The 2014 World Congress on
Advances in Civil, Environmental, and Materials Research (ACEM14)
Busan, Korea, August 24-28, 2014

Gust-induced fatigue cycle counts - sensitivity to dynamic response, wind climate and direction

Frank H. Kemper[1)] and John D. Holmes[2)]

[1)] *Institute for Steel Structures, RWTH Aachen University, 52074 Aachen, Germany*
[1)] *kemper@stb.rwth-aachen.de*
[2)] *JDH Consulting, Mentone, Victoria 3194, Australia*
[2)] *john.holmes@jdhconsult.com*

ABSTRACT

Structural failures caused by gust-induced vibrations are still difficult to predict, as the number of influencing parameters is large and the knowledge of their exact values for a certain design task is often deficient. In order to estimate the failure risk due to fatigue it is therefore helpful to get to know the most important influencing parameters and to understand their impact on the expectable fatigue lifetime prediction. For this aim, it is important to introduce a detailed calculation strategy which is able to consider all relevant influencing parameters and which is manageable with respect to the computational effort in order to perform the desired parameter calculations. In the present paper, a frequency based computation principle for the identification of the Rainflow-like cycle count spectra is introduced and explained in detail. For a correct scaling of the stress ranges, the determination of an appropriate peak factor is discussed. Afterwards, the described methods are applied in order to analyze the influencing parameters exhaustively.

1. INTRODUCTION

Wind-excited stress variations may lead to structural failures caused by fatigue. Therefore, a proper determination of the expected gust induced vibrations and a subsequent stress analysis including rainflow cycle counting is needed. Whereas for the former, spectral methods are well established, for the latter various suggestions have been made in recent years.

In most previous studies, the well-known Rice formula has been used for the calculation of the number of cycle counts for gust-excited structures. In the paper, this analytical approach is described briefly. Subsequently, the Dirlik equation as a spectral rainflow counting method is used. The difference between both methods is shown for different dynamic properties of the considered structure. In this manner, the limit of applicability of the analytical approach is established.

The short term cycle counts are convolved with site-related wind statistics in order to obtain realistic predictions of the lifetime distribution of stress cycles. Based on the methodology presented, an extensive parameter study has been performed in order to discover the dependencies between structural and aerodynamic properties on the resulting stress-range distributions. It has been found out that the stress range distribution is mainly influenced by the shape parameter *k* of the Weibull distribution of the wind speed on site and by the structural frequency *f* and the structural damping d. Based on this conclusion, the relationship between the former influence parameters and the resulting shape of the cycle count distribution has been analyzed in detail. As a result, a description of the expected cycle count distributions for arbitrary systems can be suggested in order to simplify the dimensioning of slender structures, which are especially vulnerable to gust-induced fatigue problems.

As a last step, the methodology is used to investigate the influence of the wind direction on the risk of a fatigue failure. Taking into account direction-related Weibull distributions for the considered site, both the number of cycles and the occurring amplitudes are modified. In the paper, a general method is presented which enables a simple consideration of this effect.

2. MAXIMUM STRESS RANGE AMPLITUDE $\Delta\sigma_{Max}$

2.1 Frequency Domain Approach and Peak Factor Method

Spectral methods are favored for wind engineering problems as they allow for analytical solutions of stochastically loaded systems. Using the power spectral density of a process *Y*, the standard deviation can be determined as follows:

$$\sigma_Y = \sqrt{\int_0^\infty S_{YY}(f)df} \tag{1}$$

In order to allow for a design recommendation it is essential to predict the expected extreme values of the process for a certain recurrence period. Based on the work of Cartwright & Longuet-Higgins (1956), Davenport (1964) developed a formula for the expected or mean peak factor for the largest value of a Gaussian random process:

$$g_{Dav} = \sqrt{2 \cdot ln\left(\nu T\right)} + \frac{1}{\sqrt{2 \cdot ln\left(\nu T\right)}} \tag{2}$$

where *T* is the time period in which the extreme value occurs and *v* is the cycling rate (mean crossing rate for a Gaussian process) according to Rice (1944), derived from the zero-order and second-order spectral moments m_0 and m_2:

$$\nu = \sqrt{m_2 / m_0} \tag{3}$$

Based on this expression it is common to predict the extreme value of the stochastic process as follows:

$$\hat{Y} = \bar{Y} + g_{Dav} \cdot \sigma_Y \tag{4}$$

For the case that Y is a response process of a wind excited structure and it represents the level of stress σ at a critical structural detail, one might be interested in the expectable maximum stress amplitude $\Delta\sigma_{Max}$. Following Eq. (4), it might be reasoned that:

$$\Delta\sigma_{Max} = \Delta Y = 2 \cdot g_{Dav} \cdot \sigma_Y \tag{5}$$

Doing so, it would be assumed that the distribution of extreme values $\hat{Y}$ and $\check{Y}$ around the average $\bar{Y}$ is symmetric, which is actually not necessarily correct. However, using spectral methods, currently no better prediction seems possible. For the following reasons, the accuracy of Eq. (5) is doubted for the estimation of the expectable maximum stress amplitude $\Delta\sigma_{Max}$:

- The Peak factor according to Cartwright & Longuet-Higgins (1956) is based on the assumption of normally distributed stochastic processes. As the wind velocity is assumed to be normally distributed, the mechanical response of a structure is not normally distributed, dependent on the dynamic properties.
- The assumption of a symmetric distribution of $\hat{Y}$ and $\check{Y}$ is not valid, as the velocity process has to be squared in order to derive wind pressures. This effect has already been investigated earlier by Holmes (1981).
- Doubling the peak factor in order to estimate the stress amplitude implies a synchronous occurrence of the extreme maximum and minimum within the same time period. The reduced probability of occurrence of both values is not considered. This effect has not been investigated so far.

The actual distribution of structural responses is influenced by various aerodynamic and structural parameters. Especially the mechanical admittance function has a significant impact on the composition of the stochastic response signal. In former studies it has been often tried to consider the character of the response process based on the bandwidth of the response, expressed by the bandwidth factor ε or the irregularity factor γ:

$$\varepsilon = \sqrt{1 - \frac{m_2^2}{m_0 \cdot m_4}} \tag{6}$$

$$\gamma = \frac{m_2}{\sqrt{m_0 \cdot m_4}} \tag{7}$$

Where $\varepsilon = \sqrt{1-\gamma^2}$. Although in wind engineering the peak factor according to Eq. (2) is widely accepted, in the original paper from Cartwright & Longuet-Higgins (1956) additionally a peak factor has been suggested, which actually takes into account the bandwidth of the response process. This peak factor is given as follows:

$$g_{CLH} = \sqrt{2 \cdot ln\left[\left(1-\varepsilon^2\right)^{0.5} \nu T\right]} + \frac{1}{\sqrt{2 \cdot ln\left[\left(1-\varepsilon^2\right)^{0.5} \nu T\right]}} \tag{8}$$

Compared to g_{Dav} according to Eq. (2), the peak factor g_{CLH} leads to decreasing values for systems with a bandwidth ε tending to unity. But still a normally distributed signal is assumed. For these reasons, further investigations have been performed in order to clarify the applicability of a peak factor for the stress amplitude problem. In order to derive reliable results for realistic structural systems, in the following a time domain concept is presented which was used for this aim.

2.2 Time Domain Algorithm

For the modeling of a wind excited structure a gust spectrum is assumed according to the Eurocode 1 (which is a Kaimal Spectrum):

$$\frac{f \cdot S_{uu}(f)}{\sigma_u^2} = \frac{6,8 \cdot f \cdot L_{ux}}{\overline{u} \cdot \left(1 + 10,2 \cdot f \cdot L_{ux} / \overline{u}\right)^{5/3}} \tag{9}$$

Based on this specification, a time series has been generated using an inverse fourier transformation:

$$x(t) = \frac{1}{N} \sum_{k=1}^{N} X(k) \cdot \exp\left[\frac{2\pi i(j-1)(k-1)}{N}\right] \tag{10}$$

For the phase angle θ uniformly distributed values between $-\pi$ and $+\pi$ have been considered. In Fig. (1), the target PSD spectrum according to Eurocode 1 and the PSD which was analyzed from the generated time series are compared.

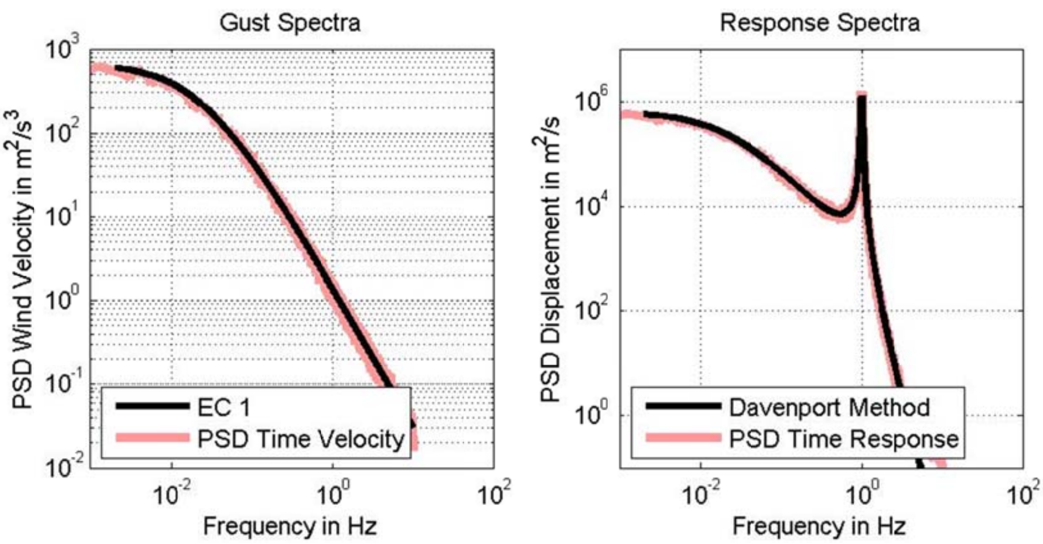

Fig. 1 Comparison of Gust Spectra: Target according to Eurocode 1 and reconstructed from Time Domain

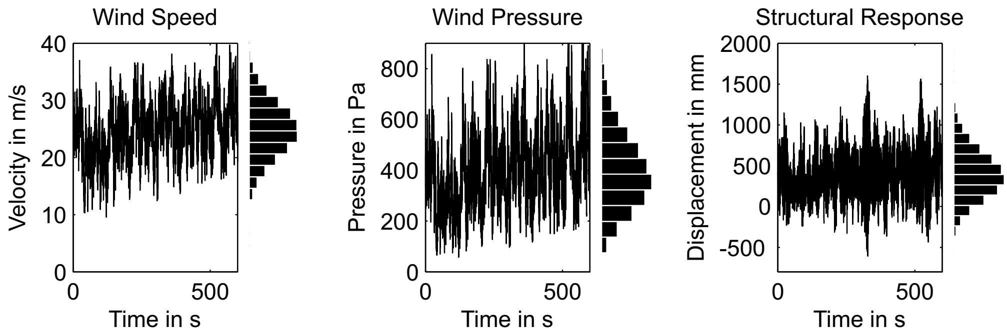

Fig. 2 Transient time series of Wind Velocity, Wind Force and Structural Response and the Distributions of all Values (ORD), Structural System with f=1.0 Hz and δ=0.1

Based on the transient time series of the wind velocity the time series of wind force can be derived assuming a quasi-steady load admittance:

$$S_{ff}(f) = \rho^2 \cdot c_f^2 \cdot A^2 \cdot \bar{u}^2 \cdot S_{uu}(f) \tag{11}$$

Finally the consideration of the mechanical behavior of an arbitrary structure succeeds based on an incremental Newmark time-step integration for a given structural frequency f and a logarithmic damping value δ. In Fig. (2) examples of resulting time series are plotted with the associated distributions of all values.

2.3 Distributions of all Values, local Peaks and Extremes

For a detailed discussion of the character of stochastic processes, the following types of distributions have to be distinguished:

- All values of a process (Ordinary Range Distribution: ORD)
- Local Peaks of a process (Peak Range Distribution: PRD)
- Extreme Values of process (Extreme Value Distribution: EVD)

For the determination of the first and second distribution the consideration of a single time series is sufficient. For the third distribution, the absolute extreme values of a multiplicity of time series with the same stochastic properties are used. In Fig. (3), a comparison of the different distribution types is plotted for two different structural systems. With respect to the objections to the usage of the peak factor according to Eq. (2) in section 2.1, the following results can be stated based on these simulations:

- The ordinary range distribution OR of the wind velocity is Gaussian for the wind velocity and nearly Gaussian for wind force and structural response
- The EVD distributions of $\check{Y}$ and $\hat{Y}$ are not generally symmetric in regard to force and response

Based on the presented simulation environment, a thorough analysis of peak factors has been carried out.

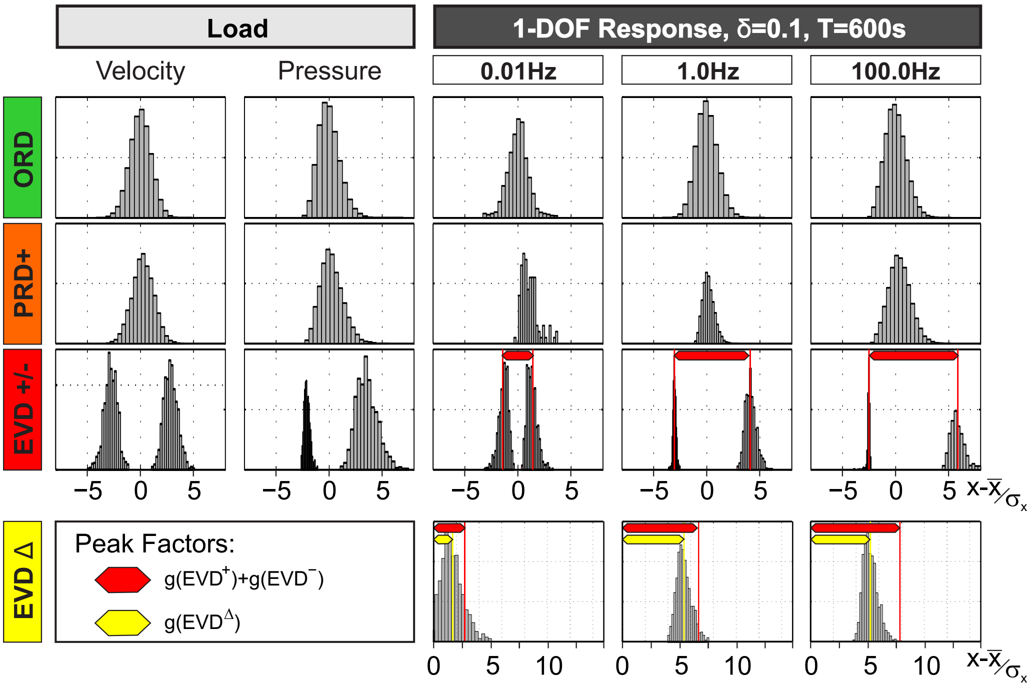

Fig. 3 Comparison of Distributions for different Structures

2.4 Peak Factors for different Applications

As illustrated in the previous section, the distributions of peak ranges and extreme value ranges differ significantly. Whereas the former is of interest with respect to the frequency of cycle counts (stress range spectra), the latter is decisive for the modeling of extreme structural stresses. However, considering extreme stress amplitudes, it is meaningful to take into account the extreme spans of amplitudes, occurring within a given event *i*:

$$\Delta Y_i = \hat{Y}_i - \check{Y}_i \tag{12}$$

The EVD of $\Delta\sigma_i$ leads finally to a peak factor which is more realistic for the fatigue damage problem. For different statistical values, the peak factors *g* based on *N* time series can be derived as follows:

$$g(\hat{Y}) = \left[\frac{1}{N}\sum_{i=1}^{N}\hat{Y}_i - \frac{1}{N}\sum_{i=1}^{N}\overline{Y}_i\right] / \sqrt{\frac{1}{N-1}\left[\left(\sum_{i=1}^{N}Y_i^2\right) - \frac{1}{N}\left(\sum_{i=1}^{N}Y_i\right)^2\right]}$$
$$= \left[\mu(\hat{Y}) - \mu(Y)\right] / \sigma(Y) \tag{13}$$

$$g(\check{Y}) = \left[\mu(\check{Y}) - \mu(Y)\right] / \sigma(Y) \tag{14}$$

$$g(\Delta Y) = \left[\frac{1}{N}\sum_{i=1}^{N}\left(\hat{Y}_i - \check{Y}_i\right) - \frac{1}{N}\sum_{i=1}^{N}\overline{Y}_i\right] / \sqrt{\frac{1}{N-1}\left[\left(\sum_{i=1}^{N}Y_i^2\right) - \frac{1}{N}\left(\sum_{i=1}^{N}Y_i\right)^2\right]}$$
$$= \left[\mu(\Delta Y) - \mu(Y)\right] / \sigma(Y) \tag{15}$$

Based on these peak factors, the statistical extremes can be reconstructed based on mean value and standard deviation of the corresponding process:

$$\hat{Y} = \overline{Y} + g(\hat{Y}) \cdot \sigma(Y) \tag{16}$$

$$\check{Y} = \overline{Y} + g(\check{Y}) \cdot \sigma(Y) \tag{17}$$

$$\Delta Y = \overline{Y} + g(\Delta Y) \cdot \sigma(Y) \tag{18}$$

Using a time domain algorithm, the peak factors according to Eq. (13) to (15) have been determined for a wide set of structural parameters. In detail, 225 single degree of freedom systems with frequencies *f* between 0.01 Hz and 100 Hz and logarithmic damping decrements *δ* between 0.001 and 1.0 have been computed. For each system a number of *N*=1000 transient calculations has been carried out in order to identify the extreme value distributions (EVD) of maxima, minima and span amplitudes. All calculations have been performed with three different time lengths of *T*=60s, 600s and 6000s.

In Fig. (3) and Fig. (4) the results of the transient simulations are plotted as peak factors for the span amplitudes $g(EVD^{\Delta})$. The results are compared to the doubled peak factor according to Eq. (2), which is the Davenport peak factor and according to the doubled Eurocode recommendation. Obviously, the peak factors $g(EVD^{\Delta})$ are generally lower than the mentioned alternative approaches.

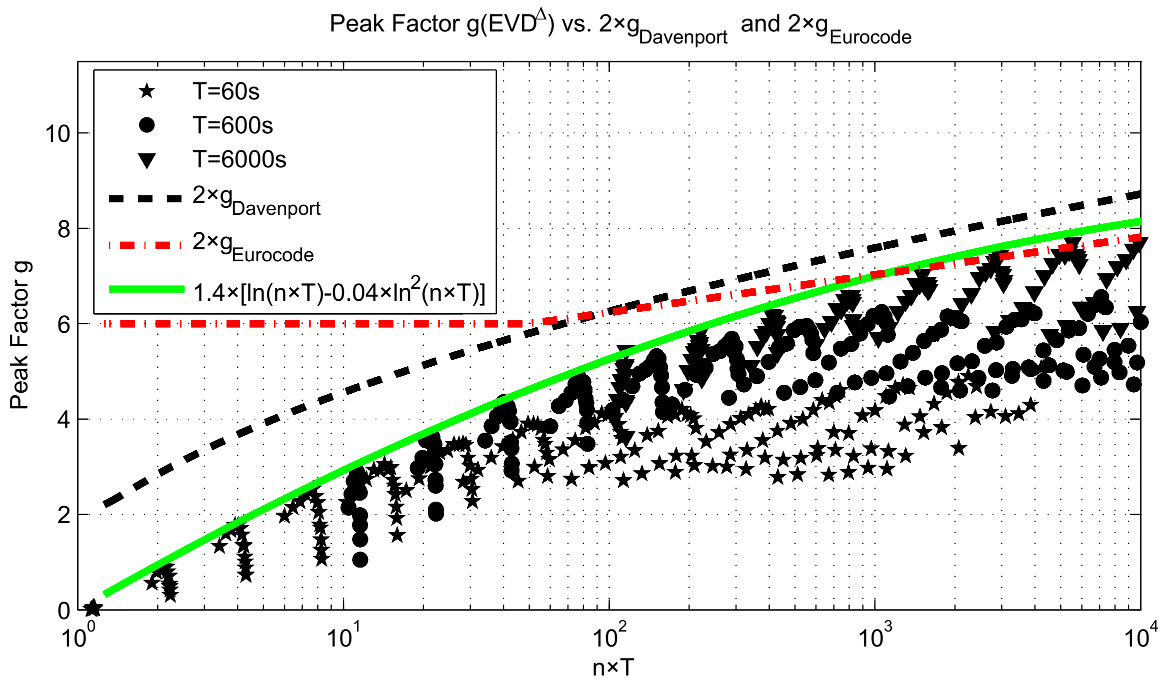

Fig. 4 Comparison of Peak Factors: Extreme Spans EVD^{Δ} out of *N*=1000 Time Domain Simulations for each System with different Time Lengths (60s, 600s and 6000s), according to Davenport (doubled) and to Eurocode (doubled)

As an enveloping, safe side suggestion for $g(EVD^{\Delta})$ the following formula has been derived:

$$g(EVD^{\Delta}) = 1.4 \cdot \left[ln(\nu \cdot T) - 0.04 \cdot ln^2(\nu \cdot T) \right] \tag{19}$$

The illustration in Fig. (4) makes clear, that dependent on structural frequency and damping value of the considered system, a significant further potential could be achieved. Currently this dependency has not been analyzed in detail and will be subjected to future analyzes. Nevertheless, using Eq. (19), a remarkable reduction of the considered stress range value $\Delta\sigma_{Max}$ can already be realized.

3. CYCLE COUNT DISTRIBUTION

For detailed fatigue calculations not only the level of the maximum stress range is decisive. Furthermore the relation between lower stress ranges and their frequency of occurrence is important in order to calculate realistic damage values and the associated expected lifetimes.

3.1 Recommendation of Eurocode 1

A very simple relationship between the stress ratio $\Delta\sigma/\Delta\sigma_{Max}$ (in percent) and the number of cycles N is given in the current version of the Eurocode 1:

$$\frac{\Delta\sigma}{\Delta\sigma_{max}}(N) = 0.7 \cdot log^2 N - 17.4 \cdot log\ N + 100 \tag{20}$$

It is noticeable that Eq. (20) does neither take into account any further information with respect to the actual dynamic properties of the considered system nor concerning the characteristics of the approaching wind field. It is therefore interesting to have a deeper look on the safety of this equation taking into account various influence parameters are considered. This comparison is made in section 4.

3.2 Number of load Repetitions

Another, first numerical approach to estimate the number of cycle counts due to gusty wind loading has been proposed by Davenport (1966), using the number of exceedances of a certain load level. Therefore, he used a Weibull distribution to describe average wind speeds (averaged over 10 minutes to 1 hour), which is appropriate for synoptic wind climates. Based on this distribution, the probability of exceedance of a given wind speed u is given as:

$$P(u) = \exp\left[-\left(\frac{u}{c}\right)^k\right] \tag{21}$$

where c is the scale factor and k the shape factor. The associated probability density function is given by:

$$p(u) = \frac{\delta P(u)}{\delta u} = \frac{k}{c} \cdot \left(\frac{u}{c}\right)^{k-1} \exp\left[-\left(\frac{u}{c}\right)^k\right] \tag{22}$$

With respect to the fluctuating structural stresses it is assumed that the relation can be described by a power law function:

$$\sigma(u) = A \cdot u^n \tag{23}$$

The exponent *n* has a value of *n*=2.0 when a structure reacts quasi-statically to wind loads, but can take a higher value (up to about 2.5) if there is significant resonant dynamic response (Holmes, 2007). It is recommended to derive an appropriate value for *n* based on the power spectral density of the structural response, see Eq. (1). Additionally, a structural calculation considering the static system behavior with respect to the displacement-stress relation of the structural detail is needed.

The probability density of positive level crossing of a process *Y* is given by the level-crossing formula according to Rice:

$$p(Y) = \exp\left[-\frac{Y^2}{2\cdot\sigma(Y)^2}\right] \tag{24}$$

With respect to the double range amplitudes $\Delta Y = 2\cdot Y$ and under consideration of Eq. (23) it follows:

$$p_{\Delta\sigma}(u) = \exp\left[-\frac{\Delta\sigma^2}{8\cdot A^2\cdot u^{2n}}\right] \tag{25}$$

The number of stress ranges *n(Δσ)* within a time period *T* is therefore:

$$n(\Delta\sigma) = \nu\cdot T\int_0^\infty \exp\left[-\frac{\Delta\sigma^2}{8\cdot A^2\cdot u^{2n}}\right]du \tag{26}$$

Considering the probability of the expected wind speed on site based on Eq. (21), the total numbers of stress amplitude repetitions is now:

$$\begin{aligned} N(\Delta\sigma) &= \nu\cdot T\cdot\int_0^\infty \frac{dP(u)}{du}\cdot p_{\Delta\sigma}(u)du \\ &= \nu\cdot T\cdot\frac{k}{c^k}\cdot\int_0^\infty u^{k-1}\exp\left[-\left(\frac{u}{c}\right)^k\right]\cdot\exp\left[-\frac{\Delta\sigma^2}{8\cdot A^2\cdot u^{2n}}\right]du \\ &= \nu\cdot T\cdot\frac{k}{c^k}\cdot\int_0^\infty u^{k-1}\exp\left[-\left(\frac{u}{c}\right)^k-\frac{\Delta\sigma^2}{8\cdot A^2\cdot u^{2n}}\right]du \end{aligned} \tag{27}$$

In his original paper, Davenport used the first structural frequency f_0 instead of the level crossing rate ν. For structural responses with narrow bandwidth, to which this method is limited due to the usage of the Rice formula, no significant difference can be expected. However, Holmes has suggested to assume an averaged cycling rate dependent on the wind speed scale factor *c* (Holmes, 2012). For the value of *v* it is thereafter recommended to consider a mean cycling rate for a wind speed of *u*=*c*, which is given by:

$$\nu_c = \nu_1 \cdot c^b \tag{28}$$

where v_1 is notionally the cycling rate when the mean wind speed is equal to *u*=1m/s and *b* is an exponent with a value of about *b*=0.5.

At this point it is important to clarify the most important restriction on which the described and often used method is based on. The Rice formula according to Eq. (24) is valid only for narrow band responses. With respect to gust excited structures this is equal to lightly damped systems with low natural frequency.

In the following section a newer method is presented, which allows for a more detailed consideration of stochastic responses in order to determine stress range amplitudes.

3.3 General Frequency based Approach

3.3.1 Cycle Counting within a short Time Period

Especially in the field of mechanical engineering an empirical approach for estimating the Rainflow distribution based on the power spectral density is meanwhile well established. Dirlik derived this method based on continuous random processes and subsequent numerical Rainflow counting (Dirlik, 1985). He discovered an empirical relationship between the frequency range description of the signal and the Rainflow distribution.

His studies were directed to stationary, ergodic random processes with arbitrary bandwidth. Consistently, the Dirlik probability density function of peak values of a process with any bandwidth is a weighted sum of a normal distribution and a Rayleigh distribution. In previous studies of Wirsching & Shehata (1977), a similar approach was chosen, but there an attempt was made to describe the Rainflow distributions by Weibull functions. The simulations of Dirlik, as well as own simulations showed, that the Rainflow distributions of peaks often differ significantly from a Weibull distribution. For this reason, the Wirsching-factor is not realistic in case of higher response bandwidth Kemper & Feldmann (2011a). With Dirlik's formula, the density of the number of cycle counts *N′* of a given stress level *Δσ* can be computed to:

$$N'(\Delta\sigma) = \nu \cdot T \cdot f_D(\Delta\sigma) \tag{29}$$

$$f_D(\Delta\sigma) = \frac{\dfrac{D_1}{Q} \cdot e^{\frac{-Z(\Delta\sigma)}{Q}} + \dfrac{D_2 \cdot Z(\Delta\sigma)}{R^2} \cdot e^{\frac{-Z(\Delta\sigma)^2}{2 \cdot R^2}} + D_3 \cdot Z(\Delta\sigma) \cdot e^{\frac{-Z(\Delta\sigma)^2}{2}}}{2 \cdot \sqrt{m_0}} \tag{30}$$

where:

$$x_m = \frac{m_1}{m_0} \cdot \sqrt{\frac{m_2}{m_4}} \qquad R = \frac{\gamma - x_m - D_1^2}{1 - \gamma - D_1 + D_1^2} \tag{31a}$$

$$D_1 = \frac{2 \cdot (x_m - \gamma^2)}{1 + \gamma^2} \qquad D_2 = \frac{1 - \gamma - D_1 + D_1^2}{1 - R} \tag{31b}$$

$$D_3 = 1 - D_1 - D_2 \qquad Z(\Delta\sigma) = \frac{\Delta\sigma}{2 \cdot \sqrt{m_0}} \tag{31c}$$

$$Q = \frac{1.25 \cdot (\gamma - D_3 - D_2 \cdot R)}{D_1} \tag{31d}$$

are auxillary values.

The accuracy of the method has been verified with respect to gust excited structures by comparing the results to corresponding transient simulations. In all cases, the discrepancies between transient Rainflow count and Dirlik formula were negligible (Kemper & Feldmann, 2011b). Even geometrical nonlinear structures can be analyzed based on the presented spectral approach. The necessary stochastic description of the mechanical system has been described by (Kemper & Feldmann, 2011c).

Making use of the PSD based cycle count algorithm already presented, the assessment of stress cycles is enabled. But so far, only the impact of a short-term extreme wind on the stress range distribution has been considered. For the computation of the expected cycles during the designated life-time with respect to fatigue phenomena, the prevalent wind events during this time period have to be taken into account as well.

3.3.2 Consideration of Long-term Wind Characteristics

The resulting cycle count spectrum during the foreseen lifetime originates from a convolution of short termed stress range spectra and a long-termed wind statistics. It can be computed by the following expression:

$$N'_{Life}(\Delta\sigma) = \int_0^{\bar{u}} \frac{T_{Life}}{T_{ref}} \cdot p(u) \cdot N'(\Delta\sigma, u) du \tag{32}$$

where: T_{Life} is the designed Lifetime, T_{ref} is the average time of the mean wind speed, $p(u)$ is the probability density function of the wind velocity and $N(\Delta\sigma,u)$ is the stress range density for a given mean wind speed within the average time period. The latter expression is computed based on the Dirlik formula according to section 3.3.

The probability density function for the frequency of occurrence of mean wind speeds can be derived based on the Weibull-distribution according to Eq. (22). Site dependent values for these parameters can be taken from the European Wind Atlas (Troen & Petersen, 1989).

Based on appropriate assumptions for the site dependent extreme wind impact and the probability of exceedance of the prevalent wind speeds, a realistic consideration of fatigue impact is possible. Using the direction related Weibull Parameter, even a directional design is possible.

3.3.3 Influence of Wind Directions

In the previous section 3.3.2 it was assumed that the approaching wind is unidirectional. Both, the probability of occurrence of wind speeds and the dependency between wind load and resulting stress response have been formulated under this presumption.

In case that detailed information concerning the direction related Weibull parameters for N_Φ defined wind sectors $\Delta\Phi_i$ are available, the probability of density of wind speeds is described as:

$$p(u,\Delta\Phi_i) = p(\Delta\Phi_i)\cdot\frac{k_i}{c_i}\left(\frac{u}{c_i}\right)^{k_i-1} exp\left[-\left(\frac{u}{c_i}\right)^{k_i}\right] \tag{33}$$

where $p(\Delta\Phi_i)$ is the probability of a wind direction out of sector $\Delta\Phi_i$. Within each wind sector, the scale and shape parameters c and k are considered individually. Based on this description, the stress range density N'_{Life} can now be computed more realistic as follows:

$$N'_{Life}(\Delta\sigma) = \sum_{i=1}^{N_\Phi} p(\Delta\Phi_i)\int_0^{\bar{u}} \frac{T_{Life}}{T_{ref}}\cdot p(u,\Delta\Phi_i)\cdot N'(\Delta\sigma,u,\Delta\Phi_i)du \tag{34}$$

In Eq. (34), the influence of the wind direction is considered for the probability density of wind speeds and for the probability density of stress range amplitudes. Concerning fatigue analyzes, generally certain structural details at a defined positions are of interest for the design process. The load-response relation for a certain position depends on the load direction at least in a static sense, in most cases (for non-rotational symmetric systems) additionally in a dynamic sense.

Assuming a axisymmetric structural system (e.g. a chimney), the following relation between wind direction Φ and structural response Y can be expected:

$$Y/Y_{max} = cos(\Phi) \tag{35}$$

In case of structural components which are shadowed by other, larger building parts (e.g. a column of cantilevered roof), the relation according to Eq. (35) might only be valid in a limited range of flow directions. In that case, certain wind sectors may theoretically not contribute to the response of the considered structural detail. For usage in fatigue concepts, it is advisable to formulate a specific sector related reduction ratio:

$$f(\Delta\Phi_i) = Y_i/Y_{max} \tag{36}$$

As this reduction ratio affects the direction related structural response, it can be used directly as a scaling factor towards the standard deviation of the response, e.g. according to Eq. (1):

$$\sigma_Y(\Delta\Phi_i) = f(\Delta\Phi_i)^2 \cdot \sigma_Y \tag{37}$$

Additional efforts are necessary, when a direction related variation of the dynamic properties are to be considered. In that case, already the mechanical admittance function has to be formulated, dependent on the direction.

4. INFLUENCING PARAMETERS

4.1 Evaluation of Influences

In this section the presented computation methods according to section 3.2 and section 3.3 are used together in order to determine the influencing parameters on the expected cycle count distribution. The general concept according to section 3.3 is applied as reference method. Hence, occuring divergences are caused by the simplified assumptions inherited in the approach of section 3.2.

In order to allow a general usage of the cycle count distribution, a normalization of the curve is necessary. It is postulated, that the maximum stress amplitude $\Delta\sigma_{Max}$ occurs exactly once in the designated lifetime. This demand requires the consideration of the cumulative function N_{Life}, as only in this notation a frequency of occurrence of N=1 can be found. The cumulative function is defined as:

$$N_{Life}(s) = \int_0^s N'_{Life}(t)\, dt \tag{38}$$

As a result, the N_{Life}-Function allows a quantifiable description of the influences on the damage life. The individual cumulative stress cycle count distributions can now be normalized by the maximum amplitude $\Delta\sigma_{Max}$ at N=1 and finally be fitted by a logarithmic polynomial 2nd order:

$$\frac{\Delta\sigma}{\Delta\sigma_{max}}(N_{Life}) = \alpha \cdot log^2 N_{Life} + \beta \cdot log\ N_{Life} + 100 \tag{39}$$

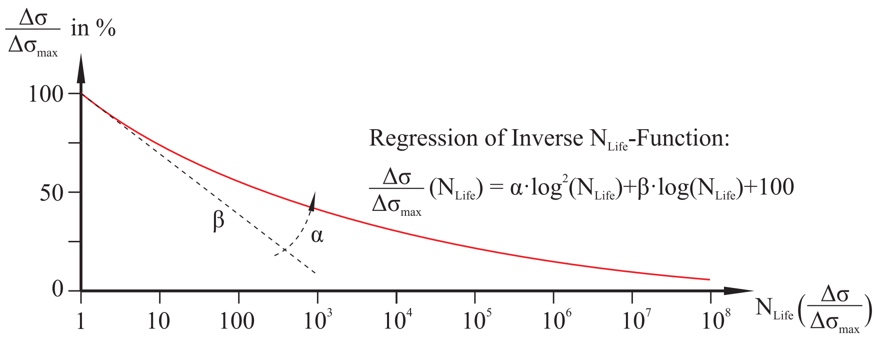

Fig. 5 Regression of the Cycle Count Distribution

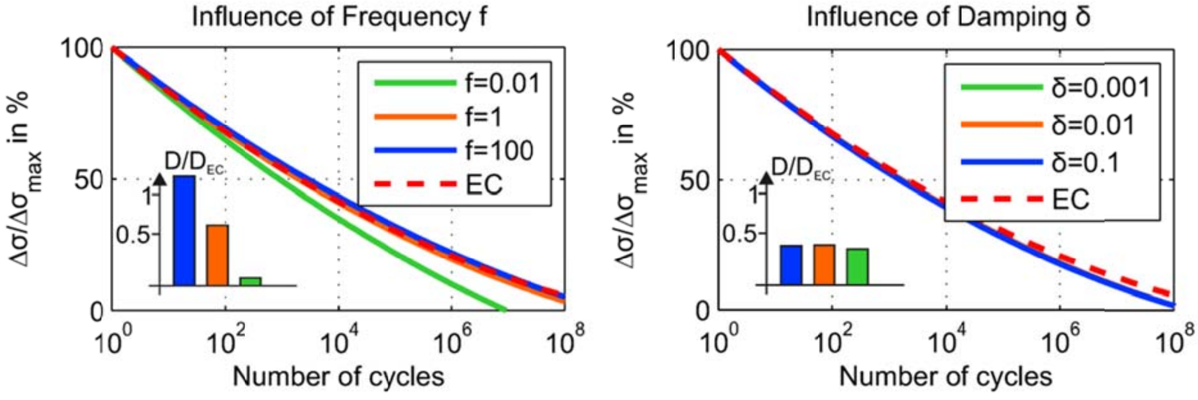

Fig. 6 Influence of the Dynamic Parameters for Frequency *f* and Damping *δ* on the Cycle Count Distribution and the relative Damage Values within 50 years (*c*=5 m/s, *k*=2)

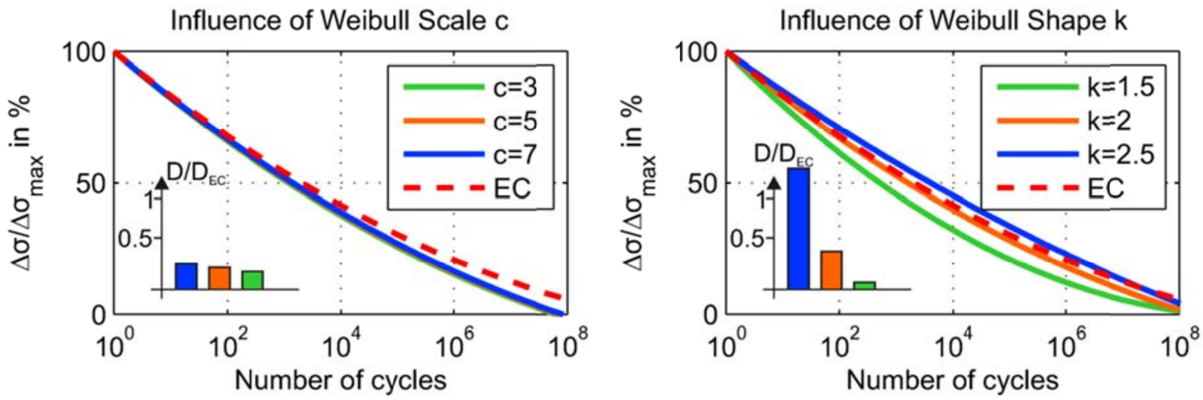

Fig. 7 Influence of the Weibull Parameters for Scale *c* and Shape *k* on the Cycle Count Distribution and the relative Damage Values within 50 years (*f*=0.5 Hz, *δ*=0.05)

In Eq. (39) is the normalized stress $\Delta\sigma/\Delta\sigma_{Max}$ given in percent. As a special case of this relation, the recommendation of the Eurocode according to Eq. (20) is covered with α=0.7 and *β*=-17.4. A general distinction has to be made concerning parameters, which affect the amplitude (influence on $\Delta\sigma_{Max}$) of the cycle count distribution and those which affect the distribution (shape expressed by Eq. (39)).

4.2 Influence of Structure and Weibull Distribution

In Fig. (6), the influence of the structural parameters on the shape of the cycle count distribution is plotted. For structures with a natural Eigenfrequency below *f*=1.0 Hz, the influence of the logarithmic damping decrement *δ* on the shape of the N_{Life}-Function has found to be negligible. In Fig. (7), the influence of different Weibull parameters is compared to each other. The shape parameter *k* has a much greater influence than the scale parameter *c*.

For the most important parameters, the Eigenfrequency *f* and the shape parameter *k* of the Weibull distribution further analyses have been performed. In Fig. (8), the impact on the N_{Life} regression parameters is visualized.

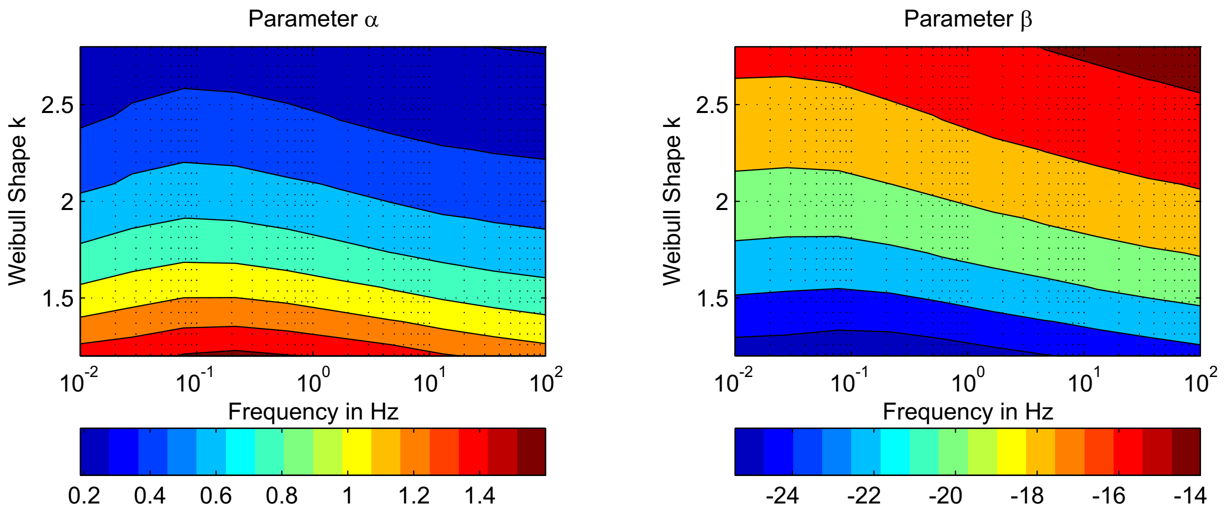

Fig. 8 Influence of the Structural Frequency *f* and the Scale Factor *k* on the Shape of the Cycle Count Distribution N_{Life}

4.3 Influence of Directionality

The consideration of directionality in order to achieve more realistic results is related to the wind direction on the one hand and to the activation of structural stresses at a certain structural detail on the other hand.

In Fig. (9), the considered parameters for both influencing aspects are illustrated. Three cases are considered for the statistical wind directions on site. As a first simple case it is assumed that the Weibull parameters do not vary dependent on the wind directions and that the probability of all wind directions is uniformly distributed.

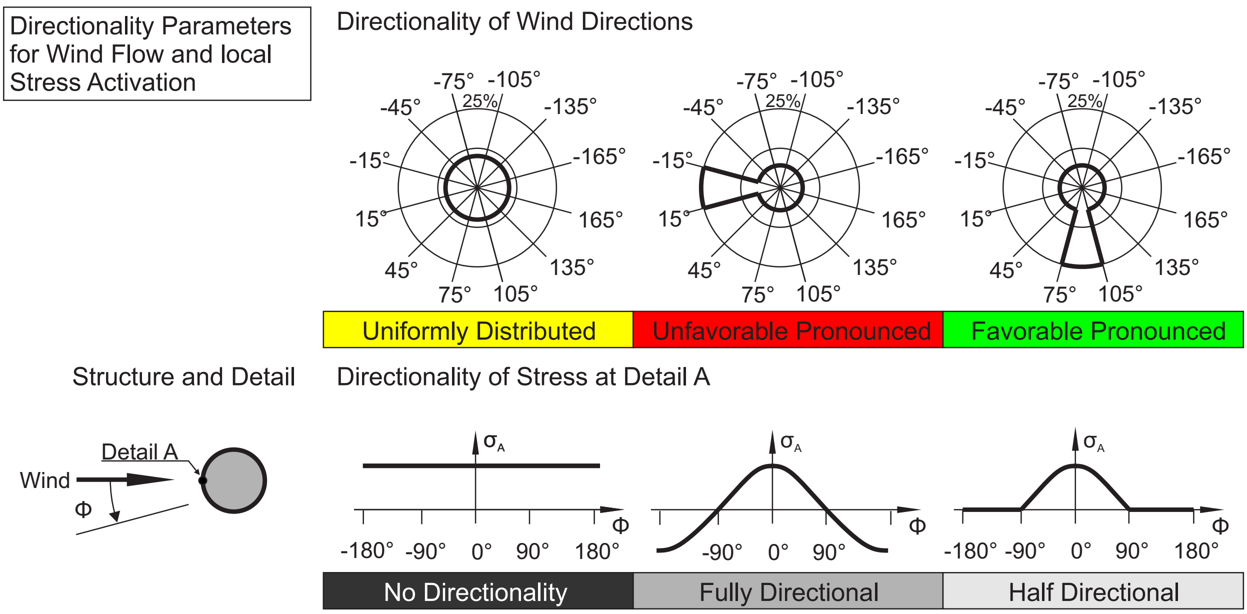

Fig. 9 Considered Parameters for the Directionality of Wind Effects

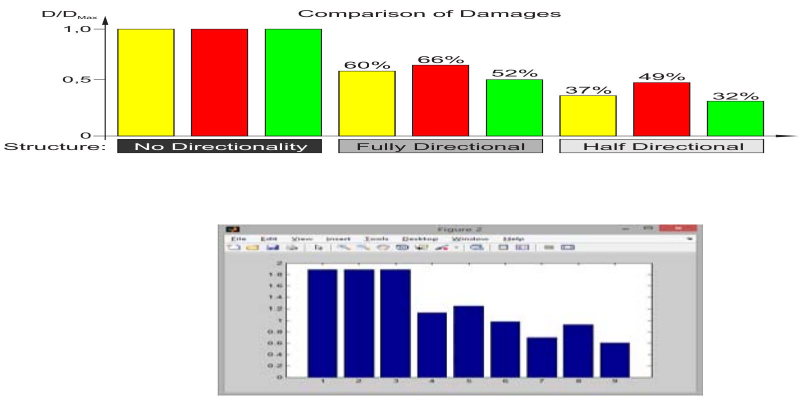

Fig. 10 Considered Parameters for the Directionality of Wind Effects

In this case only the assumed load-response relation is relevant compared to the unidirectional case. As a second and third case it is assumed that there exists a predominant wind direction of $\Delta\Phi$=0°±15° with p=0.25, all the other directions are assumed to be uniformly distributed with p=0.068. The predominant direction is unfavorable with respect to the structural detail (second case) and it is favorable in the third case.

Furthermore, three assumptions are made with respect to the directionality of the load-response functions. As a first case, directionality is neglected. Secondly a rotational symmetric system is assumed, where the stress reduction ratio $f(\Delta\Phi)$ can be expressed as a cosine function, according to Eq. (35). As a final case, a shaded system is assumed, where the cosine relation is only valid within a sector of $\Delta\Phi$=±90°. For the other wind directions, $f(\Delta\Phi)$ is supposed to be zero.

For all combinations of the described influencing parameters the cycle count distributions have been calculated with the before described methods. For an easy comparison of the results, the associated relative damages are plotted in Fig. (10). For the damage calculation the elemental Miner rule has been used.

4. CONCLUSIONS

For the prognosis of the fatigue behavior of gust excited structures a realistic cycle count distribution is needed. For the scaling of the expected maximum dynamic amplitudes different peak factors are compared. Based on numerical simulations of various systems in the time domain, peak factors for the extreme span of amplitudes are computed.

Different methods have been described to determine a realistic shape of the cycle count distributions. Based on parameter variations it has been shown that especially the structural frequency f and the shape coefficient k of the Weibull distribution influence the shape of the cycle count distribution. Compared to the Eurocode recommendation, the determined results lead in most cases to lower damage results.

Finally, the influence of directionality has been investigated. In most cases this dependency is neglected for simplification reasons. However, both the wind direction and the load-response behavior of the structure are actually directional. For some simple cases this influence has been computed. With respect to the expectable fatigue behavior, a consideration of directionality effects can lead to significantly lower damage values (reduction up to 70%).

REFERENCES

Cartwright, D. E., & Longuet-Higgins, M. S. (1956), The statistical Distribution of the Maxima of a Random Function. Proceedings of the Royal Society of London, 237(1209), 212–232.

Davenport, A. G. (1964), Note on the Distribution of the largest Value of a random Function with Application to Gust Loading. ICE Proceedings, 28(2), 187–196.

Davenport, A. G. (1966), The Estimation of Load Repetitions on Structures with Application to Wind induced Fatigue and Overload. In: RILEM (ed), RILEM International Symposium on the Effects of Repeated Loading of Materials and Structures.

Dirlik, T. (1985), Application of computers to fatigue analysis. Ph.D. thesis, Warwick University, Warwick.

Holmes, J. D. (1981), Non-Gaussian Characteristics of Wind Pressure Fluctuatians. Journal of Wind Engineering and Industrial Aerodynamics, 7, 103–108.

Holmes, J. D. (2012), Wind-induced Fatigue cycle counts - sensitivity to wind climate and dynamic response. In: Vlrag, Z. (ed), 7th ICCSM International Congress of Croatian Society of Mechanics.

Holmes, J. D. (2007), Wind loading of structures. 2nd. ed. edn. London: Taylor & Francis.

Kemper, F. H., & Feldmann, M. (2011a), Appraisement of Fatigue Phenomena due to Gust induced Vibrations based on closed-form Approaches. In: ICWE 13.

Kemper, F. H., & Feldmann, M. (2011b), Fatigue life prognosis for structural elements under stochastic wind loading based on spectral methods: Part 1: Linear structures. Pages 1629–1635 of: Roeck, G. de (ed), Proceedings of the 8th International Conference on Structural Dynamics, EURODYN 2011.

Kemper, F. H., & Feldmann, M. (2011c), Fatigue life prognosis for structural elements under stochastic wind loading based on spectral methods: Part 2: Nonlinear structures. Pages 1636–1643 of: Roeck, G. de (ed), Proceedings of the 8th International Conference on Structural Dynamics, EURODYN 2011.

Rice, S. O. (1944), Mathematical analysis of random noise. Bell System Technical Journal, 23,24, 282–332,46–156.

Troen, I., & Petersen, E. L. (1989), European wind atlas. Roskilde: Risø National Laboratory.

Wirsching, P. H., & Shehata, A. M. (1977), Fatigue under wide Band random Stresses using Rainflow Method. Journal of Engineering Materials and Technology, 205–211.