.D. thesis, The Hong Kong University of Science and Technology, Hong Kong.

Zhang, J., Tang, W. H., Zhang, L. M. (2010), "Efficient probabilistic back analysis of slope stability model parameters", *Journal of Geotechnical and Geoenvironmental Engineering*, **136**(1), 99–109.

Zhang, J., Zhang, L. M., Tang, W. H. (2011), "Slope Reliability Analysis Considering Site-Specific Performance Information", *Journal of Geotechnical and Geoenvironmental Engineering*, **137**, 227-238.