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On flag flutter as a two-degree of freedom system

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ABSTRACT

Flatter of a flag is investigated in order to extract wind power from the flapping. In order to flap a flag regularly as large amplitude, a flag with an aspect ratio of 2.17 connects loosely to a support at its four corners. In the case, the flag flaps as a large amplitude in H-90 mode of sway and pitch. The frequency of the flatter increases linearly against wind speed and the amplitudes are constant for any wind speed. The characteristics of the flatter can be modeled by a vibrating system of a flat plate with two degrees of freedom supported by a spring with no stiffness. According to the analysis of the model, the aero-dynamic coefficients in the system are constant for any wind speed.

1. INTRODUCTION

Demands for natural energy like wind power increase rapidly in recent years. Various types of wind mills have been invented and used for a long time to extract wind energy. On the other hands, several devices except for a wind mill were investigated in recent years. Isogai(2003,2011) and Matsumoto(2010,2011) proposed a device to use flatter of sway and torsion of a wing as a two degree of freedom system. The device can extract a large power from the sway flapping when the torsion is forced to vibrate by a very small power in a special frequency and a phase. Aoki(2009,2011) propose a device to use a wing-stroke mechanism.

This paper shows an investigation for flatter phenomenon of a flag in order to extract wind power from the flapping. In order to use the flapping for wind power generation, the regular and stable flapping with large amplitude in a certain frequency should be necessary. However an ordinal flag with a small aspect ratio connected to a rod at a windward side flaps unstable in a relatively complicated multiple orders traveling wave mode of a membrane. In order to obtain the regularly and stable flapping

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with large amplitude, we have carried out experiment for flags with a various aspect ratio and a way of connection. It was found that a flag can flap very regularly as large amplitude as H-90 mode of sway and pitch when a flag with an aspect ratio of 2.17 and connects loosely to a support at its four corners.

In the paper, the mode, the frequency and the amplitude of the flag are shown according to the experimental results. The analysis using the flatter theory of two degrees freedom model for sway and torsion agrees well with the experimental results.

2. EXPERIMENTAL SET UP

The experiment was carried out in the wind tunnel of DPRI, Kyoto University with a open test section of a 1m octagonal jet nozzle. The flag of which size is 37cm x 17cm and mass is 6.2g is connected by three strings to a frame shown in Figure 1. The power of the flapping is extracted by a generator connected to the flag by a rod. The amplitude of the flapping is measured by three laser displacement sensors. The movement of the core of the generator is measured by a rotation sensor. The sampling ratio is 200Hz and the sampling number is 2000. The flapping is monitored by two high speed cameras (CasioEX-ZR100) from the upper and the side or the downstream which can take 480 frames per a second in a resolution of 224 x 160.

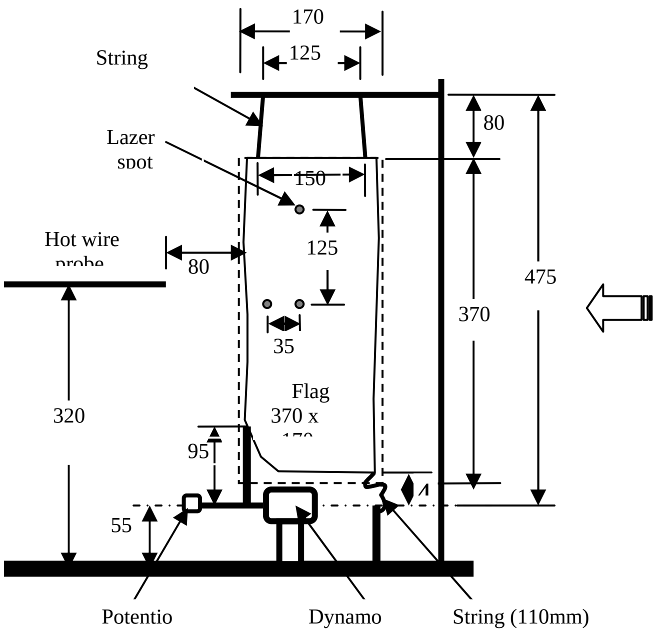

Figure 1 Experimental set up

A hot wire probe was set behind the flag to measure the velocity profile in the wake to estimate the energy extracted by the flag. The wind speed was changed from 2m/s to 12m/s ,which measured by a pitot static tube set upstream of the flag.

3. EXPERIMENTAL RESULTS

3.1 Flag motion during the flapping

Figure 2 shows the snap shots of the flag motion during a half period in a cycle of the flapping taken from the upper (left in Figure 2), the downstream (middle) and the side (right) when wind speed is 2m/sec. According to the shots from the upper, the motion of the flag seems to be similar to the mode of two degrees of freedom of sway and torsion. However, the flag is very flexible to catch wind very efficiently as shown in Figure 2. The flag inflates outside as like a sale of a yacht at time 1. As progress of time, the flag is inflating to a reverse direction from the windward side to the leeward side. When the reverse inflation reaches to the leeward end, the inflated air goes towards the wake and the flag inflates again as like a sale. The patterns of the motion do not so much changed for any wind speed.

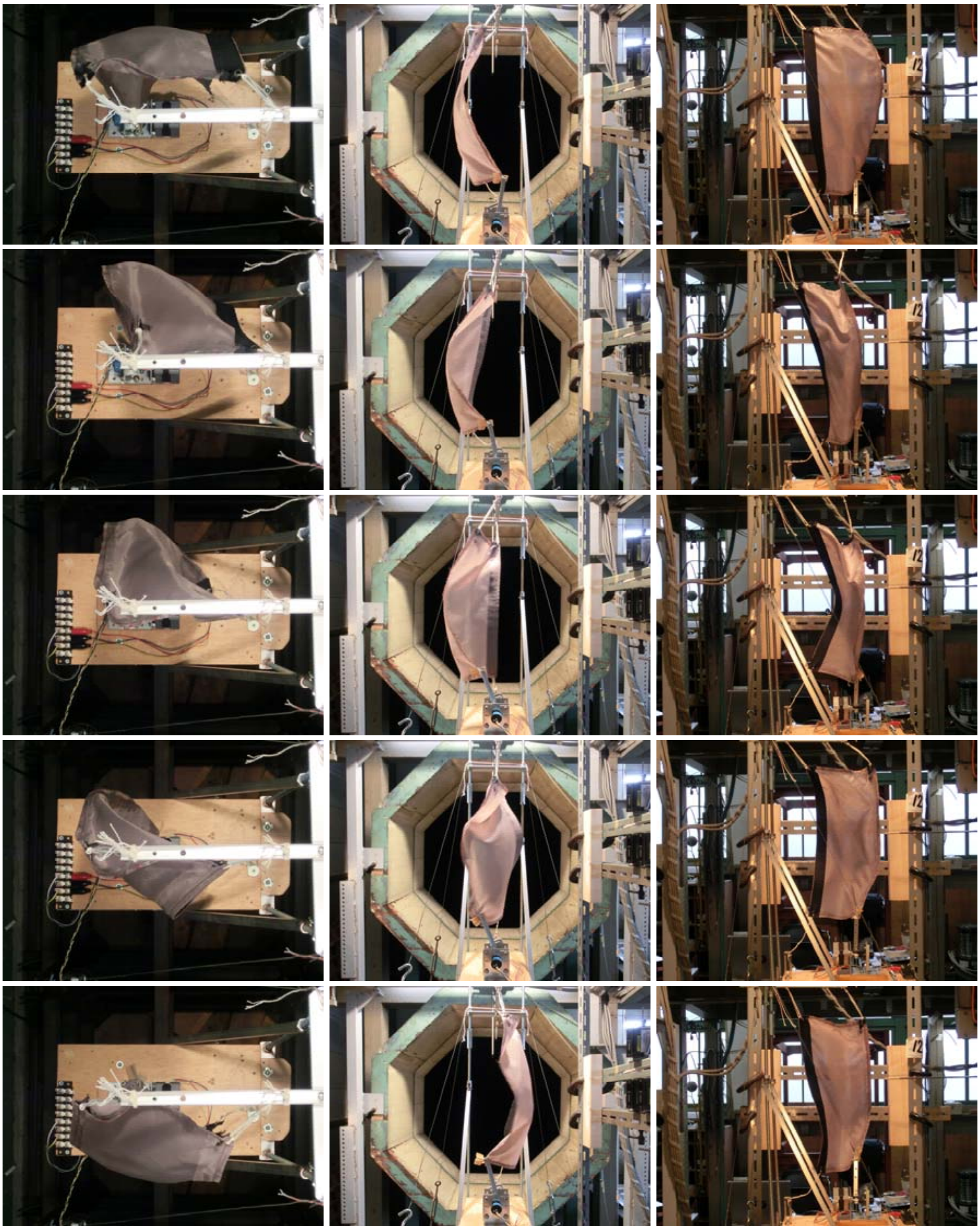

Figure 2 Snap shots during the flapping, wind speed is 2m/s. left shots is taken from the upper, middle shots from the downstream, right shots from the side.

In order to show clearly the patterns of the flapping motion, the outline of the snap shots taken from the upper is traced and is shown in the left figures of Figure 3.

The right figures show a superposition of the H-90 flatter mode of sway and torsion of a flat plate and the traveling mode of a sinusoidal wave with the same natural period. The superposition agrees well with the motion of the flag. Therefore, this flag can be modeled by this simple model.

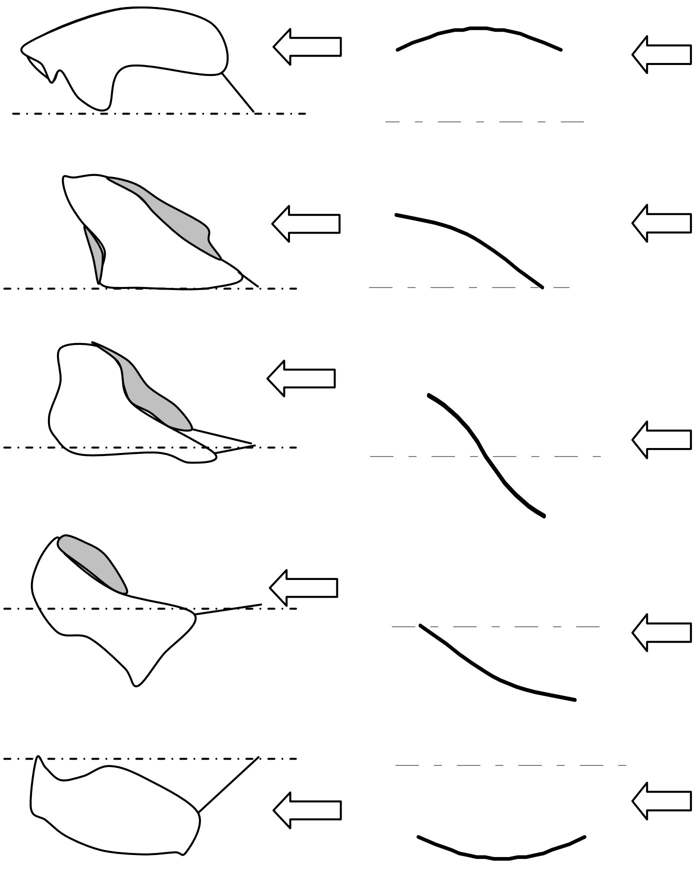

Figure 3 Sketch of the motion flag and the superposition of H-90 mode and the traveling wave model.

3.2 Time history during the flapping

Figure 4 shows the time history of the fluctuations of the velocity, the rotation of the generator and the displacement at the leeward of the flag. The wind velocity behind the flag fluctuates synchronously with the flapping, but the signal of the wind velocity fluctuation includes a lot of pulse-like fluctuation which might be induced by the jet accompanying with the flapping. On the other hands, the rotation of the core of the generator is more stable and regular than the displacement of the flag.

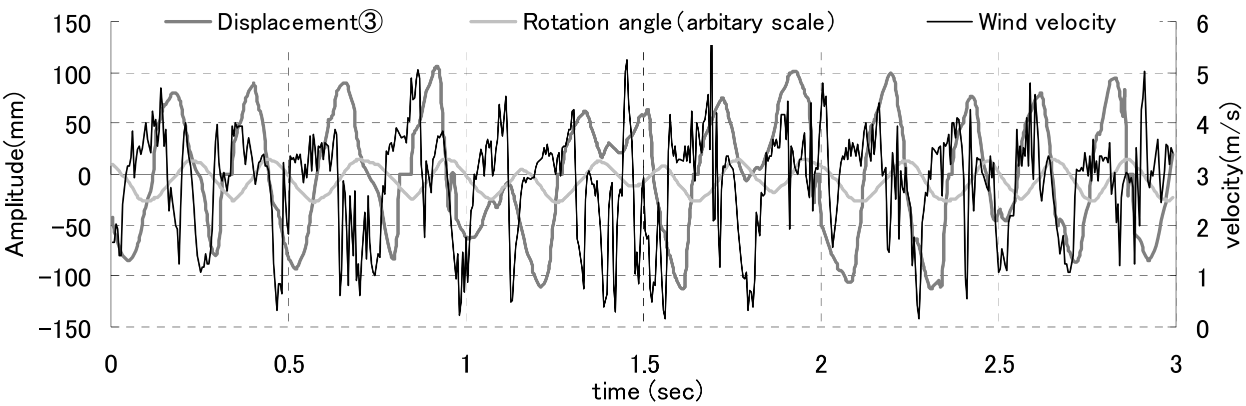

Figure 4 Time history of the displacement of the flag, the rotation angle of the core of the generator and the wind velocity fluctuation behind the flag.

Figure 5 show the same figure as Figure 4 except for the wind velocity fluctuation. In Figure 5, the fluctuation shows the moving average of 10 consecutive samplings. It is better to catch the synchronization of the three signals.

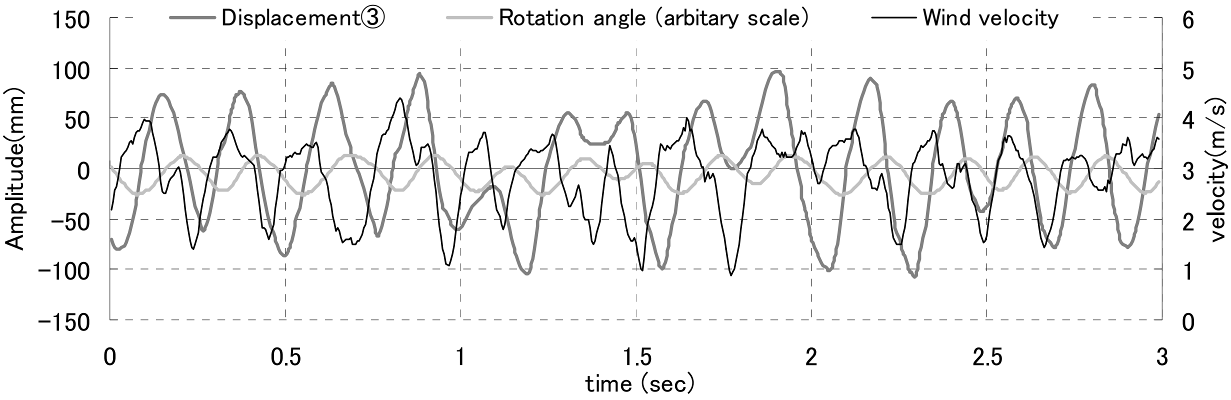

Figure 5 Time history of the displacement of the flag, the rotation angle of the core of the generator and the wind velocity fluctuation behind the flag after moving average of the 10 consecutive samplings.

3.3 Wind profile in the wake

Figure 6 shows profiles of mean and rms wind velocity in lateral direction to the flow when the reference wind velocity is 5m/s. The amplitude of the flag during the flapping is 1100mm. Both of the mean and the rms wind velocity are constant between y=-800 to 800mm. The mean wind velocity decreased 20% behind the flag. Therefore, about 50% of the wind power is extracted by the flapping. The wind velocity behind the flag is instantaneously reduced to 0 as shown in Figure 4.

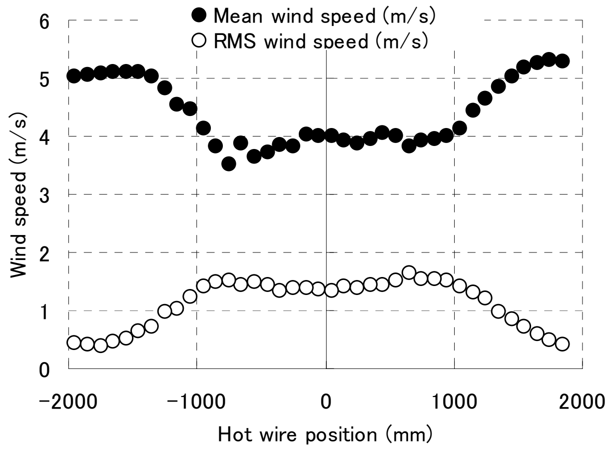

Figure 6 Profiles of mean and rms velocity

3.4 Amplitude and frequency of the flapping

Figure 7 shows rms amplitudes of the flag displacement at three points and the rotation of the core of the generator against the wind velocity. The amplitude is constant for any wind velocity. On the other hands, the frequency of the flapping increases proportionally against the wind velocity.

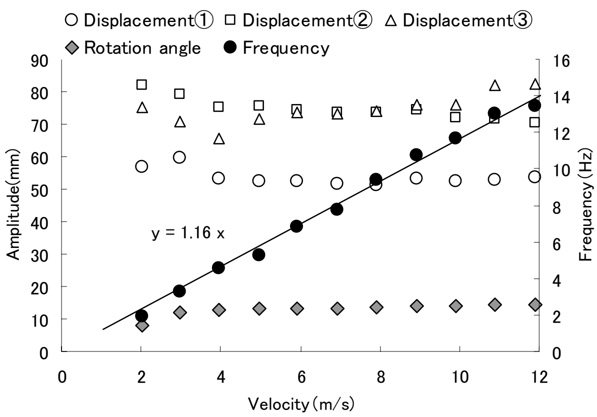

Figure 7 Rms amplitude and frequency against wind velocity.

4. DISCUSSION

4.1 Flapping as two degree of freedom system

As shown in the section 3.1, the flapping of the flag is the combination of the H-90 mode flatter and the traveling wave. In this section, the first one is considered. It is assumed that the flag is modeled by a flat plate which is supported by the spring with no stiffness and moves in sway and torsion as shown in Figure 8.

Mass and inertia of the flag are expressed by m and I respectively. Lift and moment on the flag are expressed by L and M respectively. y is displacement of the flag and θ is rotation of the flag as shown in Figure 8.

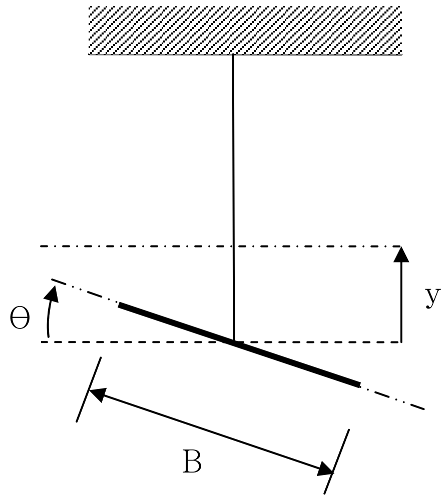

Figure 8 Flag as a flat plate with two degrees of freedom system

The equation of motion of this system is given by

$$m\ddot{y} = L \qquad I\ddot{\theta} = M \tag{1}$$

It is assumed that the lift and moment is expressed by

$$L = L_{yR}y + L_{yI}\dot{y} + L_{\theta R}\theta + L_{\theta I}\dot{\theta} \tag{2}$$

$$M = M_{yR}y + M_{yI}\dot{y} + M_{\theta R}\theta + M_{\theta I}\dot{\theta} \tag{3}$$

Substitute (2) and (3) for (1) and express as a matrix form.

$$\begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix}\begin{Bmatrix} \ddot{y} \\ \ddot{\theta} \end{Bmatrix} - \begin{bmatrix} L_{yI} & L_{\theta I} \\ M_{yI} & M_{\theta I} \end{bmatrix}\begin{Bmatrix} \dot{y} \\ \dot{\theta} \end{Bmatrix} - \begin{bmatrix} L_{yR} & L_{\theta R} \\ M_{yR} & M_{\theta R} \end{bmatrix}\begin{Bmatrix} y \\ \theta \end{Bmatrix} = 0 \tag{4}$$

Assume a solution of (4) is given by $y = y_0 e^{i(\omega t + \phi)}, \theta = \theta_0 e^{i\omega t}$ where ω is circular frequency and θ is phase, and substitute this solution for (4).

$$\begin{bmatrix} L_{yR} + \omega^2 m + i\omega L_{yI} & i\omega L_{\theta I} + L_{\theta R} \\ i\omega M_{yI} + M_{yR} & M_{\theta R} + \omega^2 I + M_{\theta I} i\omega \end{bmatrix}\begin{Bmatrix} y \\ \theta \end{Bmatrix} = 0 \tag{5}$$

The condition that the equation (5) has the non trivial solution is

$$\begin{vmatrix} L_{yR} + \omega^2 m + i\omega L_{yI} & i\omega L_{\theta I} + L_{\theta R} \\ i\omega M_{yI} + M_{yR} & M_{\theta R} + \omega^2 I + M_{\theta I} i\omega \end{vmatrix} = 0 \tag{6}$$

The real part Δ_R and imaginary part Δ_I of the equation (6) expressed as the equations (7) and (8) are,

$$\Delta_R = \omega^4 mI - \omega^2\left\{-mM_{\theta R} - IL_{yR} + L_{yI}M_{\theta I} - L_{\theta I}M_{yI}\right\} + L_{yR}M_{\theta R} - L_{\theta R}M_{yR} = 0 \tag{7}$$

$$\Delta_I = \omega^2\left(-IL_{yI} - mM_{\theta I}\right) - \left(L_{yI}M_{\theta R} + M_{\theta I}L_{yR} - L_{\theta I}M_{yR} - L_{\theta R}M_{yI}\right) = 0 \tag{8}$$

The expressions of the lift and the moment change to the following for convenience of later discussion.

$$L = \frac{1}{2}\rho U^2 B\left\{L'_{yR}\frac{y}{B} + L'_{yI}\frac{\dot{y}}{U} + L'_{\theta R}\theta + L_{\theta I}'\frac{B\dot{\theta}}{U}\right\} \tag{9}$$

$$M = \frac{1}{2}\rho U^2 B^2\left\{M'_{yR}\frac{y}{B} + M'_{yI}\frac{\dot{y}}{U} + M'_{\theta R}\theta + M'_{\theta I}\frac{B\dot{\theta}}{U}\right\} \tag{10}$$

From the equation (8), the frequency of the flapping is given by

$$\omega^2 = \frac{1}{2}\rho U^2 B^2 \frac{L'_{yI}M'_{\theta R} + M'_{\theta I}L'_{yR} - L'_{\theta I}M'_{yR} - L'_{\theta R}M'_{yI}}{-IL'_{yI} - mB^2 M'_{\theta I}} \tag{11}$$

According to the equation (11), if the aerodynamic coefficient is constant, the frequency is proportional to the velocity as shown in the section 3.4. Therefore, the coefficients might be constant for any velocity. The aerodynamic coefficient is generally a function of reduced velocity, $U/\omega B$, but when the frequency is proportional to the velocity, the reduced velocity is constant. In the case of a flat plate, $I = \frac{B^2}{12}m$, then

$$\omega^2 = -q\frac{L'_{yI}M'_{\theta R} + M'_{\theta I}L'_{yR} - L'_{\theta I}M'_{yR} - L'_{\theta R}M'_{yI}}{\left(\frac{1}{12}L'_{yI} + M'_{\theta I}\right)m} \tag{12}$$

The frequency is proportional to root of the mass.

Ratio of the sway and the torsion and phase delay between the sway and the torsion can be estimated by the equation (5).

$$\frac{y}{\theta} = \frac{i\omega L_{\theta I} + L_{\theta R}}{-L_{yR} - \omega^2 m - i\omega L_{yI}} = A + iB\,,\ \frac{y_0}{\theta_0} = \sqrt{A^2 + B^2}\ ,\ \phi = \tan^{-1}\left(\frac{B}{A}\right) \tag{13}$$

When the phase delay is $\pi/2$,

$$L_{\theta R}\left(L_{yR} + \omega^2 m\right) + \omega^2 L_{\theta I}L_{yI} = 0 \tag{14}$$

and then,

$$\omega^2 = -\frac{1}{2}\rho U^2 \frac{L'_{\theta R}L'_{yR}}{L'_{\theta R}m + \frac{\rho B^2}{2}L'_{\theta I}L'_{yI}} \tag{15}$$

From the equations (12) and (15),

$$L'_{\theta I}L'_{yI} = 0 \tag{16}$$

Consequently,

$$\omega^2 = -\frac{1}{2}\rho U^2 \frac{L'_{yR}}{m} \tag{17}$$

According to the experimental result of the relation between the frequency and the velocity, $L'_{yR} = 1.47$ is obtained.

In the similar way, the following relationship is deduced.

$$M'_{\theta I}M'_{yI} = 0 \tag{18}$$

$$\omega^2 = -\frac{1}{2}\rho U^2 \frac{12M'_{\theta R}}{m} \tag{19}$$

From the equations (17) and (19),

$$L'_{yR} = 12M'_{\theta R} \tag{20}$$

On the other hands, the solution satisfied both the equations (16) and (18) is,

$$M'_{yI} = 0, L'_{\theta I} = 0 \tag{21}$$

The ratio of the amplitude of the sway and the torsion R_a is

$$\frac{U}{\omega B}\frac{BL'_{\theta R}}{L'_{yI}} = \frac{B\omega}{U}\frac{BM'_{\theta I}}{M'_{yR}} = R_a \tag{22}$$

According to the experiment, $R_a \approx 1$ and $\omega B / U = 1.24$ and then

$$\begin{aligned} L'_{\theta R} &= \frac{\omega B}{U} L'_{yI} = 1.24 L'_{yI} \\ M'_{yR} &= \frac{\omega B}{U} M'_{\theta I} = 1.24 M'_{\theta I} \end{aligned} \tag{23}$$

Above the results, the equation of motion of the flapping is given by

$$\begin{aligned} m\ddot{y} &= \frac{1}{2}\rho U^2 B\left\{1.47\frac{y}{B} + L'_{yI}\left(\frac{\dot{y}}{U} + 1.24\theta\right)\right\} \\ I\ddot{\theta} &= \frac{1}{2}\rho U^2 B^2\left\{0.122\theta + M'_{\theta I}\left(\frac{B\dot{\theta}}{U} + 1.24\frac{y}{B}\right)\right\} \end{aligned} \tag{24}$$

5. CONCLUSIONS

(1) The frequency of the flapping increases proportionally to the wind velocity.
(2) The amplitudes of the sway and the torsion during the flapping are constant for any wind velocity.
(3) The ratio of the reduced amplitude of the sway and the torsion is about 1 for this flag with the aspect ratio of 2.18.
(4) The wind velocity behind the flag fluctuates synchronously with the flapping.
(5) The mean wind velocity reduces to 80% for the range of the double amplitude of the flapping.
(6) The flapping of the flag is expressed by the combination of the H-90 sway and torsion mode flatter and the traveling wave on the flag with the same period of the flatter.
(7) The flatter mode of the flag can be modeled by the two degrees of freedom supported by the spring with no stiffness.
(8) The analysis shows that the aerodynamic coefficients for the system are constant for any wind velocity.
(9) The frequency of the flatter is proportional to the root of the ratio of aerodynamic stiffness and the mass of the flag.

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