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Group Dynamics of a Building Cluster: A Nonlinear Model

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ABSTRACT

It has been shown that building interactions can play an important role in the dynamic response of a structure. Existing approaches can be categorized as either lumped-mass models or finite element methods. The latter are computationally expensive and only possible for relatively small-size cluster configuration. To alleviate the computational requirements a generic nonlinear mathematical model has been developed to analyze the interactions between adjacent structures via the soil. The current analysis focuses on a small-size cluster, but the model is expandable to an arbitrary number of nodes both in the horizontal and vertical plane. In this work we derive the governing equations of the system, including models for superstructures (shear-wall model), soil (hysteretic) and actuation (real-life earthquake data). A comparison between the results of the nonlinear system to those of the linear one shows significant frequency shifting. Furthermore, this work gives insight of the group dynamic behavior in comparison to the performance of a stand-alone structure.

1. INTRODUCTION

The recent series of earthquakes in Canterbury, New Zealand has prompted questions regarding the completeness of current design codes and methods. A major assumption in current design codes is the neglecting of interactions between neighboring structures. These building interactions are henceforth referred to as Soil Structure Soil Interactions (SSSI). The theory around SSSI has been developing since the 1970's [1]. According to more recent works by Uenishi [2] and Gueguen et al. [3] SSSI can play significant roles in the structural response of a building subjected to strong ground motion, especially for an array of structures on softer soil conditions [3]. Both groups [2, 3] have recently developed and analyzed the dynamics of linear analytical models, which subsequently revealed global mode shapes for one-dimensional (1D) array configurations and uncovered the mechanisms causing unusual damage patterns in earthquakes such as in the Friuli, Italy earthquake (1976) [2]. A more detailed overview about existing models that focus on neighboring structure interactions and the so called 'town effect' is given by Uenishi [2]. His review of existing literature states that most approaches aim at the 'macroscopic' (global) city-scale point of view, e.g. [4], and that a systematic study in view of the 'microscopic' (local) behavior of each building (resonator in the array) is missing.

The fundamental, scientific study of coupled oscillators goes all the way back to the seventeenth century [5, 6], where Christiaan Huygens observed mutual synchronization of pendulum clocks coupled via a common base. With the rise of nano-technology in the twentieth century, collective phenomena of coupled systems have been re-discovered and studied for mainly nano- and micro-mechanical systems [6-10]. Unlike the focus in macroscopic applications (e.g. the study of town effects of civil structures), the focus in studying nano- and micro-arrays lies on the individual performance of each resonator within the cluster rather than the global behavior. Thus, that which appears to be unknown for coupled macro-structural arrays, namely the desired individual performance within a cluster, has been studied elsewhere and for other applications, and hence, the existence of certain occurring phenomena such as energy transfer and global vibration modes are not new. Generally, the coupling mechanisms of micro- and macro-structure arrays alike follow nonlinear characteristics. It is well known that for nonlinear coupled resonators there occur special phenomena, such as bifurcation points, multiple coexisting solutions, energy transfer, loss of stability leading into chaotic (unpredictable) behavior as well as intrinsic locking of individual members [11-13].

In order to study the individual dynamic performance of a single building while the same is subject to seismic group dynamics, this paper introduces a simple 2D mathematical model and a 1D analysis of a superstructure cluster and a hysteresis model capturing the soil interactions. This work presents preliminary results and a comprehensive, systematic, nonlinear analysis will be published elsewhere.

2. THE MATHEMATICAL MODEL OF SUPERSTRUCUTRE, SOIL AND ASSEMBLY

The mathematical, computational model of a coupled 4x1x2 building cluster is derived in three parts: the superstructures, the soil and the coupled assembly. The small-size cluster is deduced from a generic approach that allows for an arbitrary number of floors as well as neighbor buildings. The formulation only considers nearest neighbor interactions via the soil.

2.1 Superstructure Model and Eigenvalue Problem

The superstructures have been modeled using a linear shear building model with translational degrees of freedom in the horizontal xy-plane, see Figure 1a. The foundation mass of each building is representative of the surrounding soil mass as well as the buildings foundation. Figure 1a shows the shear wall model applied to a single building system, whereas Figure 1b depicts the array configuration. It should be noted that although Figure 1b suggests a cluster of four two-storey buildings, each building is designed for a select fundamental natural frequency (or period), thus representing a variety of real-life civil structures, independent of size and number of floors. Hence, we will refer to the degrees of freedom as ‘base’ and ‘top’ of structures, rather than ‘floors’.

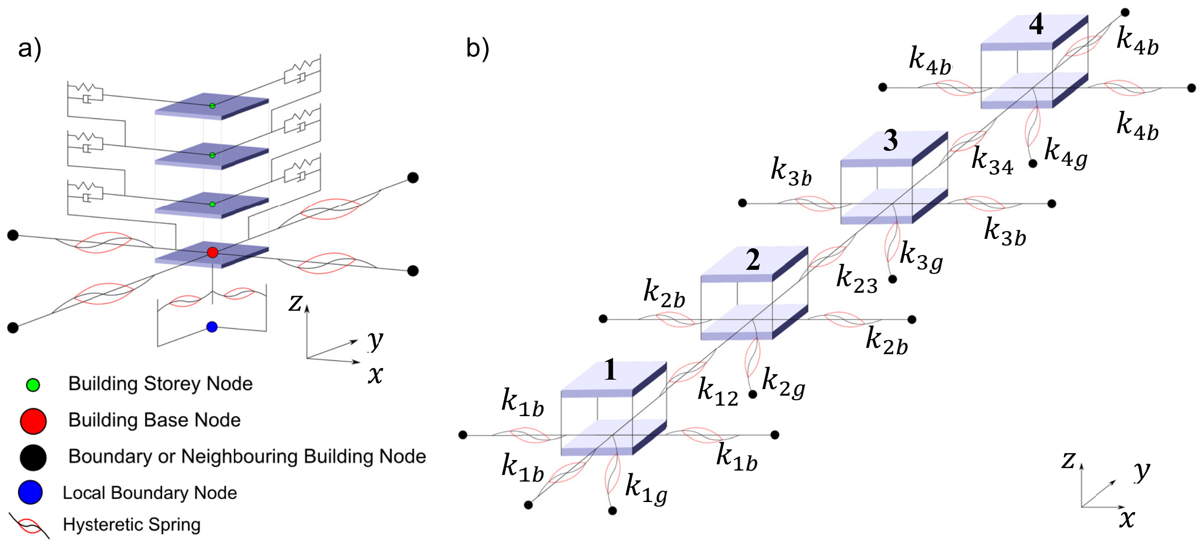

Figure 1: Schematics of four coupled two-storey buildings; a) detailed unit representing one building and its connection points; b) 4x1x2 cluster configuration.

The equations are formulated with respect to an absolute reference frame and thus, the earthquake signal is introduced as displacement excitation at any combination of the outer boundary nodes, marked with black and blue dots in Figure 1. Throughout this paper the building base nodes (depicted in red in Figure 1a) are assumed to lie in the horizontal plane, restricting building interactions to be only in the horizontal plane.

The undamped eigenvalue problem of a comparable 4x1x2 cluster considers only shear-type springs and is expressed in the classical way

$$(\boldsymbol{K} - \omega^2 \boldsymbol{M})\widehat{\boldsymbol{q}} = \boldsymbol{0}, \tag{1}$$

with the stiffness matrix

$$\boldsymbol{K} = \begin{bmatrix} \alpha_1 & -k_1 & -k_{12} & 0 & 0 & 0 & 0 & 0 \\ -k_1 & k_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -k_{12} & 0 & \alpha_2 & -k_2 & -k_{23} & 0 & 0 & 0 \\ 0 & 0 & -k_2 & k_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -k_{23} & 0 & \alpha_3 & -k_3 & -k_{34} & 0 \\ 0 & 0 & 0 & 0 & -k_3 & k_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & -k_{34} & 0 & \alpha_4 & -k_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & -k_4 & k_4 \end{bmatrix},$$

The mass matrix $\boldsymbol{M} = \mathrm{diag}\{m_{11}\ \ m_{21}\ m_{22}\ m_{31}\ m_{32}\ m_{41}\ m_{42}\}$ and the displacement amplitudes of the base and top displacements $\widehat{\boldsymbol{q}} = \{\hat{x}_{11}\ \hat{x}_{12}\ \hat{x}_{21}\ \hat{x}_{22}\ \hat{x}_{31}\ \hat{x}_{32}\ \hat{x}_{41}\ \hat{x}_{42}\}^{\boldsymbol{T}}$. The stiffness coefficients $k_{ig}, k_{ib}, k_{ij}, k_i$ denote the ground stiffness, the boundary stiffness, the inter-building stiffness and the shear wall stiffness parameters, whereas m_{i1}, m_{i2} stand for the masses of the base and the top, respectively. Numerical values and the α_i parameters of the stiffness matrix are provided in the Appendix.

2.2 Nonlinear, Hysteresis Soil Model

The soil behavior is modeled using an extended Masing model, see Figure 2. In this model the soil stress is a function of soil strain as well as the time history of the soil displacement. Figure 2 depicts an arbitrary characteristic of a possible soil behavior. Realistic parameters that determine the soil properties of interest would need to be deduced from selected core samples in the area, which is not pursued in this paper. Instead estimated parameters are used to study the dynamic behavior qualitatively.

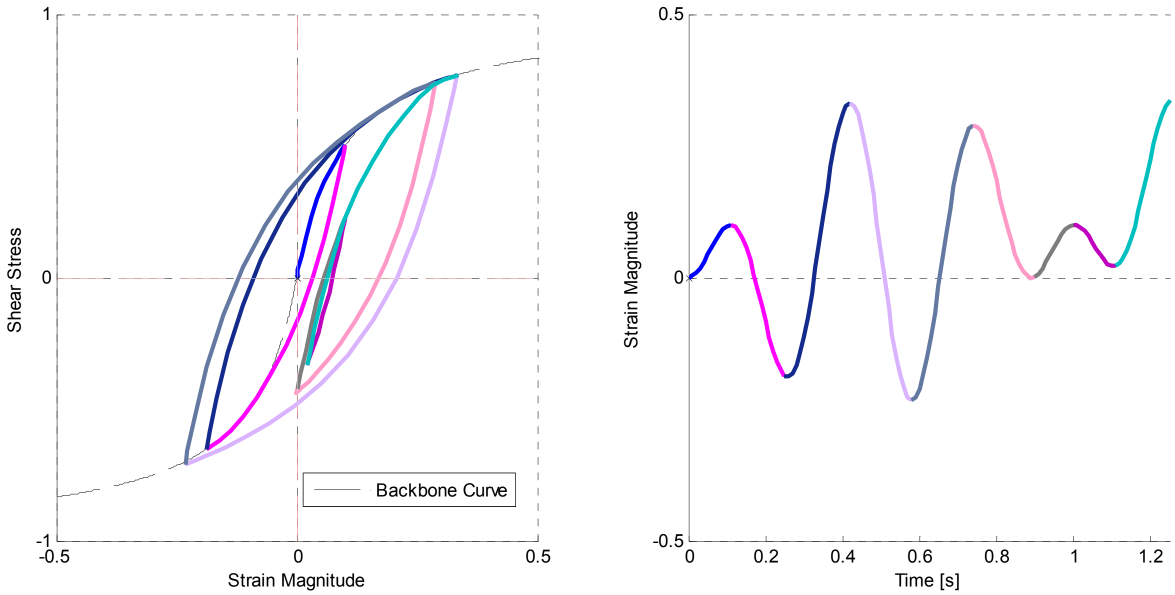

Figure 2: Hysteresis schematics of soil mechanisms.

Initially, the stress/strain state starts at $(0, 0)$ and upon increased loading follows the back bone curve (denoted by dashed black line). If a reversal of the loading direction occurs (indicated by a change in color) then the stress/strain behavior departs the backbone curve and follows a modified stress/strain curve. The stress/strain behavior returns to the backbone curve upon intersecting with the same, and also returns to the previous curve if crossing it.

2.3 Governing Equations of the Assembly (4x1x2 building cluster and soil)

The coupled cluster of buildings is created by arbitrarily grouping a set of nodes in the horizontal plane. The building foundation nodes (marked in red in Figure 1a) are surrounded by a set of reference nodes, which serve as excitation locations. As the intensity of excitation of the system is determined according to the hysteresis characteristic, it is convenient to solve the system in the time domain by numerical integration. At each time step all base nodes are computed to determine the soil strains and the resulting forcing on each building in the array. The instantaneous forces are then applied to the respective buildings and integrated over a single time step. Figure 3 presents the computation algorithm of the coupled, linear and nonlinear cluster model.

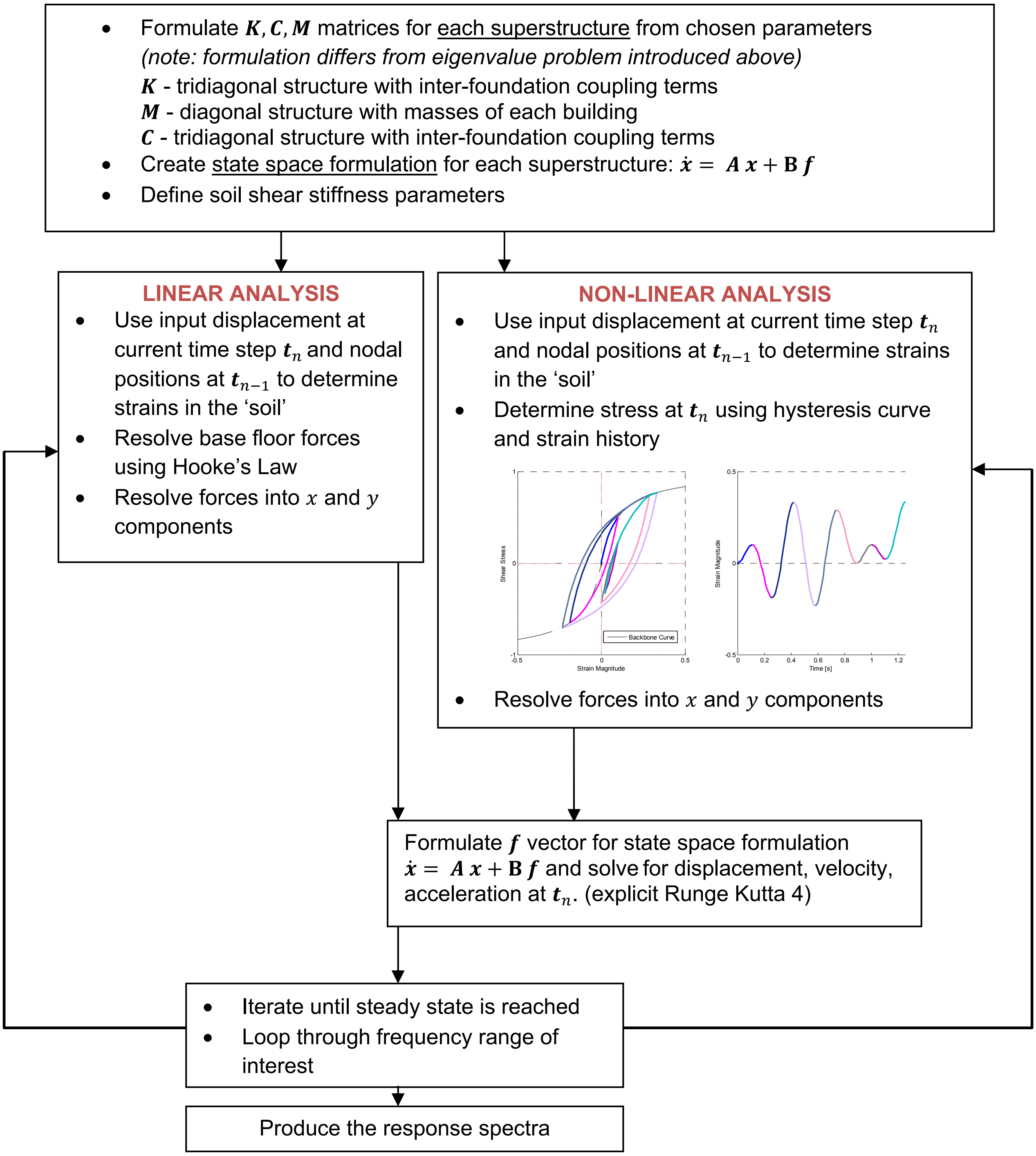

Figure 3: Solution flowchart: Simple harmonic motion

3. ANALYSIS

3. 1 Frequencies and Vibration Modes of Building Cluster

A first analysis is that of considering the eigenvalue problem of the linear system (see Section 2.1). Eq. (1) has solutions for precisely eight eigenvalues of ω. The related eigenvectors $\hat{\boldsymbol{q}}$ determine the synchronized motion of the cluster at these values, including intrinsic building mode shapes as well as global in- and out-of-phase inter-building motion patterns. Figures 3 and 4 show the collective vibration modes of the cluster for stiff and soft soil properties, respectively.

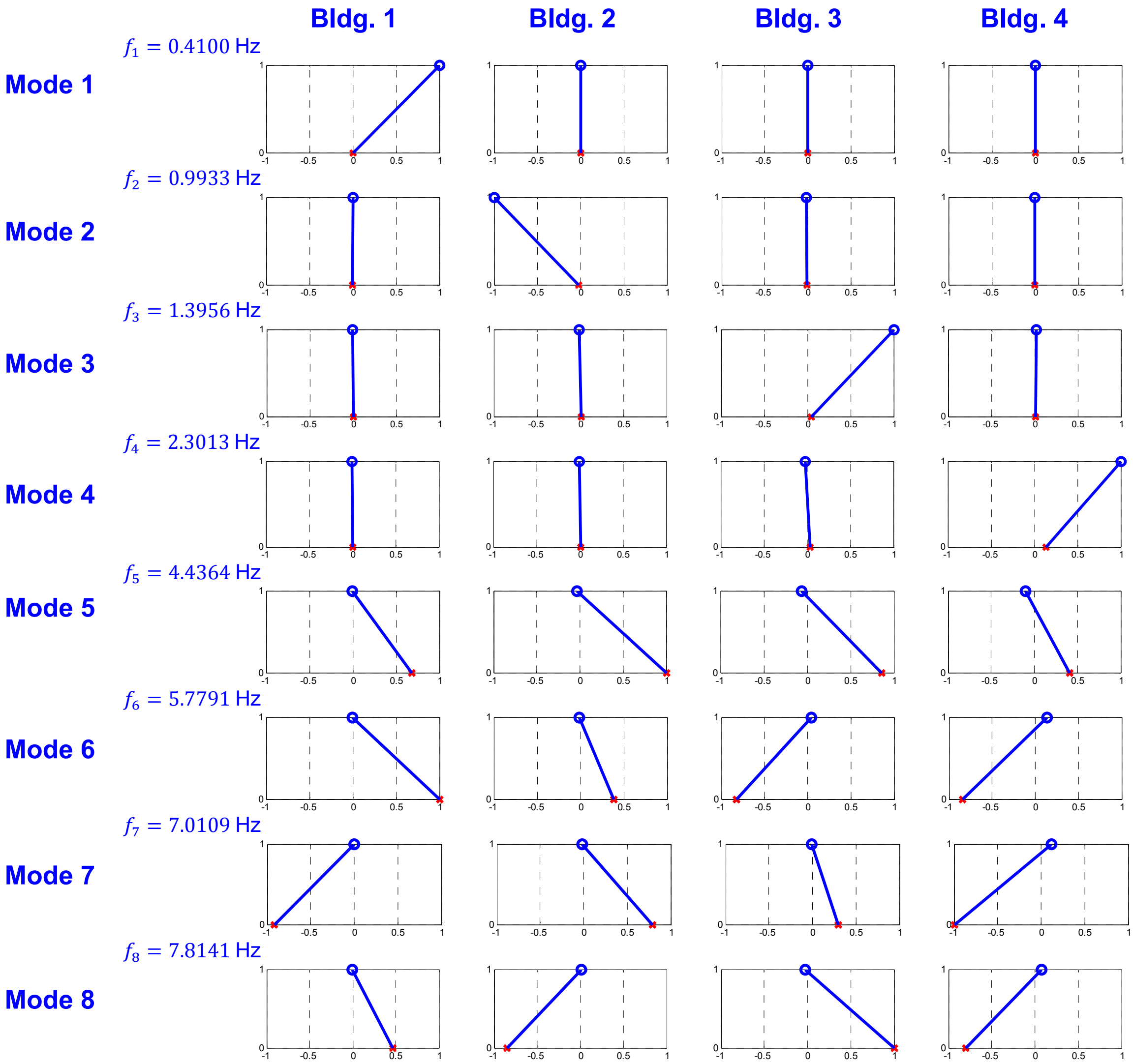

Figure 4: Global mode shapes of the 4x1x2 building cluster for stiff soil properties; red crosses: base motion, blue circles: top motion; modes are scaled to the maximum value of modal building displacement of each mode.

For a building cluster on stiff soil, the behavior for lower frequencies (modes 1-4) decouples, which means that the dynamics of one building has no or little influence on the dynamic behavior of another, neighboring building. For higher frequencies (modes 5-8), the common soil couples the behavior of buildings and the cluster shows assembly rather than individual vibration modes. In contrary, Fig. 5 shows that for a cluster on soft soil each building moves simultaneously in an assembly or global mode for lower frequencies (modes 1-4).

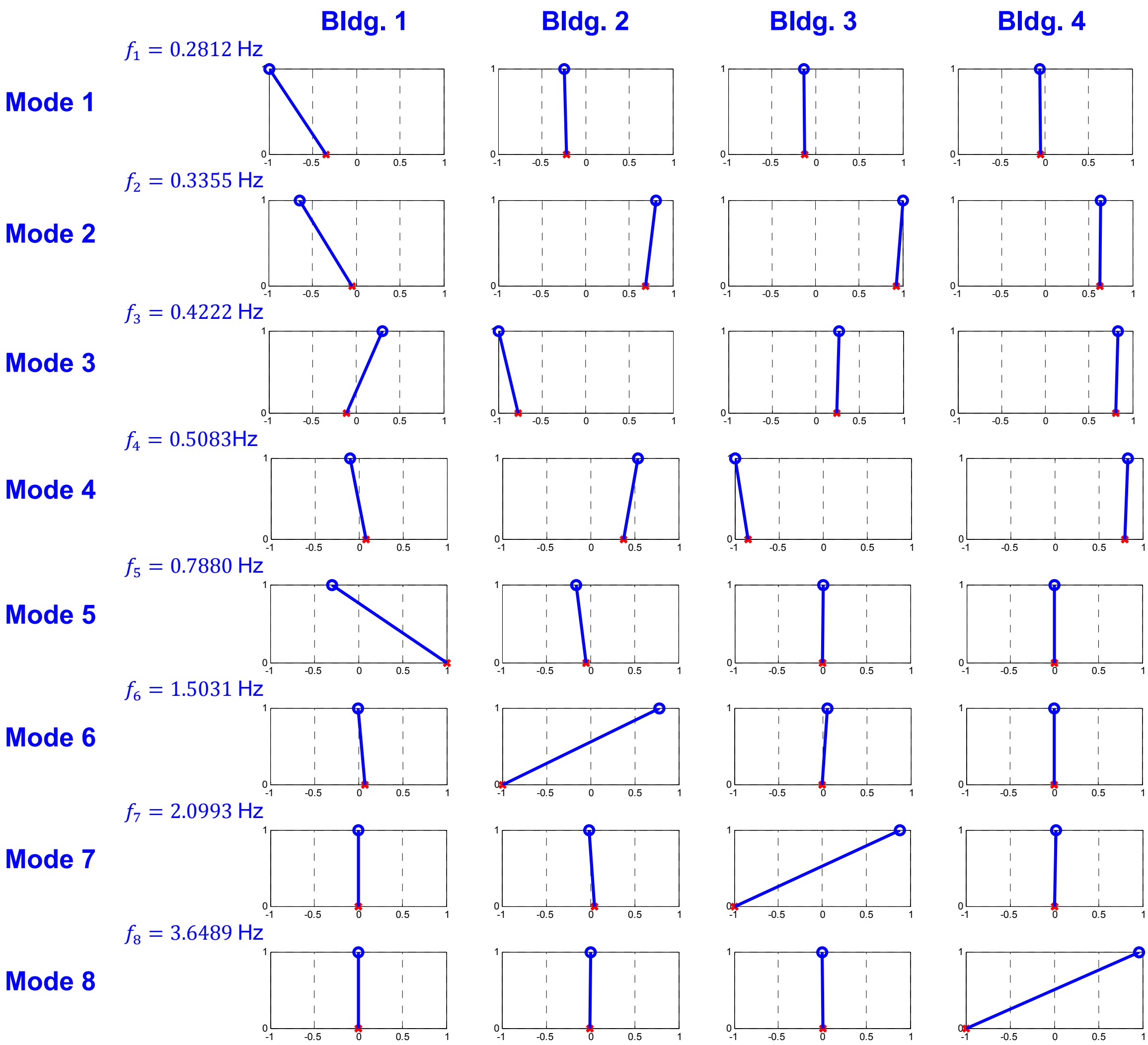

Figure 5: Global mode shapes of the 4x1x2 building cluster for soft soil properties; red crosses: base motion, blue circles: top motion; modes scaled to the maximum value of modal building displacement of mode.

For higher frequencies (modes 5-8) the cluster behavior decouples and individual building motion is observed. This observation can be explained in terms of the eigen-dynamics of the soil. The eigen-dynamics of the soil (soft or stiff) acts like a filter. In a certain frequency range it will magnify input signals, while outside this (these) range(s) it will attenuate them, which is revealed in the observed coupled and decoupled system behavior.

3. 2 Response of the Building Cluster to Harmonic Input Functions

The eigen-behavior of the 4x1x2 building cluster including the hysteretic nature of the soil has been analyzed by means of harmonic input signals over a frequency range of 0 to 4 Hz. Figure 6 shows the response spectra of the linear and nonlinear

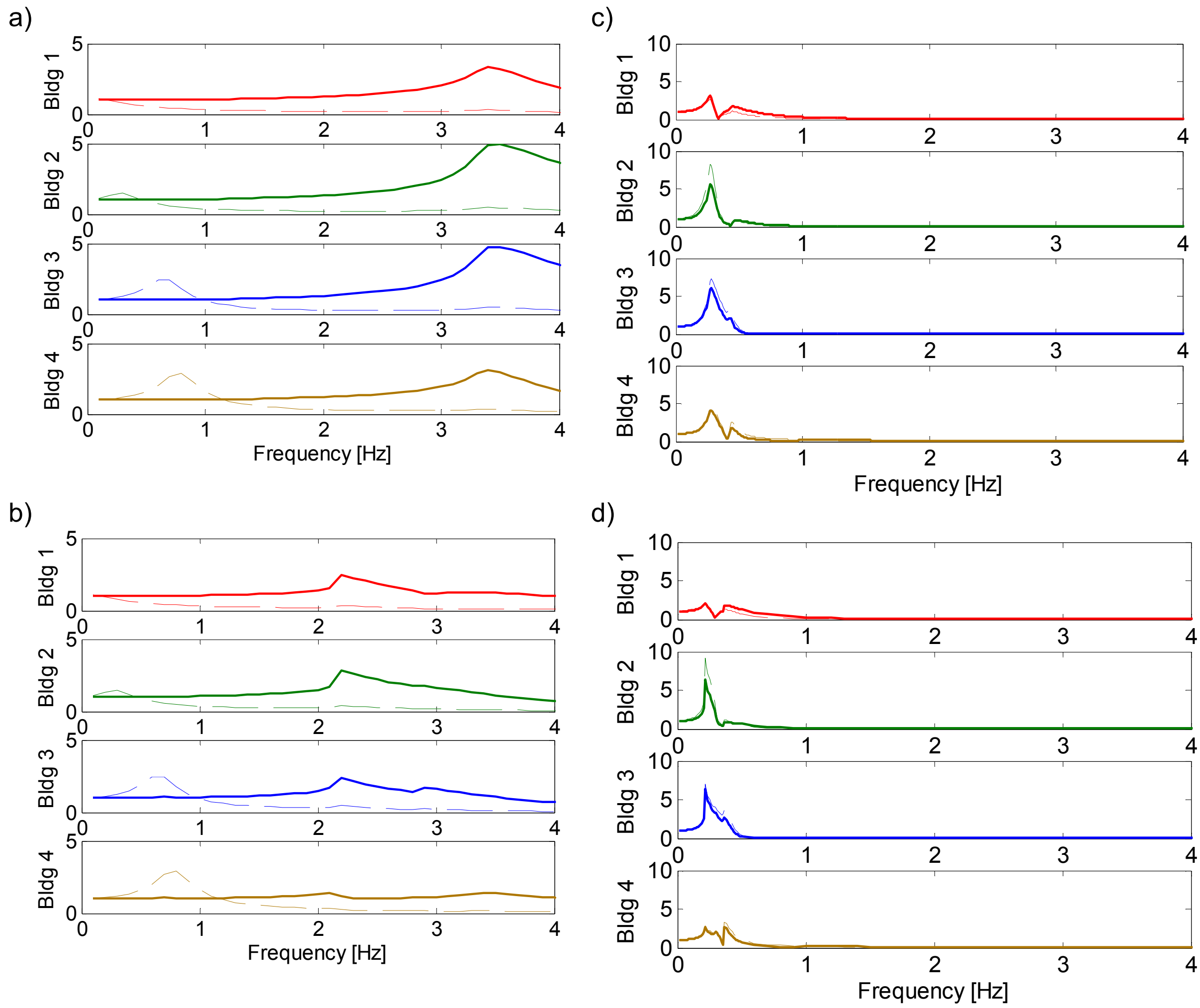

Figure 6: Comparison of frequency responses to a harmonic input for stiff (a, b) and soft (c, d) soil. a), c) small input amplitudes; b), d) large input amplitudes. Solid lines: base; dashed lines: top.

models of the cluster for stiff and soft soil properties. The linear model comprises small input amplitudes, forcing the strains to lie on the linear section of the backbone curve. The transfer functions for stiff and soft soil properties show rather distinct characteristics. The transfer behavior of the cluster on stiff soil shows a decoupled motion of individual buildings (see Figs. 6 a, b) at lower frequencies (below 1 Hz). At a frequency of approximately 3.5 Hz the cluster undergoes a motion in an assembly mode, i.e. all four buildings move in a particular synchronized motion (compare with mode shapes of the eigenvalue problem, Fig. 4). The transfer behavior of the cluster on soft soil, however, shows a coupled motion for lower frequencies while it decouples for higher frequency ranges. Investigations of the frequency responses subject to harmonic input signals validate the findings of the eigenvalue problem in the previous section.

There are several peaks (for both stiff and soft soil response curves) that were not identified. A possible answer to this question can be found by systematically studying the influence of the damping mechanisms of the whole system, including the soil, superstructures and inter-building interactions.

A close-up view of system, represented by e.g. building 3 on soft soil reveals the nonlinear nature of the behavior. Fig. 7 shows the response curves of the base for a small and large input amplitude.

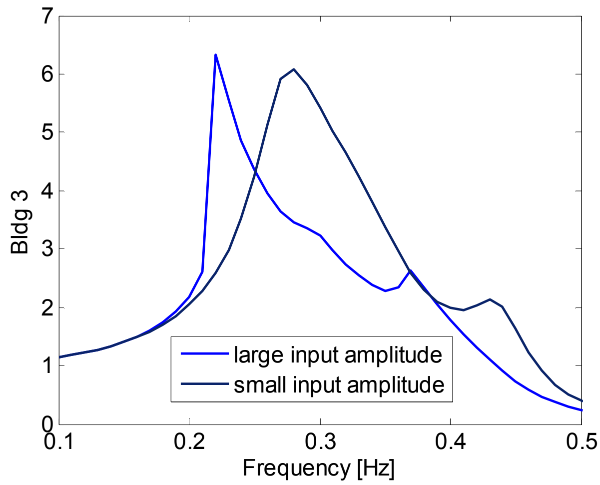

Figure 7: Comparison of response spectra of the nonlinear model for a large and small input amplitude on soft soil. Depicted is the base of building 3.

A clear shift of frequencies can be observed toward lower frequencies as well as an abrupt jump at 0.22 Hz for the case of large input amplitudes. The bending of the curves toward the left reflects the softening character of the hysteresis backbone curve. These observed nonlinear phenomena need to be further investigated in a systematic nonlinear analysis in the future.

3. 3 Response of the Building Cluster to Christchurch's 6.3 Earthquake Signal

Christchurch's 6.3 magnitude earthquake happened on the 22nd of February 2011 and had devastating impacts on residents, the city and wider area, and local and national economy [14]. While the earthquake with magnitude 6.3 on the Richter scale is not listed amongst the "top 10" in the world, the vertical peak ground acceleration measured 2.2 g is ranked second place after Japan's earthquake and tsunami one month later in 2011 (with vertical peak acceleration of 2.7 g) [15]. The horizontal displacement signal of the Christchurch 6.3 event is depicted in Figure 8. The displacement spectrum shows three distinct peaks below 1 Hz.

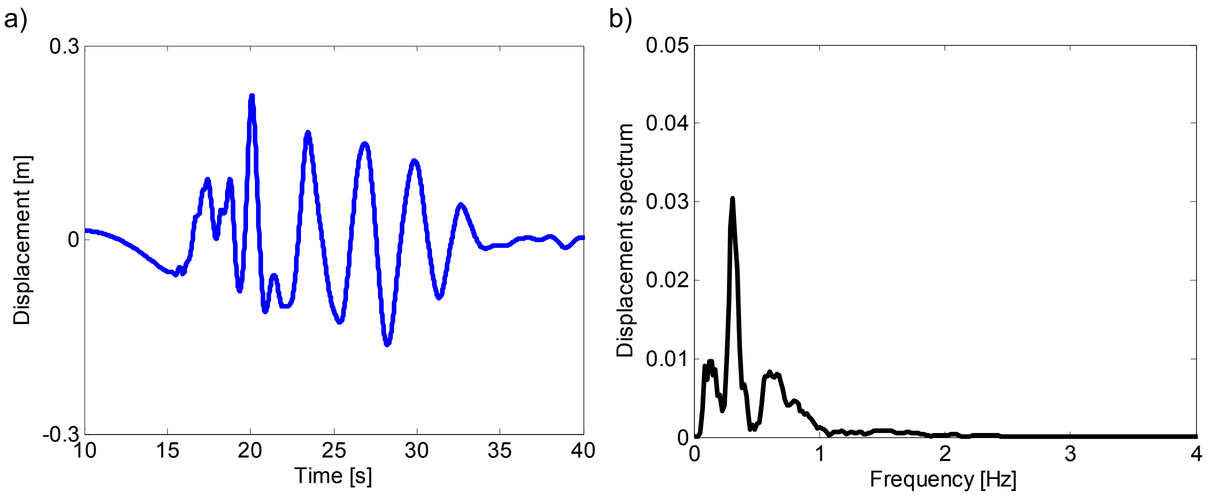

Figure 8: Horizontal displacement signal of Christchurch 6.3 earthquake; a) time signal [16]; b) displacement spectrum.

Figures 9 and 10 depict the responses of the building cluster for stiff and soft soil parameters. Figure 9 presents the response spectra of the base of the first building in the cluster.

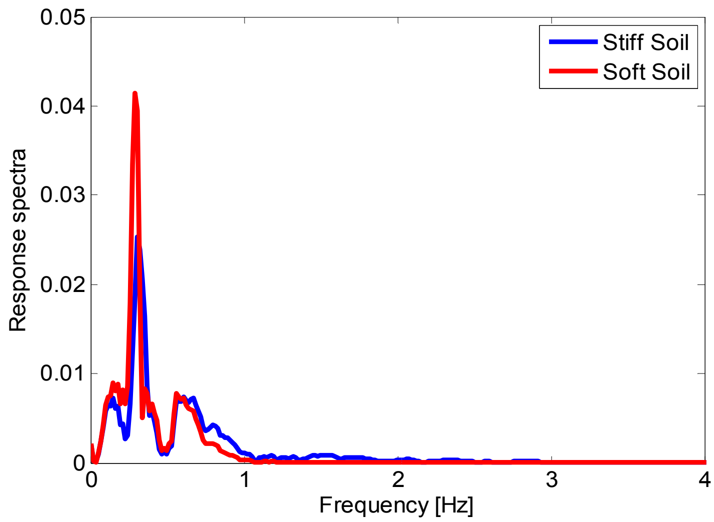

Figure 9: Comparison of response spectra of the nonlinear model for stiff and soft soil properties. Depicted is the base of building 1.

The frequency responses do not show a significant shift in frequencies from one soil condition to another, but differ in magnitude. According to Figs. 5 and 6, low frequent input signals are significantly more transmitted through the soft soil conditions than through the stiffer counterpart, which in the real scenario would be the unwanted case. Figure 10 shows the time responses of each building in the cluster on stiff and soft soil. It is immediately obvious that the buildings undergo larger displacements when placed on soft soil.

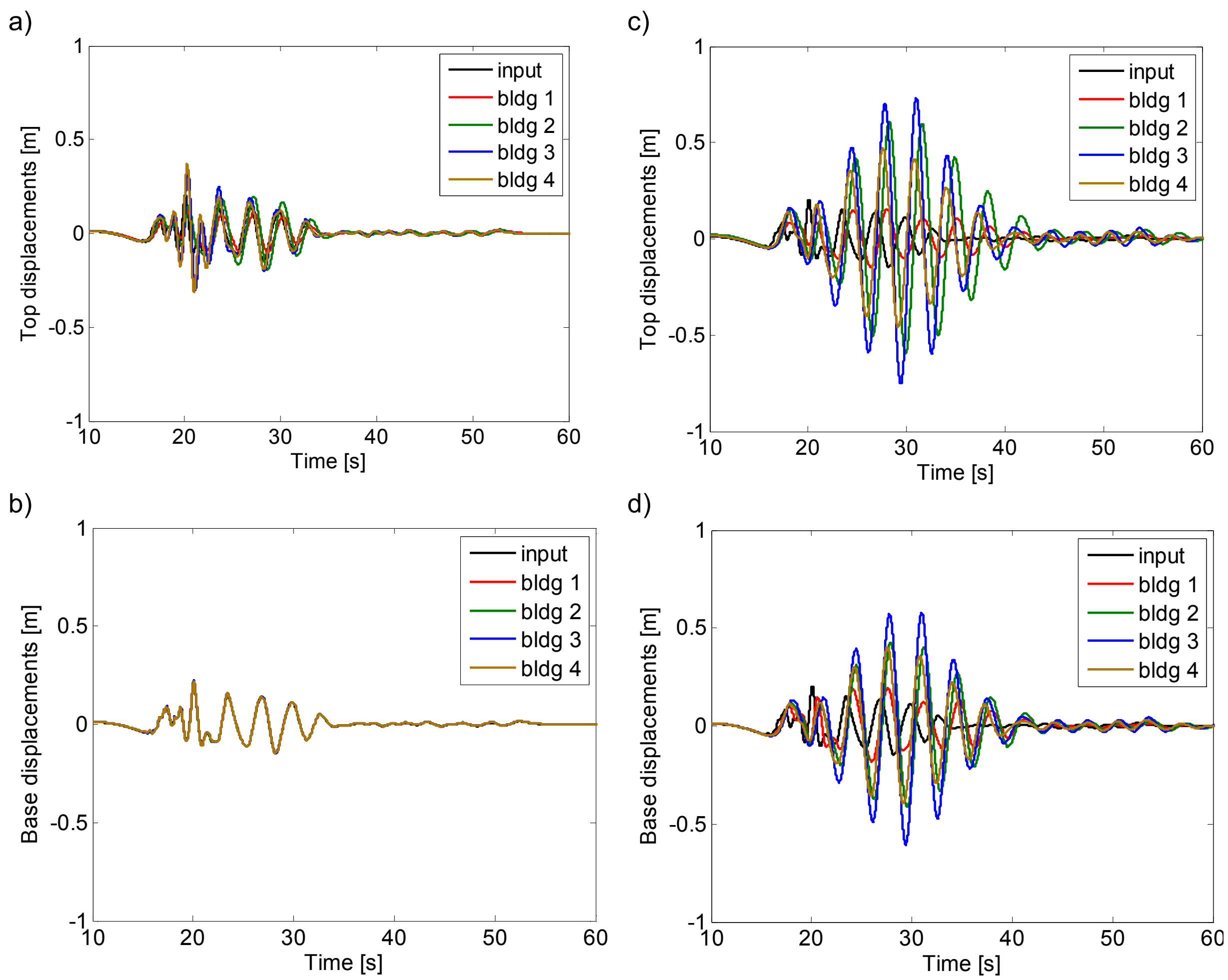

Figure 10: Comparison of time responses to Christchurch 6.3 displacement input for stiff (a, b) and soft (c, d) soil. Shown are the results of the nonlinear solver.

4. SUMMARY AND FUTURE WORK

A first 1D analysis of a 2D building cluster has been undertaken to investigate in the collective dynamic behavior by looking at natural frequencies and corresponding mode shapes, resonance curves generated by harmonic input signals and real earthquake signals. The dynamic behavior has been analyzed for stiff and soft soil properties. The analysis of the dynamic cluster behavior on soft soil revealed coupled assembly modes for lower frequencies and uncoupled, individual building modes for higher frequencies. For the case of the cluster on stiff soil the coupled assembly modes occurred at higher frequencies while the decoupled, individual modes are present at lower frequencies. Significant frequency shifts have been observed when comparing the responses between stiff and soft soil, as well as between large and small input amplitudes. Furthermore, the transfer behavior on both soft and stiff soil reveals to be highly nonlinear, which requires a more systematic, nonlinear analysis in the future.

ACKNOWLEDGMENTS

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REFERENCES

1. Wong, H.L.a.T., M. D., *Two-dimensional, antiplane, building-soil-building interaction for two or more buildings and for incident plane SH waves.* Bull Seismol Soc Am 1975. **65**: p. 1863–1885.
2. Uenishi, K., *The Town Effect: Dynamic Interaction between a Group of Structures and Waves in the Ground.* Rock Mech Rock Eng, 2010. **43**: p. 811-819.
3. Gueguen P., P.-Y.B., and Francisco J. Chavez-Garcıa, *Site-City Seismic Interaction in Mexico City–Like Environments: An Analytical Study.* Bulletin of the Seismological Society of America, 2002. **92**(2): p. 794-811.
4. Boutin, C. and P. Roussillon, *Assessment of the urbanization effect on seismic response.* Bulletin of the Seismological Society of America, 2004. **94**(1): p. 251-268.
5. Huygens, C., *Concerning the Use of Pendulum-Watches, for Finding the Longitude at Sea.* Philos. Trans. R. Soc. Lond., 1669. **4**: p. 937-953.
6. Bennett, M., et al., *Huygens's clocks.* Proceedings of the Royal Society of London Series a-Mathematical Physical and Engineering Sciences, 2002. **458**(2019): p. 563-579.
7. E. Brown, P.H.a.J.M., *Globally coupled oscillator networks. In "Perspectives and Problems in Nonlinear Science: A celebratory volume in honor of Lawrence Sirovich"*2003, New York: Springer Verlag. 183-215.
8. Danzl, P. and J. Moehlis, *Weakly coupled parametrically forced oscillator networks: existence, stability, and symmetry of solutions.* Nonlinear Dynamics, 2010. **59**(4): p. 661-680.
9. Hikihara, T., Y. Okamoto, and Y. Ueda, *An experimental spatio-temporal state transition of coupled magneto-elastic system.* Chaos, 1997. **7**(4): p. 810-816.
10. Gutschmidt, S. and O. Gottlieb, *Nonlinear dynamic behavior of a microbeam array subject to parametric actuation at low, medium and large DC-voltages.* Nonlinear Dynamics, 2012. **67**(1): p. 1-36.
11. Bromberg, Y., M.C. Cross, and R. Lifshitz, *Response of discrete nonlinear systems with many degrees of freedom.* Physical Review E, 2006. **73**(1).
12. Dick, A.J., B. Balachandran, and C.D. Mote, *Intrinsic localized modes in microresonator arrays and their relationship to nonlinear vibration modes.* Nonlinear Dynamics, 2008. **54**(1-2): p. 13-29.
13. Ilic, B., et al., *Enumeration of DNA molecules bound to a nanomechanical oscillator.* Nano Letters, 2005. **5**(5): p. 925-929.
14. Canterbury Earthquakes, R.C., *Canterbury Earthquakes Royal Commission Interim Report*, 2011: http://canterbury.royalcommission.govt.nz/Interim-Report-Contents.

15. Science., G.G., *PGA intensity map*, 2011: http://www.geonet.org.nz/var/storage/images/media/images/news/2011/lyttelton_pga/57159-2-eng-GB/lyttelton_pga.png.
16. Science., G.G., *Feb 22 2011 - Christchurch badly damaged by magnitude 6.3 earthquake*, 2011: http://www.geonet.org.nz/news/feb-2011-christchurch-badly-damaged-by-magnitude-6-3-earthquake.html.

APPENDIX

The α_i parameters of the stiffness matrix $\boldsymbol{K}$:

$\alpha_1 = k_{1g} + k_{1b} + k_{12} + k_1$

$\alpha_2 = k_{2g} + k_{12} + k_{23} + k_2$

$\alpha_3 = k_{3g} + k_{23} + k_{34} + k_3$

$\alpha_4 = k_{4g} + k_{4b} + k_{34} + k_4$

Table A1: Choice of Parameters

Symbol	Parameter description	Value	Unit
$k_{1g}, k_{2g}, k_{3g}, k_{4g}$	ground stiffness	5e6	N/m
k_{1b} , k_{4b}	boundary stiffness	5e6	N/m
k_{12}	the inter-building stiffness	5e6	N/m
k_1, k_2, k_3, k_4	shear wall stiffness parameters	[0.0667,0.404,0.83,2.59] x 10^6	N/m
m_1, m_2, m_3, m_4	Mass (all nodes)	10,000	kg