

Influence of directionality and shelter effect on fatigue life prediction of a mast structure

Haobo Wang¹⁾, *Guoqing Huang²⁾ Yanwen Su³⁾ and Xinxin Zhang⁴⁾

^{1,2,3)}*School of Civil Engineering, Southwest Jiaotong University, Chengdu, China*

⁴⁾*Department of Civil and Environmental Engineering, Texas Tech University, Lubbock, TX, USA*

²⁾*ghuang1001@gmail.com*

ABSTRACT

The basic wind pressure in current most wind design codes is based on 50-year return period, where directionality of wind speed is usually not taken into account. For the steel structures, this treatment also may miscalculate the structural fatigue life. Additionally, due to the unavailability of complete data, the possible shelter effect of wind speed appears in practice may be neglected. This effect may result in under estimate the structural fatigue life.

An assessment of long-term fatigue damage on a mast structure is presented in this paper considering the wind directionality and shelter effect. In this paper, as long as 13-year measured data from National Oceanic and Atmospheric Administration (NOAA) of USA is used. In previous study, the duration of the data used for directionality is usually 4-5 years. The preceding data provide a good opportunity to study the directionality and the possible shelter effect on the structural fatigue life estimation. The data is fitted to establish a joint probability density function of wind speed and direction. Furthermore, the influence of neglecting directionality and shelter effect is also studied. Results show that directionality may have noticeable effect on the fatigue life prediction of a mast structure. Also, shelter effect results in the remarkable influence.

1. INTRODUCTION

The fluctuating nature of wind may cause the fatigue on the structure and the accumulated damage will cause fatigue failure of structure. Based on the assumption that mean wind speed follows Rayleigh probability distribution, Homels (2002) proposed a closed-form solution to predict fatigue life under along-wind load. And with corresponding weighting to represent the wind directionality effects for wind induced fatigue damage. By the comparison of experimental measurement with theoretical calculation, Robertson et al. (2004) verified the previous closed-form solution for fatigue and researched how to choose the parameter of the solution. However, the

^{1)3,4)} Graduate Student

²⁾ Professor

directionality of wind was not considered. An advanced closed-form solution was proposed by Repetto and Solari (2004, 2009, and 2011).

In this study, as long as 13-year measured wind data is used, which differs from previous where the length of wind data is often 4 to 5 years. Long-term data means a more precise fit for wind data, which improves the accuracy of fatigue life prediction. Shelter effect was rarely studied before. This long-term data provide a good opportunity to study the directionality and the shelter effect on the structural fatigue life estimation. With the model recommended by Robertson et al. (2004), an investigation on directionality and the shelter effect of wind induced fatigue is made here.

2. PROBABILISTIC DESCRIPTION OF MEAN WIND SPEED AND DIRECTION

2.1 introductions of wind data

A 13-year wind records per minute from 2000 to 2012 are selected from the database of Automated Surface Observing System (ASOS) operated by National Oceanic and Atmospheric Administration (NOAA) of USA. The wind records come from weather station of ASOS located at Allentown, Lehigh, PA.

2.2 joint probability density function of wind speed and direction

It is necessary to find the joint distribution of the wind data to predict the fatigue life of a lighting column. Due to its convenience of a two-parameter distribution, Weibull distribution is more widely used to fit wind data than the other distributions. Here Weibull distribution is chosen to fit the wind data. The cumulative distribution function (CDF) and probability density function (PDF) of the Weibull form are given as

$$P_u(U) = 1 - \exp \left[- \left(\frac{U}{c} \right)^k \right] \quad (1)$$

$$f_u(U) = \frac{k}{c} \left(\frac{U}{c} \right)^{k-1} \exp \left[- \left(\frac{U}{c} \right)^k \right] \quad (2)$$

where U is the wind speed; k (>0) is a shape factor; and c (>0) is a scale factor with the same unit as wind speed.

For wind-induced fatigue is closely related to wind direction, a joint distribution must be used for this study. To establish the joint distribution, two assumptions are adopted here as follows (Xu et al. 2009):

(1) The distribution of the component of wind speed for any given wind direction follows Weibull distribution;

(2) The independence of wind distribution in different wind directions can be reflected by the relative frequency of occurrence of wind:

$$\begin{aligned} P_u(U, \theta) &= P_\theta(\theta) P_{u,\theta}(U|\theta) = P_\theta(\theta) \left(1 - \exp \left[- \left(\frac{U}{c(\theta)} \right)^{k(\theta)} \right] \right) \\ &= \iint f_\theta(\theta) f_{u,\theta}(U, c(\theta), k(\theta)) dud\theta \end{aligned} \quad (3)$$

$$f_{u,\theta}(U, c(\theta), k(\theta)) = \frac{k(\theta)}{c(\theta)} \left(\frac{U}{c(\theta)} \right)^{k(\theta)-1} \exp \left[- \left(\frac{U}{c(\theta)} \right)^{k(\theta)} \right] \quad (4)$$

$$P_\theta(\theta) = \int_0^\theta f_\theta(\theta) d\theta \quad (5)$$

in which $0 \leq \theta < 2\pi$; $P_\theta(\theta)$ is the relative frequency of occurrence of wind in direction θ . $P_\theta(\theta)$, $c(\theta)$ and $k(\theta)$ can be estimated by using wind data recorded by the weather station.

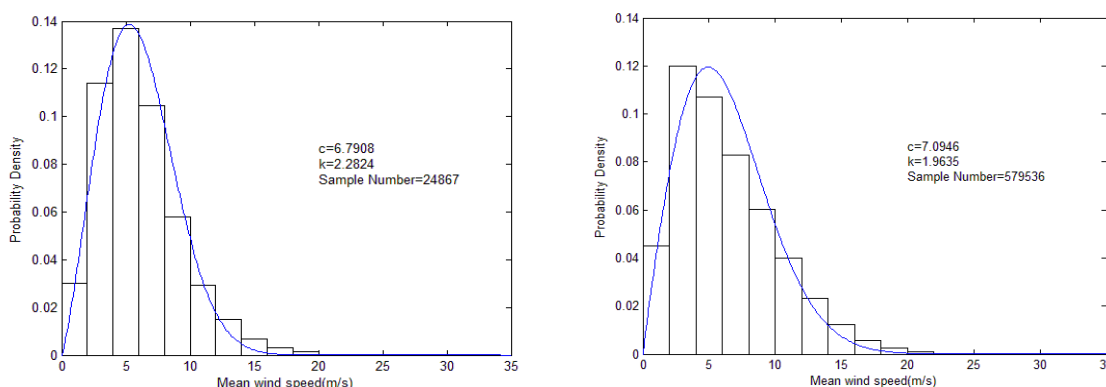
2.3 Statistical analysis of wind data

The original wind records with 1-min duration are converted into 10-min mean wind speeds. Because the wind speed is composed of magnitude and direction, it is treated as the vector. The magnitude of vector is the wind speed value, and the angle of vector is the wind direction. The magnitude and direction of 10-min mean wind speed can be obtained through the vectorial combination of the averaged projections of all wind speeds in 10 min on two axes. All the wind data are classified into 16 sectors of the compass with an interval of $\Delta\theta = 22.5^\circ$ based on 10-min mean wind speed.

The wind records are carefully checked to remove the abnormal data. Furthermore, adverse effect is prevented by eliminating the wind data whose wind speed is lower than 1 m/s. As a result, 579536 10-min wind data are acquired for calculation.

The relative frequency of mean wind speed and direction per ten minutes is listed in Table 1. The direction abbreviations (clockwise) are listed in first row. The last row in Table 1 gives the relative frequency of mean wind speed without considering the direction, and the last column shows the relative frequency of mean wind direction without considering the wind speed.

Based on the relative frequency of wind speed and direction calculated above, the theoretical expression of joint probability density function is deduced based on Eq. (3). The Weibull function is used to fit the histogram of mean wind speed per ten minutes for each wind direction; the typical results in north direction and all direction are depicted in Fig. 1.



(a) North direction

(b) All direction

Fig. 1 Weibull distribution and relative frequency of mean wind speed

Table 1 Relative frequency of mean wind speed and direction per ten minutes

| Direction | 0 - 2 | 2 - 4 | 4 - 6 | 6 - 8 | 8 - 10 | 10 - 12 | 12 - 14 | 14 - 16 |
|-----------|-------|--------|--------|--------|--------|---------|---------|---------|
| N | 0.258 | 0.980 | 1.177 | 0.897 | 0.499 | 0.252 | 0.128 | 0.057 |
| NNE | 0.334 | 1.108 | 1.440 | 1.330 | 0.906 | 0.498 | 0.224 | 0.117 |
| NE | 0.493 | 1.587 | 1.661 | 1.240 | 0.925 | 0.570 | 0.351 | 0.174 |
| ENE | 0.696 | 1.288 | 0.658 | 0.405 | 0.248 | 0.118 | 0.053 | 0.023 |
| E | 0.892 | 1.576 | 0.732 | 0.534 | 0.353 | 0.196 | 0.097 | 0.036 |
| ESE | 1.047 | 2.180 | 1.184 | 1.083 | 0.978 | 0.799 | 0.490 | 0.260 |
| SE | 0.947 | 2.033 | 1.350 | 1.253 | 1.207 | 1.018 | 0.683 | 0.397 |
| SSE | 0.761 | 1.663 | 1.302 | 1.256 | 1.239 | 1.068 | 0.781 | 0.493 |
| S | 0.760 | 2.088 | 1.791 | 1.589 | 1.470 | 1.109 | 0.691 | 0.389 |
| SSW | 0.823 | 2.827 | 2.736 | 1.946 | 1.362 | 0.859 | 0.437 | 0.192 |
| SW | 0.644 | 2.561 | 3.019 | 1.825 | 1.034 | 0.508 | 0.240 | 0.080 |
| WSW | 0.400 | 1.139 | 1.250 | 0.988 | 0.668 | 0.394 | 0.208 | 0.090 |
| W | 0.260 | 0.720 | 0.887 | 0.746 | 0.475 | 0.255 | 0.105 | 0.044 |
| WNW | 0.219 | 0.645 | 0.674 | 0.421 | 0.204 | 0.087 | 0.039 | 0.023 |
| NW | 0.226 | 0.721 | 0.706 | 0.460 | 0.214 | 0.084 | 0.027 | 0.015 |
| NNW | 0.241 | 0.836 | 0.844 | 0.601 | 0.308 | 0.147 | 0.065 | 0.028 |
| Total | 9.002 | 23.952 | 21.411 | 16.573 | 12.089 | 7.962 | 4.619 | 2.418 |

(Continued)

| Direction | 16 - 18 | 18 - 20 | 20 - 22 | 22 - 24 | 24 - 26 | 26 - 28 | 28 - | Sum |
|-----------|---------|---------|---------|---------|---------|---------|-------|---------|
| N | 0.026 | 0.014 | 0.003 | 0.001 | 0.000 | 0.000 | 0.000 | 4.291 |
| NNE | 0.047 | 0.022 | 0.012 | 0.006 | 0.002 | 0.000 | 0.000 | 6.044 |
| NE | 0.068 | 0.028 | 0.011 | 0.002 | 0.001 | 0.000 | 0.000 | 7.111 |
| ENE | 0.007 | 0.004 | 0.002 | 0.003 | 0.002 | 0.001 | 0.001 | 3.508 |
| E | 0.013 | 0.004 | 0.001 | 0.001 | 0.000 | 0.000 | 0.000 | 4.435 |
| ESE | 0.130 | 0.048 | 0.013 | 0.006 | 0.001 | 0.001 | 0.000 | 8.220 |
| SE | 0.196 | 0.082 | 0.034 | 0.012 | 0.007 | 0.004 | 0.003 | 9.225 |
| SSE | 0.250 | 0.123 | 0.046 | 0.015 | 0.003 | 0.001 | 0.000 | 9.002 |
| S | 0.212 | 0.098 | 0.046 | 0.020 | 0.006 | 0.002 | 0.001 | 10.273 |
| SSW | 0.078 | 0.040 | 0.014 | 0.004 | 0.001 | 0.000 | 0.000 | 11.320 |
| SW | 0.037 | 0.011 | 0.005 | 0.001 | 0.001 | 0.000 | 0.000 | 9.966 |
| WSW | 0.031 | 0.013 | 0.003 | 0.001 | 0.001 | 0.000 | 0.001 | 5.186 |
| W | 0.014 | 0.009 | 0.005 | 0.001 | 0.000 | 0.000 | 0.000 | 3.522 |
| WNW | 0.016 | 0.007 | 0.006 | 0.002 | 0.001 | 0.000 | 0.000 | 2.343 |
| NW | 0.007 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 2.462 |
| NNW | 0.015 | 0.006 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 3.091 |
| Total | 1.148 | 0.510 | 0.201 | 0.075 | 0.026 | 0.008 | 0.006 | 100.000 |

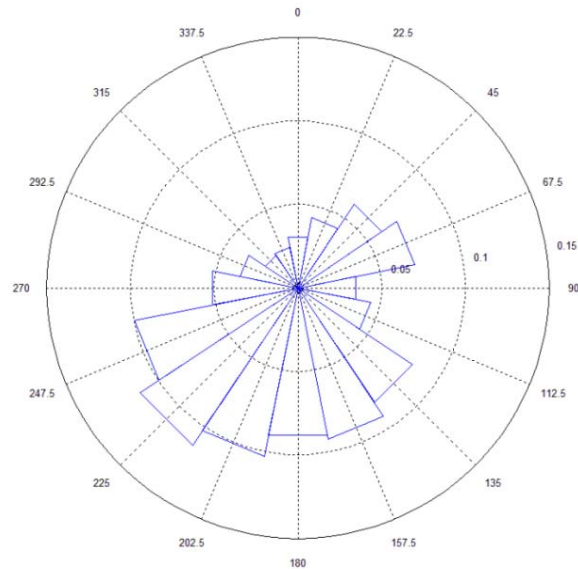
The Weibull parameters identified for each wind direction are listed in Table 2. For the convenience of subsequent calculation, the data given in Table 2 are fitted by the following harmonic functions (Coles and Walshaw 1994):

$$f_{\theta}(\theta) = a^f + \sum_{m=1}^{n_f} b_m^f \cos(m\theta - c_m^f) \quad (6)$$

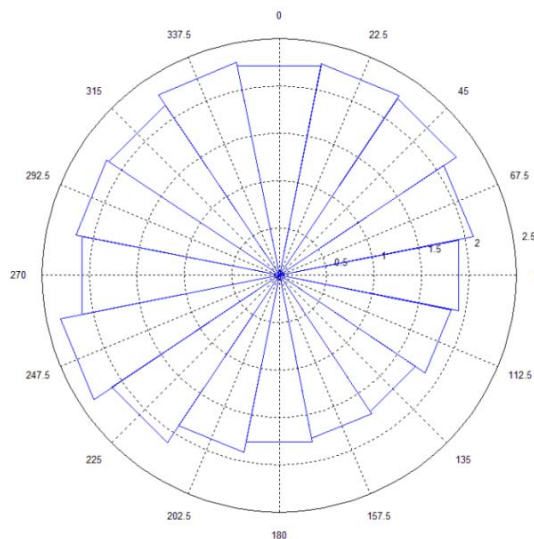
$$c(\theta) = a^c + \sum_{m=1}^{n_c} b_m^c \cos(m\theta - c_m^c) \quad (7)$$

$$k(\theta) = a^k + \sum_{m=1}^{n_k} b_m^k \cos(m\theta - c_m^k) \quad (8)$$

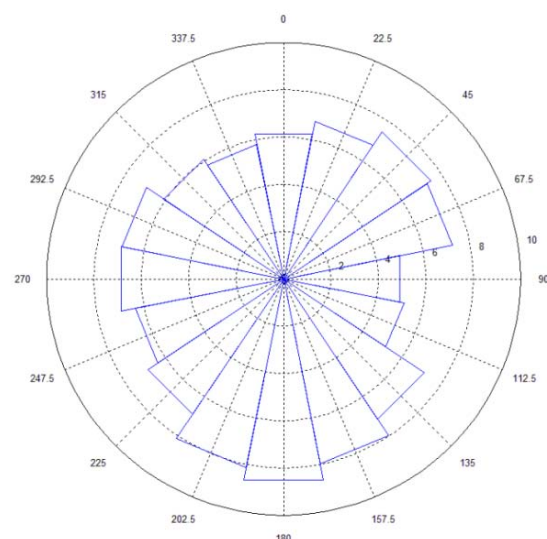
where a , b_m and c_m are the coefficients to be determined, whose superscripts f , c and k respectively denote the relative frequency, the scale and shape parameters, which are given in Fig. 2 and Table 2; and n_f , n_c and n_k are the order of harmonic functions.



(a) Relative frequency of wind direction



(b) Weibull shape parameter



(c) Weibull scale parameter

Fig. 2 Relative frequency of wind direction and Weibull scale and shape parameters

Table 2 Parameters for different wind direction sectors

| Direction | record number | $c(\theta)$ | $k(\theta)$ | $f_{\theta}(\theta)$ | Goodness of fit |
|-----------|---------------|-------------|-------------|----------------------|-----------------|
| N | 24867 | 6.791 | 2.282 | 0.043 | 0.972 |
| NNE | 35030 | 7.480 | 2.250 | 0.060 | 0.979 |
| NE | 41213 | 7.278 | 2.092 | 0.071 | 0.966 |
| ENE | 20328 | 5.005 | 1.926 | 0.035 | 0.901 |
| E | 25703 | 5.191 | 1.853 | 0.044 | 0.895 |
| ESE | 47637 | 7.144 | 1.732 | 0.082 | 0.927 |
| SE | 53464 | 7.960 | 1.756 | 0.092 | 0.947 |
| SSE | 52171 | 8.671 | 1.796 | 0.090 | 0.960 |
| S | 59533 | 8.128 | 1.905 | 0.103 | 0.963 |
| SSW | 65604 | 6.897 | 2.129 | 0.113 | 0.961 |
| SW | 57759 | 6.379 | 2.356 | 0.100 | 0.968 |
| WSW | 30052 | 6.999 | 2.121 | 0.052 | 0.969 |
| W | 20412 | 6.960 | 2.187 | 0.035 | 0.976 |
| WNW | 13579 | 6.079 | 2.166 | 0.023 | 0.958 |
| NW | 14268 | 5.804 | 2.297 | 0.025 | 0.966 |
| NNW | 17916 | 6.261 | 2.254 | 0.031 | 0.966 |
| Total | 579536 | 7.095 | 1.963 | 1.000 | 0.956 |

3. FATIGUE DAMAGE DATA

The fatigue damage is calculated for mean wind speed from 5 m/s to 35 m/s with an interval of 5 m/s. The damage data from Robertson et al (2004) is used here, as shown in Fig. 3. The figure shows fatigue damage values for lighting column with the correction for natural frequency, face area and Reynolds number effect. The damage data is calculated based on one hour duration in Fig. 3. In this study, these data have been converted into those based on 10-min by divided by 6.

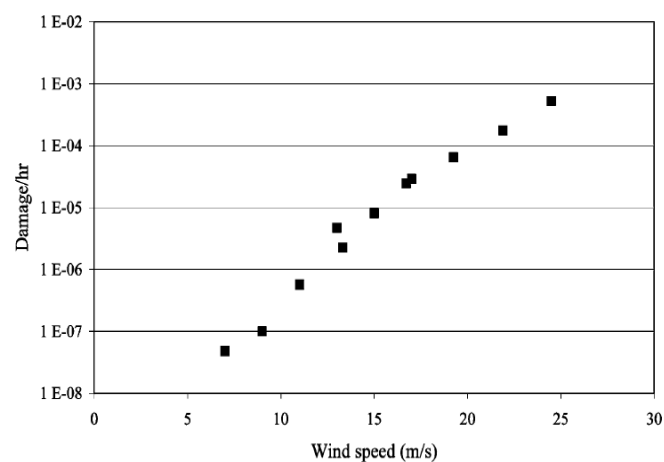


Fig. 3 Damage data for enlarged-diameter column (cited from Robertson et al 2004)

4. FATIGUE DAMAGE ANALYSIS

To assess the yearly fatigue damage, the distribution of occurrence sequence of 52560(=365×24×6) 10-min blocks is needed. In j th direction range Ω_j , the number of 10-min wind data can be reflected by:

$$n_{0,j} = n_0 \int_{\Omega_j} f_{\theta}(\theta) d\theta \quad (9)$$

where $n_0 = 52560$ is the total number of 10-min wind data in 1 year; $f_{\theta}(\theta)$ is the relative frequency of wind direction (see Eq. (6)). After the total number of wind data in j th direction was acquired, the number of wind data in i th wind speed range within the j th direction range can be calculated by:

$$n_0(i, j) = n_{0,j} [P_u(5(i), \theta_j) - P_u(5(i-1), \theta_j)] \quad (10)$$

where $P_u(U, \theta)$ is the cumulative distribution function (see Eq. (3)).

By Eqs. (3) and (10), the distribution of 10-min wind data within corresponding wind direction and speed range can be calculated and the results are shown in Table 3. The fatigue damage value corresponding to each wind block can be found from Fig. 3. The accumulative fatigue damage can be calculated by the summation of fatigue damage corresponding to each wind block.

Table 3 Distribution of the number of wind record in 1 year

| Direction | 0-5 | 5-10 | 10-15 | 15-20 | 20-25 | 25-30 | 30-35 | Total |
|-----------|-------|-------|-------|-------|-------|-------|-------|-------|
| N | 800 | 1214 | 268 | 10 | 0 | 0 | 0 | 2292 |
| NNE | 830 | 1510 | 536 | 48 | 1 | 0 | 0 | 2925 |
| NE | 1160 | 1601 | 452 | 35 | 1 | 0 | 0 | 3249 |
| ENE | 1787 | 1344 | 180 | 6 | 0 | 0 | 0 | 3317 |
| E | 1869 | 1231 | 169 | 7 | 0 | 0 | 0 | 3276 |
| ESE | 1385 | 1382 | 448 | 69 | 6 | 1 | 0 | 3291 |
| SE | 1096 | 1386 | 723 | 219 | 42 | 6 | 1 | 3473 |
| SSE | 1137 | 1541 | 842 | 258 | 52 | 7 | 1 | 3838 |
| S | 1429 | 1961 | 783 | 133 | 10 | 1 | 0 | 4317 |
| SSW | 1859 | 2357 | 484 | 21 | 0 | 0 | 0 | 4721 |
| SW | 2039 | 2471 | 345 | 6 | 0 | 0 | 0 | 4861 |
| WSW | 1875 | 2313 | 389 | 11 | 0 | 0 | 0 | 4588 |
| W | 1649 | 1869 | 326 | 11 | 0 | 0 | 0 | 3855 |
| WNW | 1371 | 1256 | 125 | 2 | 0 | 0 | 0 | 2754 |
| NW | 870 | 592 | 24 | 0 | 0 | 0 | 0 | 1486 |
| NNW | 159 | 141 | 17 | 0 | 0 | 0 | 0 | 317 |
| Total | 21315 | 24169 | 6111 | 836 | 112 | 15 | 2 | 52560 |

By combining the damage data under different wind speed and distribution of wind block in Table 3, yearly damage data in each direction are acquired. The total fatigue damage value and some large damage directions are shown in Table 4.

Table 4 Yearly cumulative fatigue damage

| Total | SSE | SE | S | ESE | NNE | NE | SSW | WSW |
|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| 2.30×10^{-2} | 5.31×10^{-3} | 4.53×10^{-3} | 3.08×10^{-3} | 1.71×10^{-3} | 1.54×10^{-3} | 1.25×10^{-3} | 1.18×10^{-3} | 8.99×10^{-4} |

If the directionality is taken into account, the fatigue life is decided by damage of the largest fatigue direction; if not, the fatigue life is decided by the total damage. When the directionality of wind is considered, the fatigue life of the column is $T_{SSE}=1/D_{SSE}=188.32$ (year); when the directionality is not considered, the fatigue life is $T_{Total}=1/D_{Total}=43.48$ (year). The fatigue life of column under consideration of directionality T_{SSE} is 4.33 times of fatigue life without considering directionality T_{Total} . It is clear that considering direction or not has significant influence on fatigue life.

In this study, shelter effect of wind is also considered. When shelter effect is taken into consideration, wind data are selected from the daily maximum average wind speed per ten minutes with directionality. Wind data under the consideration of shelter effect are fitted by following the procedure in section 2.3. The corresponding parameters are not listed here. With the fitting of the wind data, the corresponding 10-min wind block distribution is listed in Table 5.

Table 5 Distribution of the number of wind speed under shelter effect

| Direction | 0-5 | 5-10 | 10-15 | 15-20 | 20-25 | 25-30 | 30-35 | Total |
|-----------|-----|------|-------|-------|-------|-------|-------|-------|
| N | 111 | 735 | 904 | 246 | 10 | 0 | 0 | 2006 |
| NNE | 117 | 1164 | 1820 | 468 | 10 | 0 | 0 | 3579 |
| NE | 143 | 946 | 1237 | 397 | 23 | 0 | 0 | 2746 |
| ENE | 29 | 186 | 239 | 78 | 5 | 0 | 0 | 537 |
| E | 52 | 449 | 734 | 275 | 16 | 0 | 0 | 1526 |
| ESE | 94 | 999 | 2155 | 1222 | 128 | 1 | 0 | 4599 |
| SE | 104 | 1066 | 2531 | 2000 | 431 | 17 | 0 | 6149 |
| SSE | 96 | 908 | 2119 | 1822 | 505 | 33 | 1 | 5484 |
| S | 82 | 857 | 2025 | 1544 | 305 | 10 | 0 | 4823 |
| SSW | 89 | 1150 | 2700 | 1369 | 88 | 0 | 0 | 5396 |
| SW | 116 | 1623 | 3183 | 823 | 10 | 0 | 0 | 5755 |
| WSW | 136 | 1562 | 2307 | 383 | 2 | 0 | 0 | 4390 |
| W | 99 | 913 | 1139 | 179 | 1 | 0 | 0 | 2331 |
| WNW | 47 | 536 | 682 | 74 | 0 | 0 | 0 | 1339 |
| NW | 12 | 267 | 372 | 12 | 0 | 0 | 0 | 663 |
| NNW | 25 | 489 | 691 | 32 | 0 | 0 | 0 | 1237 |

Based on the distribution of 10-min wind block in Table 5 and fatigue damage data, yearly damage data under the consideration of shelter effect can be calculated. The larger fatigue damage directions and damage values are listed in Table 6. The damage values in the other directions are small, so it is not listed here.

Table 6 Yearly cumulative fatigue damage under shelter effect

| SE | SSE | S | SSW | ESE | SW | NNE | WSW | NE |
|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| 3.18×10^{-2} | 3.02×10^{-2} | 2.43×10^{-2} | 2.13×10^{-2} | 1.90×10^{-3} | 1.51×10^{-2} | 8.64×10^{-3} | 8.41×10^{-3} | 6.94×10^{-4} |

Under the consideration of shelter effect, the fatigue life of column is decided by damage of the largest fatigue direction. Based on the Table 5, the fatigue life is $T_{shelter}=1/D_{SE}=31.48$ (year) considering shelter effect. Compared $T_{shelter}$ with T_{SSE} , the fatigue life with no shelter effect $T_{SSE}=188.32$ (year) is 5.98 times of fatigue life with the

consideration of shelter effect T_{shelter} . It is clear that shelter effect leads to a prominent influence on fatigue life.

5. CONCLUSION

In this study, the fatigue life of a lighting column was predicted by the combination of the joint probability distribution of wind speed and direction and fatigue data. Based on the long-term measured wind records, the joint distribution of wind speed and direction was established. The cumulative fatigue damage of the lighting column was finally estimated by the summation of fatigue damage corresponding to each wind block. The results show that:

(1) The directionality has significant influence on fatigue life prediction. The fatigue life of considering directionality is 4.33 times of that neglecting directionality. Thus, there is a distinct directionality to fatigue damage.

(2) The fatigue life with no shelter effect is 5.98 times of fatigue life considering shelter effect. It can be seen that shelter effect also has a significant influence on wind-induced fatigue.

Acknowledgements

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