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Probabilistic Study into the Impact of Soil Spatial Variability on Soil Consolidation by Prefabricated Vertical Drains

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ABSTRACT

Soil consolidation by prefabricated vertical drains (PVDs) relies on several soil properties that are spatially variable such as the coefficients of permeability and volume compressibility. However, available design methods assume a single best estimate of the degree of consolidation based on "average" soil properties that are used to define an "equivalent" homogeneous soil. For heterogeneous soils, however, this assumption can result in desired (predicted) degree of consolidation that may not be reached at the required time, leading to unreliable and uneconomical solutions. To date, a few studies have been carried out to investigate the effects of spatial variability on soil consolidation and more research is immensely needed. In this paper, the effects of spatial variability of soil permeability and volume compressibility on consolidation of soft soil by PVDs are investigated stochastically by combining the local average subdivision (LAS) method of the random field theory and the Monte-Carlo finite-element simulations. The results indicate that spatial variability of soil permeability and volume compressibility within an affected soil mass significantly affects the degree of consolidation achieved via PVDs and hence the amount of soil improvement.

1. INTRODUCTION

Construction over soft soils, which have low bearing capacity and excessive compressibility, often requires a pre-construction treatment of the existing soft subsoils in order to improve its strength and stiffness, thus, eliminating the undue risks of excessive post construction deformations and associated instability. Although a number of soft soil improvement techniques are currently available, the use of prefabricated vertical drains (PVDs) with preloading has become the most popular method as it is cost effective and environmentally friendly (Indraratna et al. 2003). Despite the fact that the theoretical design aspects of soil consolidation by PVDs are well established (e.g. Barron 1948; Hansbo 1981), satisfactory agreement between the theoretical predictions of consolidation and the actual observed values is hardly achieved, especially for heterogeneous soils. The degree of consolidation achieved by PVDs is greatly controlled by some soil properties (e.g. soil permeability and volume compressibility) that are highly variable from one point to another in the ground and potentially induce uncertainty in

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their characterization. The inherent variation of soil properties with respect to spatial location is known as soil spatial variability and is due to the uneven soil micro fabric, complex characteristics of geological deposition and stress history. In order to properly acknowledge and quantify the soil spatial variability in geotechnical engineering analysis and design, probabilistic modeling techniques that treat the soil properties as random variables are more realistic. Unlike deterministic analyses, the probabilistic analyses explicitly take into account the variable nature of soil properties, based on their statistical characteristics, and thus provide more physical insights into the levels of risk associated with the obtained degree of consolidation.

The formulation and solution of stochastic problems are often very complicated. The review of relevant literature has indicated that although the fact that the impact of spatial variability of soil properties on soil consolidation has long been realized by many researchers (e.g. Pyrah 1996; Rowe 1972), it has never been previously considered in a systematic, scientific manner in design and little research has been made in this area. Given the complexity of the problem, a few studies that consider soil spatial variability in soil consolidation have been found in the literature. However, the existing studies either deal with the vertical drainage only (i.e. without PVDs) in 1D and 2D geometries (e.g. Badaoui et al. 2007; Freeze 1977; Hong 1992; Huang et al. 2010; Hwang and Witczak 1984) or analyze soil consolidation via PVDs but only consider the uncertainty associated with the testing errors in measuring the soil properties while the soil spatial variability has not been investigated account (e.g. Hong and Shang 1998; Zhou et al. 1999). In this paper, a parametric study that investigates the effects of soil spatial variability in treatment of ground improvement by PVDs is presented in a coupled Biot consolidation (Biot 1941), where the coefficient of permeability, k, and coefficient of volume compressibility, m_v, are separately treated as random variables.

2. STOCHASTIC APPROACH OF SOIL CONSOLIDATION BY PVDs

Among several approaches to model stochastic problems, the use of deterministic finite element analysis with stochastic input soil parameters in a Monte Carlo framework has gained much popularity in recent years (Elkateb et al. 2002). Similar scheme is employed in this study to investigate the effects of soil spatial variability on the behavior of soil consolidation by PVDs. The approach merges the local average subdivision (LAS) method (to generate random permeability fields) and finite element modeling (to calculate soil consolidation by PVDs) into a Monte Carlo framework using the following steps:

1. Create a virtual soil profile for the problem in hand which comprises a grid of elements that are assigned design values of soil properties different from one element to another across the grid. The virtual soil profile allows arbitrary distributions of soil properties to be realistically and economically modeled;
2. Incorporate the generated soil profile into a finite element modeling of soil consolidation by PVDs; and
3. Repeat Steps 1 and 2 many times using the Monte Carlo technique so that a series of consolidation responses can be obtained from which the statistical distribution parameters and probability of achieving a target degree of consolidation can be estimated and analyzed.

The above steps are applied to a consolidation problem of an axisymmetric unit cell of geometry (see Fig. 1): L = 1.0 m, r_e = 0.85 m, r_w = 0.05 m, where L is the maximum vertical drainage distance; r_e is the radius of equivalent soil cylinder with impermeable perimeter or the radius of zone of influence; and r_w is the equivalent radius of the drain. As the detailed description of the above steps can be found elsewhere (Bari et al. 2012), only a brief discussion is presented below.

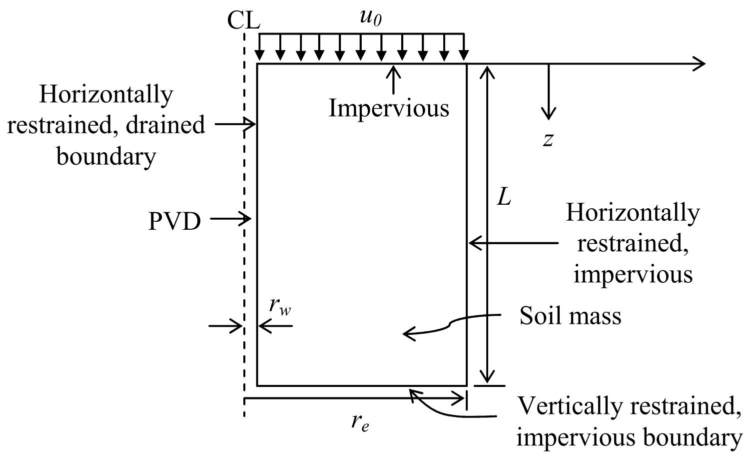


Fig. 1 Schematic diagram of soil cylinder with prefabricated vertical drain

2.1. Generation of Virtual Soil Profiles

As mentioned earlier, k and m_v are considered to be random variables in the present study (note that to obtain accurate results, k and m_v cannot be embodied into a single coefficient of consolidation), and are characterized in terms of their mean (μ), standard deviation (σ), probability distribution, and correlation length (θ). In selecting the probability distribution of k and m_v, the authors reviewed a broad range of literature (e.g. Badaoui et al. 2007; Freeze 1977; Huang et al. 2010) and concludes that it is reasonable to assume lognormal probability distribution both for k and m_v. Since the same approach is used to to generate both k and m_v, only the procedure to generate the random soil permeability is summarized here.

In the process of simulating the lognormally distributed random field of k, correlated local averages of standard normal random field $G(x)$ are first generated with zero mean, unit variance and spatial correlation function using LAS technique (Fenton and Vanmarcke 1990). The correlation coefficient between k measured at a point x_1 and a second point x_2 is specified by a correlation function, $\rho(\tau)$, where $\tau = |x_1 - x_2|$ is the absolute distance between the two points. An isotropic (i.e. the spatial correlation lengths in the horizontal and vertical directions are taken to be equal) exponentially decaying (Markovian) spatial correlation function is used in this research as follows (Fenton and Griffiths 2008):

$$\rho(\tau) = \exp\left(-\frac{2|\tau|}{\theta_k}\right) \tag{1}$$

The correlation length (also known as scale of fluctuation) given in Eq. (1), describes the limit of spatial continuity of spatial correlation. Thus, a large value of θ_k indicates a smoothly varying field, whereas a small value of θ_k implies an erratic field. It is worthy to note that spatial correlation length is estimated with respect to the underlying normally distributed random field.

As k is assumed to be characterized statistically by a lognormal distribution, the correlated standard normal random field, $G(x)$, generated by LAS method is then transformed into a lognormal distribution using the following transformation function:

$$k_i = \exp\{\mu_{\ln k} + \sigma_{\ln k} G(x_i)\} \tag{2}$$

where: x_i and k_i are, respectively, the vector containing the coordinates of the center of the ith element and the soil property value assigned to that element; $\mu_{\ln k}$ and $\sigma_{\ln k}$ are the mean and standard deviation of the underlying normal distribution; $\mu_{\ln k}$ and $\sigma_{\ln k}$ are obtained from the specified permeability μ_k and σ_k using the following lognormal distribution transformation functions (Fenton and Griffiths 2008):

$$\mu_{\ln k} = \ln \mu_k - \frac{1}{2}\sigma_{\ln k}^2 \tag{3}$$

$$\sigma_{\ln k} = \sqrt{\ln\left(1 + \frac{\sigma_k^2}{\mu_k^2}\right)} = \sqrt{\ln\left(1 + \upsilon_k^2\right)} \tag{4}$$

where: $\upsilon_k = \sigma_k/\mu_k$ is the coefficient of variation of permeability. It should be noted that the random fields of both k and m_v are generated using the free available 2D LAS computer code (http://www.engmath.dal.ca/rfem/) implying that the scale of fluctuation in the circumferential direction is infinite (i.e. the soil properties in this direction remain constant).

2.2. Finite Element Modeling Incorporating Soil Spatial Variability

In this study, all numerical analyses are carried out under axisymmetric condition using the finite element computer program AFENA (Carter and Balaam 1995). Soil consolidation in AFENA is analyzed under Biot's consolidation theory (Biot 1941) in which the pore fluid is coupled to the solid by the conditions of equilibrium and continuity. Since a single-drain analysis is often enough to investigate the soil consolidation behavior, the effect of soil spatial variability is examined using a unit cell of soil around a single drain (see Fig. 1). It should be noted that, although the well resistance and smear effect may affect the rate of consolidation, for simplicity, the smear and well resistance are not considered in the current study as they are left for future refinement. In order to determine optimum mesh density with minimal discretization error, a sensitivity analysis for the problem under consideration is carried out. Based on the result of the sensitivity analysis, the problem is discretized into a mesh of 16 × 20 square finite elements. The applied boundary conditions for the problem under consideration are shown in Fig. 1. In soil stabilization by PVDs, soil consolidation takes place by combined vertical and horizontal (radial) drainage of water. However, in practical sense, soil consolidation due to vertical drainage is insignificant (due to large drainage length and lower permeability in the vertical direction) compared to that of the horizontal drainage, thus, only soil consolidation due to horizontal drainage is considered in the current study. The soil skeleton is modeled as a linear elastic solid and the mean value of the spatially variable permeability, μ_k, and volume compressibility, μ_{m_v}, are selected to be equal to 5×10^{-10} m/sec and 1.67×10^{-4} m^2/kN, respectively. The effect of soil spatially variability on the stochastic behavior of soil consolidation by PVDs is investigated over a range of different combinations of standard deviation, σ, and scale of fluctuation, θ. For the interest of generality, σ is presented herein in a normalized form as υ (i.e. coefficient of variation). The following values of υ and θ are considered:

- υ_k (%) = 50, 100, 200 and 400;
- υ_{m_v} (%) = 12.5, 25, 50 and 100; and
- θ = 0.125, 0.25, 0.5, 1.0, 2.0, 4.0 (both k and m_v).

It can be noticed that, υ_{m_v} is selected so as to be one quarter of υ_k. This is due to the fact that k can possess a COV (i.e. υ_k) of as high as 300%, which is much higher than that of COV of m_v (i.e. υ_{m_v}) that usually ranges from 25% to 30% (Baecher and Christian 2003; Kulhawy et al. 1991). However, the same value of θ (i.e. θ_k and θ_{m_v}) is assumed for both k and m_v for simplicity. Since little currently known about the relationship or cross-correlation between k and m_v, the stochastic independence between k and m_v is assumed.

Both the excess pore water pressure and settlement can be used in determining the average degree of consolidation for a coupled system. Since the general trend of the statistics (mean and standard deviation) of the average degree of consolidation, U, estimated either on the basis of excess pore water pressure or settlement remains the same (see e.g. Bari et al. 2012), in this study U at any particular stage of analysis is calculated in terms of excess pore water pressure with the help of the following expression:

$$U(t) = 1 - \frac{\bar{u}(t)}{u_0} \tag{5}$$

where: $U(t)$ and $\bar{u}(t)$ are, respectively, the average degree of consolidation and average excess pore water pressure at a given time t; and u_0 is the initial (uniform) excess pore water pressure. It has to be emphasized that, $\bar{u}(t)$ is obtained by performing numerical integration over the depth and width of the discretized mesh. It should also be noted that, $U(t)$ described in Eq. (5) is the average degree of consolidation over the soil domain but hereafter will be simplified by denoting it as the degree of consolidation. This is to avoid the conflict that may occur with the mean (over a suite of Monte Carlo simulations) degree of consolidation, μ_U, that will be described later in Eq. (6). By invoking each parametric combination of υ and θ into the LAS method, the lognormally distributed random fields of k and m_v at every location of the finite element mesh is generated using the transformation function in Eq. (2). A single generation of such random fields over the finite element mesh and the subsequent finite element analysis is termed "realization".

2.3. Repetition of Process Based on the Monte Carlo Technique

Following the procedures of the Monte Carlo technique, the process of generating random fields of soil properties of interest (i.e. k and m_v) and the subsequent finite element analysis for a certain υ and θ is repeated 1000 times to give reasonably stable statistics for the output quantities of interest. The above process is performed for each set of υ and θ by which the nature of the generated random soil property fields (whether uniform or erratic) is regulated. Fig. 2 shows a typical example of a discretized mesh and the corresponding soil domain represented by a grey scale of a typical permeability field realization in which the magnitude of permeability remains constant within each element but differs from one element to another. The lighter elements represent “higher” soil permeability regions, whereas the darker elements refer to “lower” soil permeability regions.

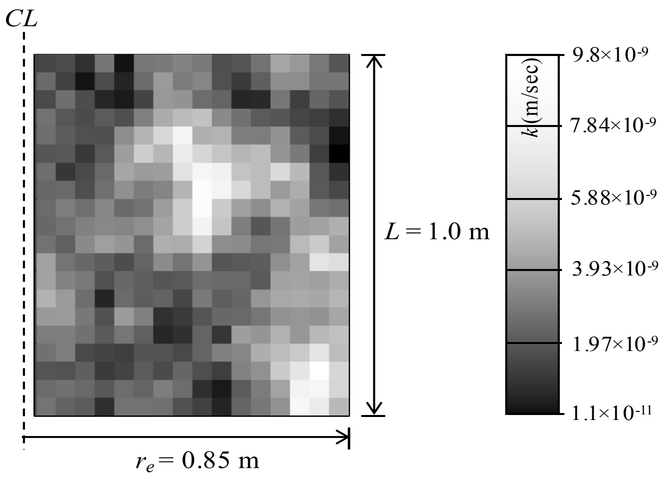


Fig. 2 Typical realization of a random permeability field for υ_k = 100% and θ_k = 0.5 ($\mu_k = 5\times10^{-10}$ m/sec)

The obtained outputs from the suite of 1000 realizations of the Monte Carlo simulation are collated and statistically analyzed to produce estimates of the mean and standard deviation of the degree of consolidation. In this study, at any given time t, the mean of the degree of consolidation based on the excess pore water pressure, μ_U, is estimated by utilizing the geometric average (considered as the representative mean) of $\bar{u}(t)$, as follows:

$$\mu_U = 1 - \exp\left[\frac{1}{n_{sim}}\sum_{i=1}^{n_{sim}}\ln\left(\frac{\bar{u}(t)}{u_0}\right)_i\right] \tag{6}$$

The standard deviation of the average degree of consolidation at any time t defined by the pore water pressure, σ_U, is estimated as follows:

$$\sigma_U = \sqrt{\frac{1}{n_{sim}-1}\sum_{i=1}^{n_{sim}}\left[(U(t))_i - \mu_U\right]^2} \tag{7}$$

where: n_{sim} is the number of Monte Carlo simulations; $(\bar{u}(t)/u_0)_i$ and $(U(t))_i$ are, respectively, the ratio of the average excess pore pressure to the initial excess pore water pressure and the degree of consolidation at any time t for the ith simulation (see Eq. (5)). The use of the geometric average of $\bar{u}$ in computing μ_U is due to the fact that, in a 2D space, the flow of water is not as strongly dominated by the low permeability regions as in 1D space. This is because in 2D space, compared to the 1D space, the flow of water has more freedom to avoid low permeability zones by detouring around them and therefore, the geometric average may be a better estimator for computing the representative mean of the average excess pore water pressures.

3. PROBABILISTIC INTERPRETATION

The estimation of the probability that a deterministic degree of consolidation overestimates the true consolidation value is one of the main objectives of the stochastic consolidation analyses. To determine such probability for a specific stochastic simulation test, it is necessary to establish the probability distribution nature of the degree of consolidation data obtained from the suite of 1000 realizations. In order to obtain a reasonable probability distribution, the degree of consolidation data obtained at any time t from the suite of 1000 realizations are transformed to $U^*(t)$, which is used as an alternative representing form used to the degree of consolidation $U(t)$. The reason for using $U^*(t)$ instead of $U(t)$ is that the obtained fit using the raw data of $U(t)$ was typically poor while a reasonable probability distribution for the obtained degree of consolidation data is better facilitated using $U^*(t)$, which gives sufficiently reasonable approximation of the degree of consolidation behavior of natural soils. $U^*(t)$ is assumed to be lognormally distributed and can be determined as follows:

$$U^*(t) = \ln\left[\frac{1}{1-U(t)}\right] \tag{8}$$

Detailed description of the analytical formulations used to derive the rationality of the lognormal distribution hypothesis for $U^*(t)$ is beyond the scope of this paper and can be found elsewhere (Bari et al. 2011). The legitimacy of the lognormal distribution hypothesis for $U^*(t)$ is examined by the well-known Chi-square test through frequency density plot of $U^*(t)$ data obtained from the 1000 realizations. This process is performed for many combinations of υ and θ at several different consolidation times. For each of the cases considered, the goodness-of-fit p-value is found to be high enough to approve the rationality of the lognormal distribution hypothesis of simulated $U^*(t)$ data. Fig. 3 illustrates a typical example of the histogram of $U^*(t)$ for the case of $\upsilon_k = \upsilon_{m_v} = 200\%$, $\theta_k = \theta_{m_v} = 0.5$ at 271.6 days, along with their fitted lognormal distributions. The goodness-of-fit test yielded p-value of 0.83, indicating strong agreement between the histogram and the fitted distribution. Therefore, the lognormal distribution is certainly an appropriate assumption to the distribution of the simulated $U^*(t)$ data.

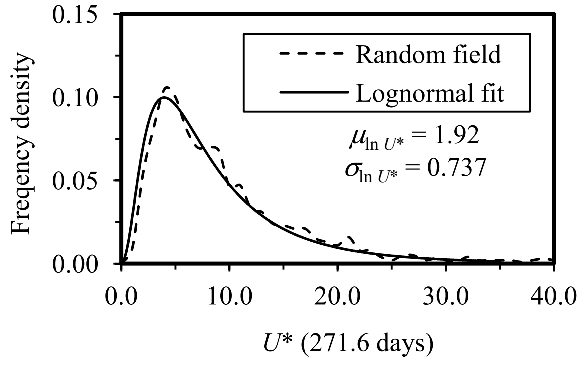


Fig. 3 Typical example of frequency density histogram of simulated $U^*(t)$ with fitted lognormal distribution for $\upsilon_k = \upsilon_{m_v} = 200\%$, $\theta_k = \theta_{m_v} = 0.5$ at 271.6 days

By accepting the lognormal distribution as a reasonable fit for $U^*(t)$, the statistical moments, μ_{U^*} and σ_{U^*} that represent the mean and standard deviation of the lognormally distributed $U^*(t)$ are calculated for each set of υ and θ from the suite of 1000 realizations using method of moments. In this study, it is assumed that the target degree of consolidation is 90% and for convenience, it is simply denoted as U_{90}. For 90% target degree of consolidation (U_{90}) (i.e. when $U(t) = 0.9$), $U^*(t) = \ln[1/(1-0.9)] = 2.3026$. Therefore, the probability of getting $U^*(t) \geq 2.3026$ (i.e. $P[U^*(t) \geq 2.3026]$) will be equal to the probability of achieving $U(t) \geq 90\%$ (i.e. $P[U(t) \geq U_{90}]$) and the $P[U(t) \geq U_{90}]$ can be estimated as follows:

$$P[U(t) \geq U_{90}] = P[U^*(t) \geq 2.3026] = 1 - \Phi\left(\frac{\ln 2.3026 - \mu_{\ln U^*}}{\sigma_{\ln U^*}}\right) \tag{9}$$

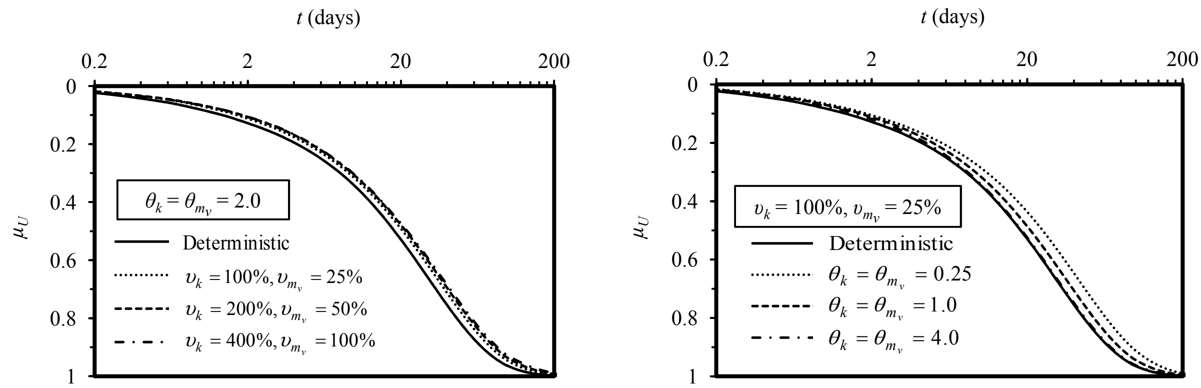
where: P [.] is the probability of its argument; $\Phi(.)$ is the standard normal cumulative distribution function; $\mu_{\ln U^*}$ and $\sigma_{\ln U^*}$ are, respectively, the mean and standard deviation of the underlying normally distributed $\ln U^*(t)$ and can be estimated from μ_{U^*} and σ_{U^*} using transformation equations between lognormal and normal distribution (see Eqs. (3) and (4)). Following the procedure set out above, probabilities of achieving 90% degree of consolidation at any time can be estimated for any combination of υ and θ, and the stochastic behavior of soil consolidation by PVDs can be investigated.

4. RESULTS AND DISCUSSION

In order to investigate the sensitivity of the statistics of the degree of consolidation and probability of achieving 90% consolidation to the statistically defined input data (i.e. υ and θ) in relation to both k and m_v, a series of axisymmetric consolidation analyses are performed. For each selected set of υ and θ, 1000 Monte Carlo simulations are performed. The obtained consolidation responses are then statistically analyzed to estimate μ_U, σ_U and $P[U \geq U_{90}]$ using the excess pore water pressure. Since the general trends of μ_U, σ_U and $P[U \geq U_{90}]$ remain unaltered over the specified range of υ and θ, only the results of a few of the tests conducted are presented in Figs. 4-6, which are believed to be sufficient to demonstrate the main features of the influence of spatial variability of k and m_v on soil consolidation by PVDs. In Figs. 4-6 μ_U, σ_U and $P[U \geq U_{90}]$ are expressed as a function of the consolidation time t. Prior to put the stochastic analyses into context, an initial deterministic solution has been performed assuming a homogeneous soil. It should be noted that the deterministic solution of this case yields U_{90} at t = 67.9 days (i.e. t_{D90} = 67.9 days).The results obtained from this study are described below.

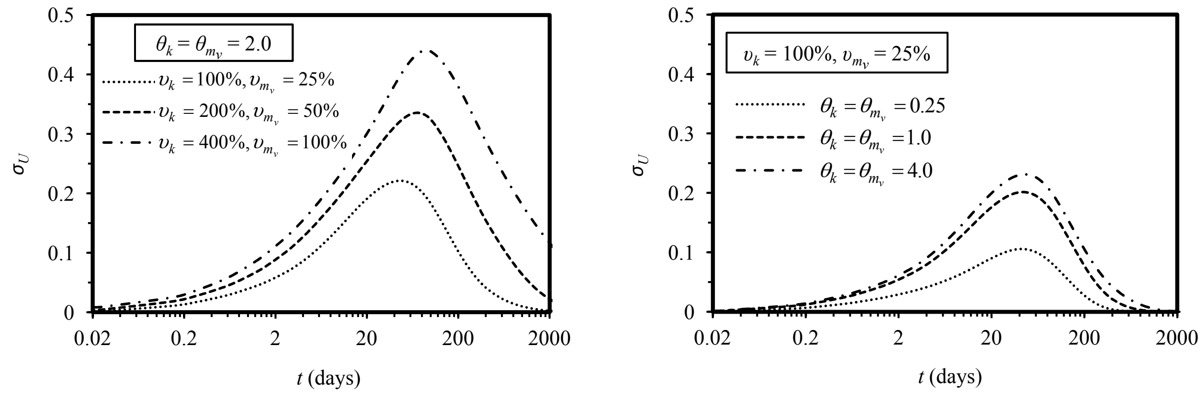
- *Effects of υ and θ on the mean and standard deviation of U*

The effect of υ on μ_U for a constant value of $\theta_k = \theta_{m_v} = 2.0$ is shown Fig. 4(a). It can be seen that at any particular consolidation time, μ_U decreases marginally with the increase of υ. At any certain time, a decrease in μ_U with the increase of υ can be explained by noting that a higher υ makes the heterogeneous system more erratic, so that the low k values and relatively higher compressible zones (as $\upsilon_{m_v} < \upsilon_k$) are bunched together in most of the simulations, resulting in a decrease in the average coefficient of consolidation. It should be noted that, this observation is opposite to that found for $\upsilon_{m_v} = \upsilon_k$ case (see Bari et al. 2012) and indicates that the variational trend of μ_U (i.e. decreases or increases with the increase of υ) with respect to υ depends on the ratio of υ_k to υ_{m_v}. Fig. 4(b) shows the effect of θ on μ_U for a fixed value of υ_k = 100% and υ_{m_v} = 25%. It can be seen that at any particular consolidation time, t, there is a gradual increase in μ_U as θ increases. It is also interesting to see that for ragged random fields with a smaller θ, the μ_U curve approaches the deterministic curve. This behavior is expected, as for small θ, both k and m_v with low and high values are distributed quite uniformly throughout the domain, implying an average coefficient of consolidation close to the deterministic coefficient of consolidation. As the random fields become smooth with higher θ, high k values and comparatively lower m_v values tend to bunch together in most of the simulations (this is possibly because k and m_v are uncorrelated). Consequently, there is an increase in the average coefficient of consolidation compared to the deterministic coefficient of consolidation and in turn the μ_U.



(a) (b)

Fig. 4 Effect of: (a) υ on μ_U for θ = 2.0; (b) θ on μ_U for υ_k = 100%, υ_{m_v} = 25%



(a) (b)

Fig. 5 Effect of: (a) υ on σ_U for θ = 2.0; (b) θ on σ_U for υ_k = 100%, υ_{m_v} = 25%

The influence of increasing υ and θ on σ_U is investigated in Fig. 5. It can be seen that σ_U increases with the increase of υ as shown in Fig. 5(a). This behavior is 'intuitive' due to the fact that the larger the value of υ, the more chance is there for a low k to come with low m_v in one simulation and vice versa for another simulation. As a result, the potential coefficient of consolidation value will be exaggerated. The effect of θ on σ_U is illustrated in Fig. 5(b) for a constant value of υ_k = 100% and υ_{m_v} = 25%. It can be seen that at any certain consolidation time, t, σ_U increases with the increase of θ. For large correlation length, σ_U is also expected to be large as there is less averaging variance reduction within each realization.

- *Effects of υ and θ on the probability of achieving* 90% *consolidation*

The effects of the spatial variability of k and m_v on the probability of achieving 90% consolidation are shown in Fig. 6. The deterministic time of achieving 90% consolidation, t_{D90}, is also shown in Fig. 6 by vertical solid lines to give $P[U \geq U_{90}]$ at that time for any combination of υ and θ.

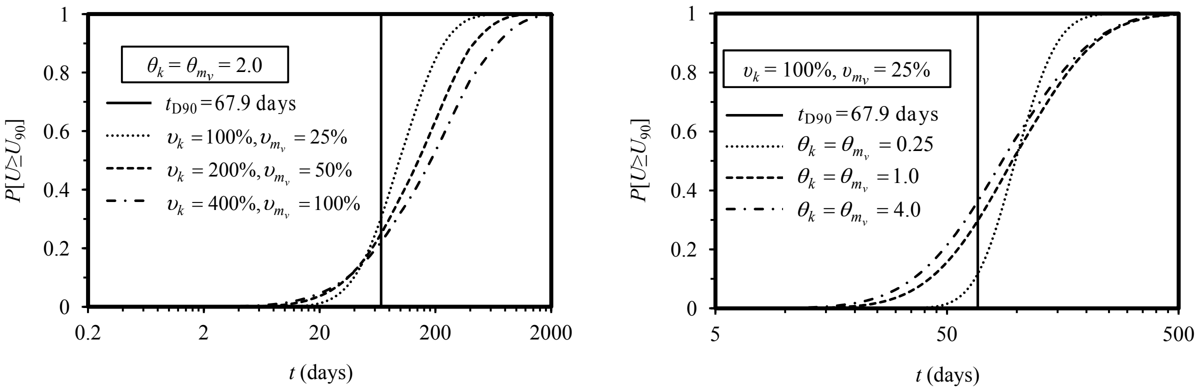


Fig. 6 Effect of: (a) υ on $P[U \geq U_{90}]$ for θ = 2.0; (b) θ on $P[U \geq U_{90}]$ for υ_k = 100%, υ_{m_v} = 25%

Fig. 6(a) illustrates the effect of varying υ on $P[U \geq U_{90}]$ at a fixed value of $\theta_k = \theta_{m_v}$ = 2.0. It can be seen that, at any certain consolidation time, $P[U \geq U_{90}]$ decreases with the increase of υ. The exception to this trend occurs before the deterministic 90% consolidation time (i.e. t_{D90}) where the role of υ has the opposite effect, with lower values of υ tending to give the lowest values of $P[U \geq U_{90}]$. This is expected because the range of values of U^* (or U) over which the frequency density curve is distributed increases as υ increases. In other words, U^* distribution ''bunching up'' at low υ rapidly excludes the area to the right of the stationary target value of U^* = 2.3026.

The effect of θ on $P[U \geq U_{90}]$ for a constant value of υ_k = 100% and υ_{m_v} = 25% is investigated in Fig. 6(b). It can be seen that, initially the time rate of $P[U \geq U_{90}]$ decreases as θ increases (e.g. θ = 1.0), then it starts to increases for large θ (e.g. θ = 4.0). This behavior can be explained by noting that, when θ = 0, the simulated soil profile will consist of an infinite number of independent 'observations' of which the average coefficient of consolidation is equal to the true mean coefficient of consolidation (or true median, if the average is a geometric average). Since the rate of consolidation depends also on the average coefficient of consolidation, it 'sees' the same true mean (or true median) value predicted by the soil profile. Consequently, the predicted mean of the degree of consolidation becomes 'perfect' when the correlation length is zero and therefore the probability of achieving a desired degree of consolidation approaches 100%. At the other extreme of θ, when $\theta = \infty$, the soil becomes uniform, having the same value everywhere. In this case, any soil profile also perfectly predicts conditions in the unit cell. At intermediate θ the soil profile becomes imperfect estimator of the conditions surrounding the PVD, and the time rate of $P[U \geq U_{90}]$ decreases. Therefore, the maximum decrease in the time rate of $P[U \geq U_{90}]$ will occur at some correlation length between 0 and ∞. The precise value depends on the geometric characteristics of the problem under consideration and the COV of spatially variable soil properties.

CONCLUSIONS

This paper has used the random field theory and finite element modeling to investigate the influence of soil spatial variability, over a range of values of coefficient of variation and scale of fluctuation, on soil stabilization by prefabricated vertical drains. Both the coefficient of permeability, k, and coefficient of volume compressibility, m_v, were treated as independent random variables and Biot consolidation analysis was applied. The results obtained from the study led to the following findings:

1. Increasing the input υ generally decreased the mean of the degree of consolidation. The standard deviation of the degree of consolidation increased with the increase of coefficient of variation;
2. Increasing the scale of fluctuation generally increased the mean and standard deviation of the degree of consolidation. However, for large θ (e.g. $\theta > 1.0$), the influence of θ on the mean and standard deviation of the degree of consolidation was marginal; and
3. The time rate of the probability of achieving 90% consolidation decreased with the increase of υ, as expected. The time rate of the probability of achieving 90% consolidation initially decreases as θ increases (e.g. $\theta = 1.0$), then it starts to increases for large θ (e.g. $\theta = 4.0$). The probability of achieving 90% consolidation at a consolidation time corresponding to the deterministically predicted 90% consolidation time was found to be always be less than 50% over the range of the statistical parameters considered.

Overall, the results obtained from this research highlight the significant influence of soil spatial variability on soil consolidation via PVDs and clearly demonstrate the benefit of stochastic analyses in routine design practice.

REFERENCES

Badaoui, M., Nour, A., Slimani, A., and Berrah, M. K. (2007). "Consolidation statistics investigation via thin layer method analysis." *Transport in Porous Media*, Vol. **67**(1), 69-91.

Baecher, G. B., and Christian, J. T. (2003). *Reliability and statistics in geotechnical engineering*, John Wiley & Sons, West Sussex, England.

Bari, M. W., Shahin, M. A., and Nikraz, H. R. (2011). "Probabilistic analysis of soil consolidation via prefabricated vertical drains." *International Journal of Geomechanics, ASCE*, under review.

Bari, M. W., Shahin, M. A., and Nikraz, H. R. (2012). "Investigation into Consolidation by Vertical Drains in Spatially Variable soils." *Geomechanics and Engineering*, submitted.

Barron, R. A. (1948). "Consolidation of in-grained soil by drain wells." *Transactions of the American Society of Civil Engineering*, Vol. **113**, 718-754.

Biot, M. A. (1941). "General theory of three-dimensional consolidation." *Journal of Applied Physics*, Vol. **12**, 155-164.
Carter, J. P., and Balaam, N. P. (1995). "Program AFENA - A general finite element algorithm: Users' Manual." Centre for Geotechnical Research, University of Sydney, Sydney.
Elkateb, T., Chalaturnyk, R., and Robertson, P. K. (2002). "An overview of soil heterogeneity: quantification and implications on geotechnical field problems." *Canadian Geotechnical Journal*, Vol. **40**, 1-15.
Fenton, G. A., and Griffiths, D. V. (2008). *Risk assessment in geotechnical engineering*, Wiley, New York.
Fenton, G. A., and Vanmarcke, E. H. (1990). "Simulation of random fields via local average subdivision." *Journal of Engineering Mechanics*, Vol. **116**(8), 1733-1749.
Freeze, R. A. (1977). "Probabilistic one-dimensional consolidation." *Journal of Geotechnical Engineering Division*, Vol. **103**(GT7), 725-742.
Hansbo, S. (1981) "Consolidation of fine-grained soils by prefabricated drains." *Proceedings of the 10th International Conference on Soil Mechanics and Foundation Engineering*, Stockholm, Sweden, 677-682.
Hong, H. P. (1992). "One-dimensional consolidation with uncertain properties." *Canadian Geotechnical Journal*, Vol. **29**(1), 161-165.
Hong, H. P., and Shang, J. Q. (1998). "Probabilistic analysis of consolidation with prefabricated vertical drains for soil improvement." *Canadian Geotechnical Journal*, Vol. **35**(4), 666-677.
Huang, J., and Griffiths, D. V. (2010). "One-dimensional consolidation theories for layered soil and coupled and uncoupled solutions by finite-element method." *Géotechnique*, Vol. **60**(9), 709-713.
Huang, J., Griffiths, D. V., and Fenton, G. A. (2010). "Probabilistic analysis of coupled soil consolidation." *Journal of Geotechnical and Geoenvironmental Engineering*, Vol. **136**(3), 417-430.
Hwang, D., and Witczak, M. W. (1984). "Multidimensional probabilistic consolidation." *Journal of Geotechnical Engineering*, Vol. **110**(8), 1059-1077.
Indraratna, B., Bamunawita, C., Redana, I. W., and McIntosh, G. (2003). "Modelling of prefabricated vertical drains in soft clay and evaluation of their effectiveness in practice." *Ground Improvement*, Vol. **7**(3), 127-137.
Kulhawy, F. H., Roth, M. J. S., and Grigoriu, M. D. (1991) "Some statistical evaluations of geotechnical properties." *Proceedings of the 6th International Conference on Applied Statistical Problems in Civil Engineering (ICASP 6)*, Mexico City, 705-712.
Pyrah, I. C. (1996). "One-dimensional consolidation of layered soils." *Géotechnique*, Vol. **46**(3), 555-560.
Rowe, P. W. (1972). "The relevance of soil fabric to site investigation practice." *Géotechnique*, Vol. **22**(2), 195-300.
Zhou, W., Hong, H. P., and Shang, J. Q. (1999). "Probabilistic design method of prefabricated vertical drains for soil improvement." *Journal of Geotechnical and Geoenvironmental Engineering*, Vol. **125**(8), 659-664.