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Alternative Approach of Pressure-Impulse Diagram for Multi-Peak Pressure

*Jong Yil Park[1)], Theodor Krauthammer[2)] and WooSeok Kim[3)]

[1)] *Department of Safety Engineering, Seoul National University of Science and Technology, Seoul, Korea*
[2)] *Department of Civil Engineering, University of Florida, Florida, US*
[3)] *Department of Civil Engineering, ChungNam National University, Korea*

[1)] jip111@seoultech.ac.kr

ABSTRACT

A new approach to estimate the structural damage under explosive loading with multiple peak pressures is presented. Integration is adopted to characterize the loading shape. The maximum displacements from the proposed method show less than 18% difference from the numerical solutions, with very efficient calculation time.

1. Problem Statement

When a structural system is subjected to an external force, the damage can be determined based on a threshold structural response (e.g., cracking, yielding, fracture, a specific displacement, etc.). The P-I diagram represents combinations of peak load and corresponding impulse of the applied force that cause a predetermined level of structural response. Generally, the simple shape of a loading history (rectangular, triangular, and exponential with zero rise time) is assumed for the most of available P-I diagrams. However, the loading history from detonations, when there are reflections (e.g., confined detonation), has multiple peaks rather than a single peak. In the classical P-I approach, one addresses a dynamic load with a single peak. However, addressing pressure pulses with multiple peaks needs to be considered.

In this study, an alternative method that can be applied for both single and multiple peak loading to determine structural damage is presented. The pressure-time history measured by sensors, or calculated from hydro-codes, is generally expressed in tabulated format of pressure and time pairs, and it can be used directly for the fast damage calculation. One of the simplest calculations for tabulated data could be integration. Thus, first and second order integrations of pressure-time histories are adopted to characterize the pressure-time history. Also, the displacement-time history

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[1)] Assistant Professor
[2)] Goldsby Professor of Civil Engineering
[3)] Assistant Professor

is assumed as a simplified function.

2. Methodology

The response function was assumed, as shown below, which is valid until the time is B:

$$\dot{x} = At(B - t)$$
$$x = \frac{AB}{2}t^2 - \frac{A}{3}t^3 \tag{1}$$

where

$$x|_{t=0} = 0$$
$$\dot{x}|_{t=0} = 0$$
$$\dot{x}|_{t=B} = 0$$

Then, the maximum displacement is:

$$x_{max} = x|_{t=B} = \frac{AB^3}{6} \tag{2}$$

The governing equation for an undamped perfectly elastic SDOF system is, as follows :

$$F(t) = M\ddot{x} + Kx \tag{3}$$

where, F(t) is the external force, M and K are equivalent mass and stiffness, respectively.

Integrating Eq (3) from t=0 to t=B, one obtains the following equation if B is less than the force duration t_d :

$$\int_0^B F(t)dt = \int_0^B (M\ddot{x} + Kx)dt$$
$$I_1 = \int_0^B Kx\,dt = K\frac{AB^4}{12} = \frac{KB}{2}x_{max} \tag{4}$$

where I_1 is the impulse of the external force.

Integrating Eq (3) twice from t=0 to t=B, one obtains the following equation if B is less than the force duration t_d :

$$\iint_0^B F(t)dt = \iint_0^B (M\ddot{x} + Kx)dt$$

$$B^2 + \frac{10}{7}\left(\frac{I_2}{I_1} - t_d\right)B - \frac{20}{7\omega^2} = 0 \quad (5)$$

Where

$$\iint_0^{t_d} F(t)dt = I_2$$

$$\omega = \sqrt{K/M}$$

Combining Eqs (4) and (5), the maximum displacement can be expressed as shown below:

$$\frac{x_{max}}{P_{max}/K} = \frac{2I_{n1}}{-S+\sqrt{S^2+\frac{20}{7(\omega t_d)^2}}} \quad \text{when } \omega t_d < \sqrt{\frac{20}{7(2S+1)}} \quad (6)$$

where P_{max} is the maximum peak pressure, and

$$S = \frac{5}{7}\left(\frac{I_{n2}}{I_{n1}} - 1\right)$$

$$I_{n1} = I_1/(P_{max}t_d)$$

$$I_{n2} = I_2/(P_{max}t_d^2)$$

3. Comparison with closed form and numerical solutions

The difference between the proposed Eq. (6) and exact solutions for a triangular loading pulse with zero rise time, a rectangular loading pulse (Krauthammer, 2008), and numerical solution for multi peak loading pulse were computed, as discussed next.

3.1 Triangular loading pulse with zero rise time and rectangular loading pulse

The differences range between 0.1 and 18% compared to the closed form solutions, as shown in Table 1 and 2.

Table 1. Triangular loading pulse with zero rise time

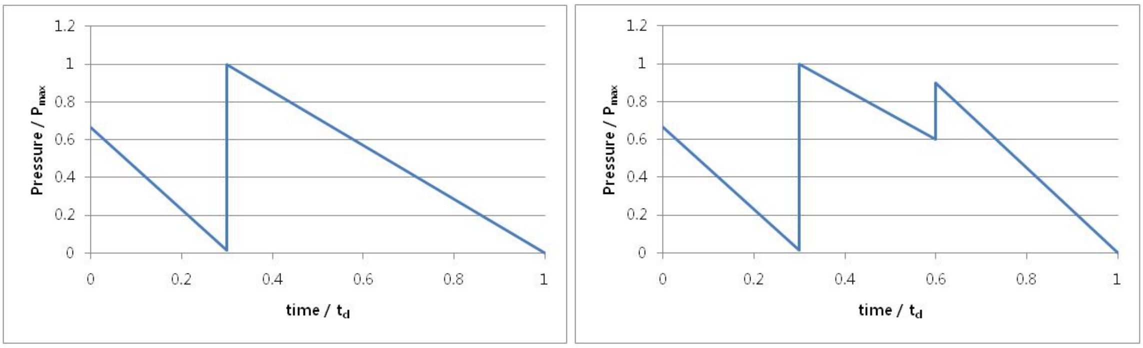
ωt_d	$\frac{x_{max}}{P_{max}/K}$		Difference (%)
	Closed Form Solution	Eq. (6)	
0.001	0.0005	0.0006	18.3
0.2509	0.1252	0.1433	14.4
0.5008	0.2487	0.2761	11.0
0.7507	0.3695	0.3996	8.2
1.0006	0.4865	0.5144	5.7
1.2505	0.5986	0.6209	3.7
1.5004	0.7044	0.7197	2.2
1.7503	0.8032	0.8112	1.0
2.0002	0.8938	0.896	0.2
2.2501	0.9755	0.9745	-0.1

Table 2. Rectangular loading pulse with zero rise time

ωt_d	$\frac{x_{max}}{P_{max}/K}$		Difference (%)
	Closed Form Solution	Eq. (6)	
0.001	0.001	0.0012	18.3
0.3509	0.3491	0.3855	10.4
0.7008	0.6865	0.7155	4.2
1.0507	1.003	0.9975	-0.6
1.4006	1.2889	1.2378	-4.0
1.7505	1.5354	1.4423	-6.1
2.1004	1.735	1.616	-6.9
2.4503	2	1.7637	-11.8
2.8002	2	1.8894	-5.5
3.1501	2	1.9966	-0.2
3.5	2	ωt_d is out of limit	

3.2 Multi peak loading pulse

Two sample multi peak loading pulses were adapted as shown in Fig. 1. The difference is between 0.1 and 18% compared to the numerical solutions (MATLAB), as shown in Table 3 and 4



(a) Case 1 (b)Case 2

Figure 1. Multi peak pressure

Table 3. Case 1

ωt_d	$\frac{x_{max}}{P_{max}/K}$ Numerical Solution	$\frac{x_{max}}{P_{max}/K}$ Eq. (6)	Difference (%)
0.001	0.0005	-0.0005	18.4
0.2609	0.118	0.1334	-13.1
0.5208	0.2342	0.2538	-8.4
0.7807	0.3478	0.3628	-4.3
1.0406	0.4576	0.4612	-0.8
1.3005	0.5623	0.5501	2.2
1.5604	0.6609	0.6301	4.7
1.8203	0.7524	0.7022	6.7
2.0802	0.8357	0.7671	8.2

Table 4. Case 2

ωt_d	$\frac{x_{max}}{P_{max}/K}$ Numerical Solution	$\frac{x_{max}}{P_{max}/K}$ Eq. (6)	Difference (%)
0.001	0.0005	0.0006	-18.4
0.2609	0.142	0.1601	-12.7
0.5208	0.2819	0.3039	-7.8
0.7807	0.4183	0.4331	-3.5
1.0406	0.5498	0.5491	0.1
1.3005	0.6747	0.653	3.2
1.5604	0.7916	0.7461	5.8
1.8203	0.8993	0.8293	7.8
2.0802	0.9964	0.9038	9.3

4. CONCLUSIONS

A new approach is presented to calculate the structural response based on first and second order integration of pressure-time history for multi peak loading. Further study of this approach is required to reduce the observed differences between the approach and closed form and numerical solutions.

REFERENCES

Theodor Krauthammer (2008), “Modern Protective Structure” CRC Press