

Application of Fourier series approach in a time domain buffeting analysis considering frequency-dependency of aerodynamic admittance function

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ABSTRACT

A time domain buffeting analysis on bluff bridge section with an aerodynamic admittance function is presented. Formulation of Fourier series approach the aerodynamic admittance function is proposed and frequency-dependency of aerodynamic admittance function is considered through this approximation. Fourier series approach (FSA) satisfies causality condition, as rational function approach(RFA) does, and trigonometric function, the basis function of FSA, is powerful to approximate complicated shaped transfer function. To verify applicability of the proposed scheme, RFA, conventional manner to consider aerodynamic admittance function is compared as a reference. To verify validity of proposed method, rectangular section is adopted. Since the results from both methods are almost same, applicability of the proposed scheme was verified. The aerodynamic admittance function of H-type section, measured from experiments, is selected for the buffeting analysis.

INTRODUCTION

To perform a time domain buffeting analysis, it is necessary to handle aerodynamic transfer functions such as flutter derivatives(FD's) and aerodynamic admittance function (AAF). Since FD's is governing factor of stability problem and has big effect on non-stationary behavior of bridge deck, it has been considered important in time domain aeroelastic analysis. On the contrary, AAF is ignored in time domain analysis because application of AAF estimates dynamic response less than that of the case without AAF. It is valid approach for rectangular type bridge section. All values of the AAF for rectangular section are less than 1 (Jancauskas and Melbourne 1986). And Sears' function, which is widely used AAF for vertical wind component, is always less than 1, also (Sears 1941). That means dynamic response with AAF of rectangular section will be cut down in whole frequency range. However, some AAFs of practical bridge sections have larger values than 1 in several frequency section (Hatanaka et al. 2002 and Diana et al. 2008). When those kind of AAF's are applied to an analysis, amplitude of dynamic response was amplified (Kim 2013). Therefore, it is potentially dangerous to neglect the effect of the AAF of bridge deck and important to consider AAF in time domain buffeting analysis.

Rational function approximation (RFA) has been widely used, as a conventional way to consider aerodynamic transfer functions in time domain. Chen (2000) carried out time

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domain buffeting analysis for 3-dimensional cable supported bridge with RFA. Applied AAF was Sears' function and it is not actual AAF of bridge deck used in analysis. As mentioned, Sears' function always reduces the dynamic response. Therefore it is required to apply measured AAF of bridge deck. There have been carried out several attempts to apply measured AAF on time domain analysis.

Hatanaka (2002) proposed a method to use measured FD's to evaluate AAF of rectangular sections. By using equivalent Theodorsen's circulation function, he defined relationship between AAF and FD's. AAF's obtained from FD's were similar with measured AAF. But this kind of indirect way to evaluate AAF is not precise. Costa (2006) and Costa (2007) have perform time domain analysis with measured AAF of rectangular section and applied RFA. Since this is direct approach with actual AAF, it is advanced way compared with Hatanaka's. But only rectangular sections were studied. Since AAF's of rectangular sections are in smooth shape and RFA is working very well in this case. But Transfer functions of the bridge section, which normally have adjunctive structures on itself, is in an intricate shape. AAF's of practical bridges has intricate shape, also (Diana et al. 2008 and Kim et al. 2008).

Caracoglia (2003) raised a question for applying RFA on bluff section. He insists that even if RFA is working well in smooth type transfer function, it has potential problems when applied to that of bluff section. In addition, Jung (2012) show that RFA application on h-type section yields erroneous results in steady state solution. To solve several limitation of RFA, Park (2014) proposed new method to consider aerodynamic transfer function using Fourier series in time domain. Fourier series is very powerful to approximate arbitrary shaped function and causality condition is strongly imposed by modified coefficients of Fourier series. Originally, this method is developed to evaluate FD's. Since both AAF and FD's are transfer function, Fourier series approximation (FSA) can be extended to consider AAF in time domain analysis. In this study, Formulation of FSA for admittance is derived and its validity is verified by comparison of RFA and FSA. And research scope only is focused on lift force and vertical wind component.

FORMULATION OF FOURIER SERIES APPROXIMATION

Lift force induced by vertical wind component is defined as follow (Strommen 2010):

$$L_w(x,t) = \frac{\rho UB}{2} (C'_L + \bar{C}_D) \chi_{Lw} w(t) \quad (1)$$

where ρ =air density; U =Mean wind velocity; B =width of bridge deck; C'_L : 1st derivative of lift coefficient; \bar{C}_D =Drag coefficient; χ_{Lw} =admittance function between Lift and vertical wind component. Since AAF, χ_{Lw} is complex function, it is possible to divide into real and imaginary part. After taking Fourier transform in both side,

$$\bar{L}_w(\omega_w) = \frac{\rho UB}{2} (C'_L + \bar{C}_D) (\chi_{Lw}^R + i \chi_{Lw}^I) \bar{w}(\omega_w) \quad (2)$$

AAF can be approximated by Fourier series. FSA hold causality condition strongly by using pair coefficient of cosine and sine functions for Fourier series (Park 2014):

$$\bar{\phi}_{Lw}^R(K) = a_{Lw}^0 + \sum_{n=1}^N a_{Lw}^n \cos \frac{n\pi}{K_{\max}} K, \quad \bar{\phi}_{Lw}^I(K) = b_{Lw}^0 K - \sum_{n=1}^N a_{Lw}^n \sin \frac{n\pi}{K_{\max}} K \quad (3)$$

where χ_{Lw}^{R*} and χ_{Lw}^{I*} = real and imaginary part of modified AAF; a_{Lw}^i and b_{Lw}^i = unknown coefficients of the modified AAF. Unknown coefficients would be obtained by solving quadratic optimization problem. Then Impulse response function (Park 2014) and Lift force with coefficients are derived as follows:

$$\Phi_{kl}(t) = a_{kl}^0 \delta(t) + b_{kl}^0 \frac{B}{U} \dot{\delta}(t) + \sum_{n=1}^N a_{kl}^n \delta\left(t - \frac{B}{U} \frac{n\pi}{K_{\max}}\right) \quad (4)$$

$$L_w(t) = \frac{\rho U B}{2} (C_L' + \bar{C}_D) \left(a_{Lw}^0 w(t) + b_{Lw}^0 \frac{B}{U} \dot{w}(t) + \sum_{n=1}^N a_{Lw}^n w\left(t - \frac{B}{U} \frac{n\pi}{K_{\max}}\right) \right) \quad (5)$$

VERIFICATION OF THE PROPOSED METHOD

To verify validity of the proposed method, airfoil section is adopted with Sears' function as an AAF. Results from RFA and FSA are compared in Fig 1. Analysis period is 600 seconds. Both results are almost same in whole analysis period. Since RFA is working well for Sears' function, if response from FSA is same with that of RFA, validity of the proposed method is verified.

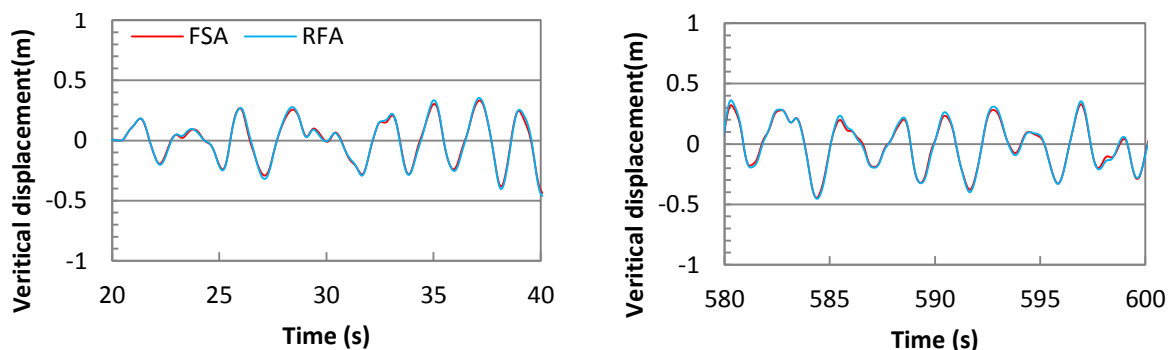


Fig. 1 Comparison of RFA and FSA

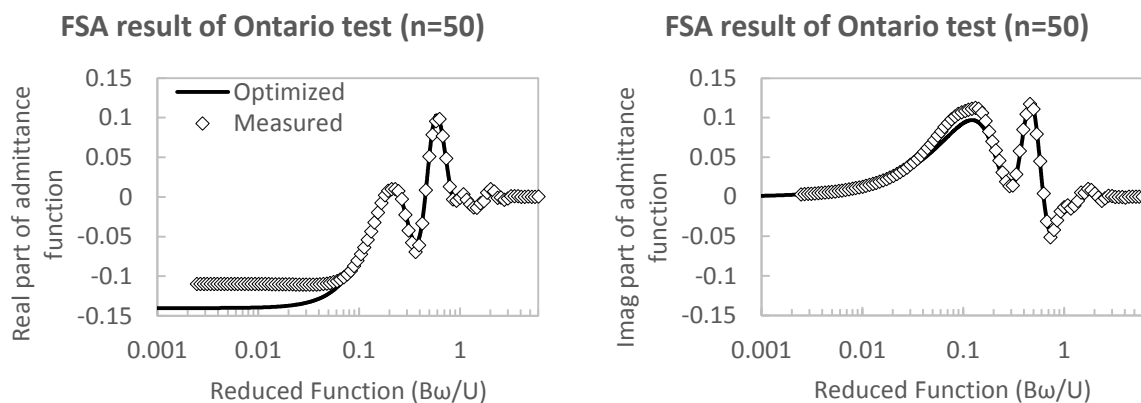


Fig 2. FSA on H-type section

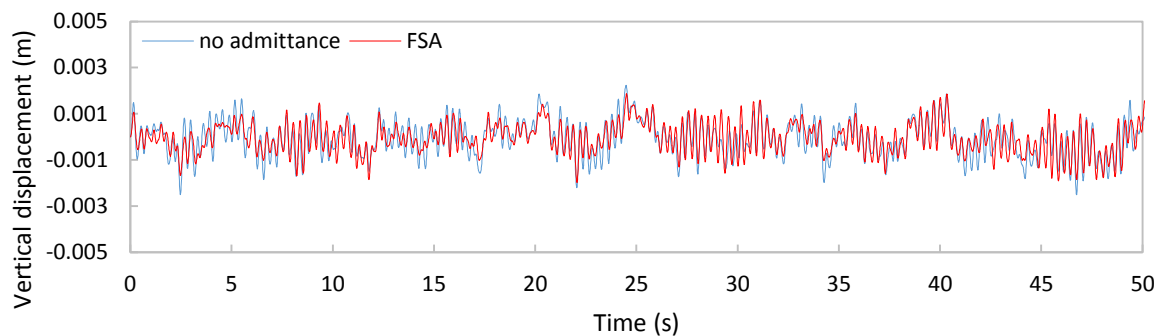


Fig 3. Vertical response of H-type section with FSA

APPLICATION OF FSA ON H-TYPE SECTION

AAF of H-type section is approximated by FSA. Acceptable fitness was obtained with fifty order of Fourier series. Approximation results are shown in the Fig 2. Time domain buffeting analysis of H-type section is shown in the Fig 3. Buffeting analysis of H-type section is successfully done with FSA. To show if the analysis is right, further study is necessary. Verification from experiment and frequency domain analysis is necessary.

CONCLUSIONS

Formulation of FSA for AAF is derived and verified with conventional method, RFA. Application of H-type bluff section is performed successfully. However, experimental and theoretical verification is necessary to show if the analysis result is right.

ACKNOWLEDGEMENT

This research was supported by the grant (13CCTI-A052531-06-000000) from the Ministry of Land, Infrastructure and Transport of Korean government through the Core Research Institute at Seoul National University for Core Engineering Technology Development of Super Long Span Bridge R&D Center.

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