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# High accuracy numerical and signal processing approaches to extract flutter derivatives

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## ABSTRACT

Process of differentiation is based on displacement information in structural dynamics. Rotation data is constructed by vertical data for experimental efficiency and high or low frequency effects of fluid and equipment can be affected to results. From those reasons, errors have no choice but to be accumulated when constructing velocity and acceleration. As the results, numerical and signal processing approaches must be considered to obtain solutions. This study adopts some methods to deal with error control. Specifically, compact differencing method which uses contiguous term is adopted. To obtain solutions in system identification, GLS (General least square) method is used to consider iteration method and nonlinear problem which error terms is included in. Noise and Non-noise data are compared for confirming accuracy.

## 1. INTRODUCTION

The discipline of aeroelasticity refers to the study of phenomenon wherein aerodynamic forces and structural motions interact significantly. Flutter is aeroelastic self-excited oscillation of a structural system.
Extraction of flutter derivatives can be done through the forced vibration technique or the free vibration technique. The free vibration technique is comparatively simple because it only requires initial displacements.
Sarkar(1992) developed the modified Ibrahim time domain (MITD) method to extract all the direct and cross-flutter derivatives from the coupled free vibration data of a 2-DOF section model. Chowdhury(2003) et al. were successful in identifying eight flutter derivatives simultaneously from noisy displacement time-histories generated under laminar and turbulent flow.
Other system identification methods that can be applied to problems in structural dynamics are least square, instrumental variable, maximum likelihood, and extended Kalman filtering. Hsia(1976) described different least squares algorithms for system parameter identification. Extended Kalman filtering techniques were used by Yamada et al(1992). Jakobsen et al.(1995) and Brownjohn et al.(2001) have used covariance

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block Hankel matrix(CBHM) method for parameter extraction of a 2-DOF system. Gu and Zhu(2000) have used an identification method based on unifying least square theory to extract flutter derivatives of a 2-DOF model.
But, approach to process raw data is more important for guaranteeing accuracy of results. So, in this study various numerical approaches is introduced and improved results can be found.

## 2. FLUTTER ANALYSIS

### *2.1 Equation of Motion*

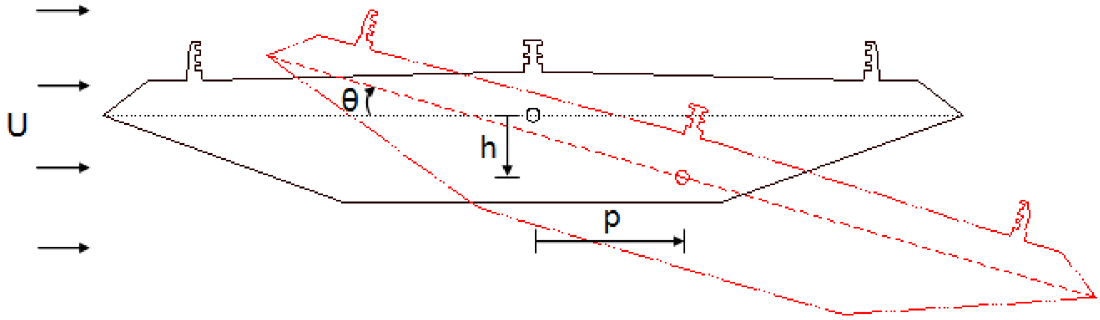



Fig. 1 Positive sign convention of 3DOF motion

In the wind, it is expressed as shown below

$$\underline{\underline{M}} \cdot \underline{\ddot{X}} + \underline{\underline{C}} \cdot \underline{\dot{X}} + \underline{\underline{K}} \cdot \underline{X} = \underline{F_{se}} + \underline{F_{ad}} \quad \cdots\cdots (1)$$

where,

$$\underline{X} = [h\ \alpha\ p]^T$$

$$\underline{\underline{M}} = \begin{bmatrix} m_h & 0 & 0 \\ 0 & I_\alpha & 0 \\ 0 & 0 & m_p \end{bmatrix}, \quad \underline{\underline{C}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}, \quad \underline{\underline{K}} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix}$$

$$\underline{F_{se}} = \begin{bmatrix} 0.5\rho U^2 B & 0 & 0 \\ 0 & 0.5\rho U^2 B^2 & 0 \\ 0 & 0 & 0.5\rho U^2 B \end{bmatrix} \begin{bmatrix} KH_1^*/U & KH_2^* B/U & KH_5^*/U & K^2 H_4^*/B & K^2 H_3^* & K^2 H_6^*/B \\ KA_1^*/U & KA_2^* B/U & KA_5^*/U & K^2 A_4^*/B & K^2 A_3^* & K^2 A_6^*/B \\ KP_5^*/U & KP_2^* B/U & KP_1^*/U & K^2 P_6^*/B & K^2 P_3^* & K^2 P_4^*/B \end{bmatrix} \begin{bmatrix} \dot{h} \\ \dot{\alpha} \\ \dot{p} \\ h \\ \alpha \\ p \end{bmatrix}$$

$\underline{F_{ad}}$ : Aerodynamic force by buffeting

If it is laminar flow, $F_{ad}$ can be eliminated. Because $F_{se}$ is consisted of displacement and velocity term, Eq. (1) can be expressed homogenous form as shown below.

$$\underline{\underline{M}} \cdot \underline{\ddot{X}} + \underline{\underline{C^{eff}}} \cdot \underline{\dot{X}} + \underline{\underline{K^{eff}}} \cdot \underline{X} = 0 \cdots\cdots (2)$$

*2.2 Sate Space Form*

Dynamic equations which are expressed in effective form like Eq. (2) can be organized as four differential equations in case of 2DOF condition. It is called state-space equation.

$$\begin{Bmatrix} \dot{X} \\ \ddot{X} \end{Bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \begin{Bmatrix} X \\ \dot{X} \end{Bmatrix} \cdots\cdots (3)$$

or

$$\dot{X} = AX$$

where, X is $2n \times 1$ and n is DOF

Procedure to get A matrix is explained in another chapter.
The solution of first order matrix differential equation is shown as below

$$X = e^{At} X_0 \cdots\cdots (4)$$

If A matrix is determined, it is obvious that velocity and acceleration can be obtained from Eq. (4). As the results, if system of structure is determined, displacement, velocity and acceleration can be evaluated. Those values extracted by Eq. (4) are theoretical value that errors are not contained.
To show accuracy of proposed method, specific system property will be adopted. According to Jakobsen et al. [1995], effective structural matrices were determined for the mean-wind speed equal to U = 10.26 m/s in field condition.

$$M = \begin{bmatrix} 2.6526 & 0 \\ 0 & 0.0189 \end{bmatrix},\ C = \begin{bmatrix} 8.9308 & -0.0799 \\ 0.4345 & 0.0386 \end{bmatrix},\ K = \begin{bmatrix} 420.1002 & -59.1805 \\ 1.7552 & 19.6592 \end{bmatrix}$$

When considering damped eigenvalue problem which is non-proportional damping, frequencies of the corresponding conditions are $w_1 = 1.991\ \mathrm{Hz}$ and $w_2 = 5.1312\ \mathrm{Hz}$. To solve equation of dynamic motion, initial condition is needed. And Initial condition is h0/B=0.03, B=387.5 mm, alpha_0=0.03 rad.

## 3. DISCRETIZATION TECHNIQUE

### *3.1 Discretization Method*

Considering dynamic problem, the equation has 2nd order which is the highest term involved with time. Generally central difference technique is used to discretize n-th order derivatives as shown below.

$$\dot{h}_{i+1} = \frac{h_{i+2} - h_i}{2\Delta t},\ \ddot{h}_{i+1} = \frac{h_{i+2} - 2h_{i+1} + h_i}{\Delta t^2} \cdots\cdots (4)$$

Discretizing with central difference technique, it needs 4 points in case of 2nd order accuracy. And to have 4th order accuracy, it require 5 points. But comparing compact discretization (pade's differencing) with central differencing, it just needs 3 points in case of 4th order accuracy. Moreover compact differencing has more accuracy than central differencing. That is, error term is proportional to $O(\Delta t^2)$ in case of central differencing, but is proportional to $O(\Delta t^4)$ in case of compact differencing. As the results, compact differencing technique has more accuracy for discretization.

$$\frac{1}{4}h'_{i-1} + h'_i + \frac{1}{4}h'_{i+1} = \frac{3}{4\Delta t}(h_{i+1} - h_{i-1}) \cdots\cdots (5)$$

$$\frac{1}{10}h''_{i-1} + h''_i + \frac{1}{4}h''_{i+1} = \frac{6}{5\Delta t^2}(h_{i+1} - 2h_i + h_{i-1}) \cdots\cdots (6)$$

### *3.2 Numerical Error*

#### *3.2.1 Simple Fourier Error Analysis*

Arbitrary periodic function, f(x), can be expressed as a linear combination of Fourier component $e^{ikt}$. Exact solution is compared with central and compact differencing. Compact differencing is well matched up with continuous value when displacement is increased.

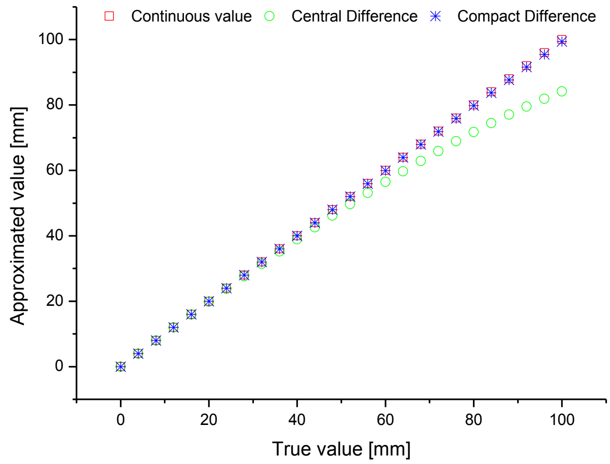
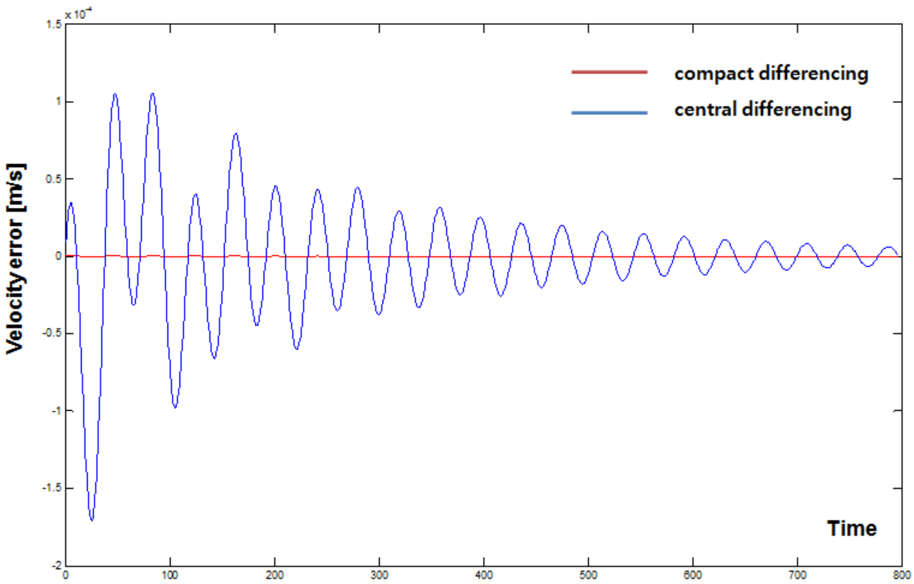



Fig. 2 Simple Fourier error according to discretization method

*3.2.2 Error of Discretization Method*

From the system property which is referred to as J. Bogunovic Jakobsen et al. [1995], theoretical solution can be evaluated. The value obtained by discretization method can be also evaluated. Difference between theoretical and discretized value is shown as Fig. 3 according to discretization method. From Fig. 3, compact differencing method is good at accuracy.



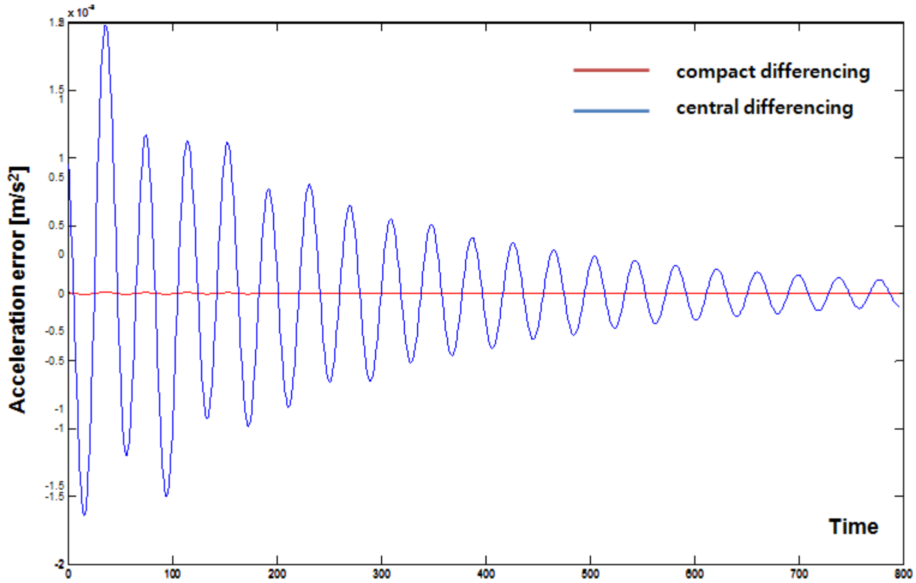



Fig. 3 Velocity and acceleration error according to discretization method

When using central differencing, constructed matrix, stiffness and damping coefficient, is shown below

$$K = \begin{bmatrix} 420.0354 & -59.0032 \\ 1.750643 & 19.61726 \end{bmatrix}, C = \begin{bmatrix} 8.933412 & -0.08015 \\ 0.433677 & 0.038579 \end{bmatrix}$$

And when adopting compact differencing, constructed matrix is shown below

$$K = \begin{bmatrix} 420.1718 & -59.1524 \\ 1.755794 & 19.65963 \end{bmatrix}, C = \begin{bmatrix} 8.930786 & -0.07985 \\ 0.434496 & 0.038599 \end{bmatrix}$$

When comparing compact differencing with central differencing, compact differencing method has high quality results. Moreover, differences of discretization method have more effect on damping coefficient.

| Error of central differencing [%] | | | | Error of compact differencing [%] | | | |
|---|---|---|---|---|---|---|---|
| K | | C | | K | | C | |
| 0.015424 | 0.299667 | -0.02925 | -0.31354 | -0.01704 | 0.047443 | 0.000152 | 0.068038 |
| 0.259609 | 0.213332 | 0.189325 | 0.053698 | -0.03385 | -0.0022 | 0.000922 | 0.001604 |

Table. 1 Error of compact differencing and central differencing

## 4. SIGNAL PROCESSING

### *4.1 Filtering*

When measuring experimental data, there is noise like systemic error, gross error and etc. For this reason, filtering method is adopted to vertical and rotation displacement data.

To simulate experiment condition which can have various errors, wind tunnel test is conducted in another condition. First vertical frequency is 1.428 Hz and first rotational frequency is 3.308. To reduce noise, each frequency must be considered simultaneously in case of 2 or 3 DOF because of coupled motion. cut-off frequency of vertical data is 1.5 Hz and rotational data is 3.5 Hz. In the MATLAB, bandpass filter - equiripple - is used and zero-phase filtering is performed by using filtfilt internal function.

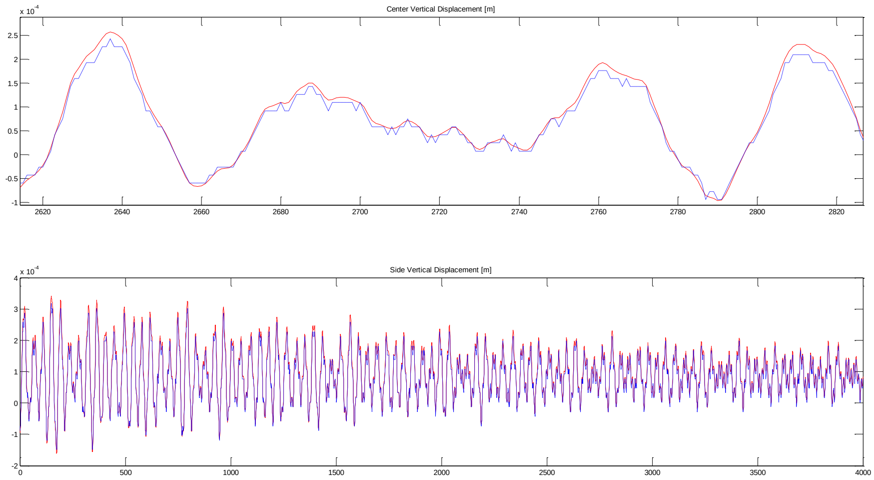



Fig. 4 Filtered signal

Like Fig. 4, noise effects are mitigated. Because high frequency noise is eliminated, data is changed into smooth curve.

*4.2 Staking*

Each signal has property such as static delay, epicentral distance and weight. Dynamic delay can be calculated by difference of vertical arrival time and velocity of refracted wave. A certain property of delay can be automatically increased by data processing. Those features can be eliminated when several measured data which is in same time are averaged. This fact can be intuitionally understood from Fig 5 that signal is superposed in same time. That is, clear line seems to be averaged value. From Fig 6, it is obvious that results that stacking is used have good quality.

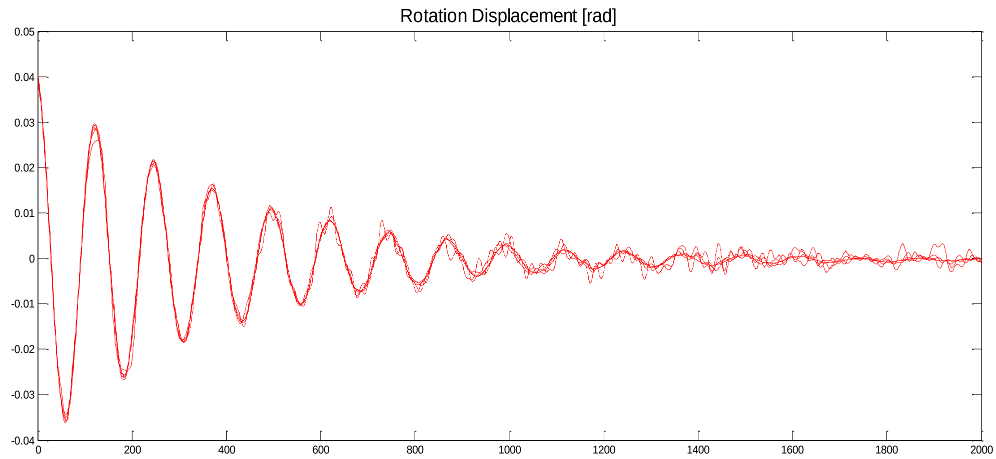



Fig. 5 Superposed signal in same time

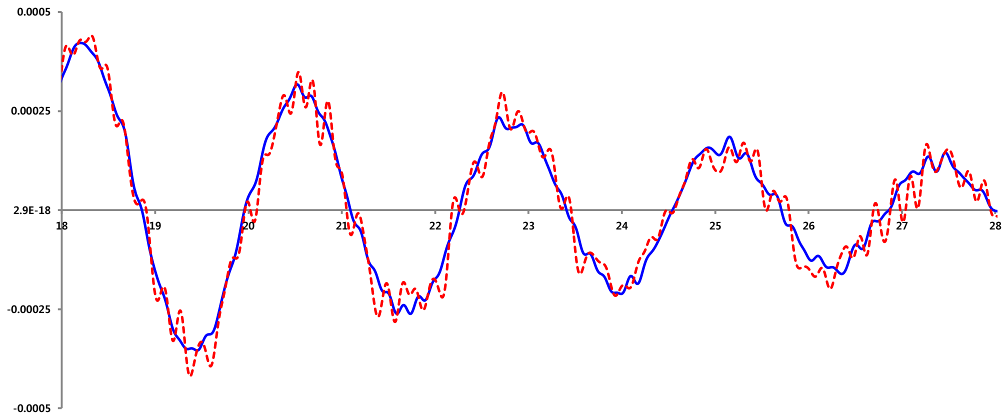



Fig. 6 Stacked signal

## 5. CONCLUSIONS

Method which can enhance data quality is introduced in case of discretization and signal processing. It is important that a series of numerical method need to be adopted in certain order. First signal processing need to be considered to overcome external error such as mechanical noise and so on. And then, compact discretization technique is used to minimize error of solution. As the results, all of those sequences can make results reasonable.

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