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A novel optimization approach for determining wind tunnel derived load combinations for tall buildings

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ABSTRACT

Wind tunnel technique has emerged as a powerful tool for predicting wind-induced loads on high-rise buildings. Aerodynamic wind loads acting on buildings generate building vibrations in two swaying and one twisting directions. Accurate assessment of wind load combinations is an extremely important issue in the design of wind sensitive tall buildings. Traditional practice for determining wind tunnel derived wind load cases lacks a robust approach and relies on somewhat subjective judgments. This paper presents a novel optimization-based framework for systematically determining wind tunnel derived wind load cases for structural design of wind sensitive tall buildings. Specifically, an optimization-based framework is developed by firstly assuming that the probability function of the three load components in each incident wind direction can be expressed by a multivariate normal distribution so that the equivalence surface of probability corresponding to a specified value of the peak factor becomes an ellipsoidal one. An optimization algorithm is then applied to search for a convex polyhedral hull which serves as a design envelope covering all ellipsoidal thresholds in all incident wind directions. Individual load cases can be given in terms of the coordinates of the vertexes on the optimized design envelope. The objective function is to achieve more precise prediction of wind load combinations by minimizing the volume of the polyhedral design envelope in consideration of the minimum deviation of the polyhedral envelope surfaces from the ellipsoidal thresholds. The Pareto optimal theory is further integrated with this new method allowing for determining the appropriate total number of wind load cases for structural design of a building. One full-scale 30-story building example is used to illustrate the effectiveness and practical application of the proposed optimization-based technique for evaluating peak resultant wind load cases.

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1. INTRODUCTION

Accurate assessment of wind-induced load effects plays a pivotal role in the structural design of high-rise buildings. The wind tunnel technique has long been recognized as an accurate and comprehensive experimental method for estimating wind loads for tall buildings. The high frequency base balance (HFBB) test has become one of the most common wind tunnel testing techniques capable of measuring aerodynamic wind loads that generate three-dimensional building vibrations in two translational and one twisting directions. For tall buildings with significant coupled lateral and torsional responses, the estimation of peak resultant load effects is an extremely important issue in the assessment of building performances under wind excitation.

Isyumov (1982) proposed an approach for estimating the peak resultant load effects resulting from wind forces in the two swaying directions and the wind induced torque by using the SRSS rule with empirical joint action factors. Solari and Pagnini (1999) provided an analytical evaluation scheme of vectorial load effects from alongwind and crosswind load components which were considered uncorrelated. A dodecagon representing the envelope of critical load conditions was constructed, in which an elliptical threshold defined by the maximum and minimum values of a single process was enclosed. AIJ Recommendations (2004) assumed a bivariate normal distribution for crosswind and torsional load components in which the equivalence line of probability could then be interpreted as an elliptical line with a sloped major axis due to the consideration of correlation. An octagon enclosing the elliptical line served as an envelope to represent critical load combinations.

The above studies focused on the determination of peak resultant load effects for one single process. In wind tunnel tests, attack angles between the approaching wind and the building may vary from 0 to 360 degrees resulting in many single processes. Therefore, critical load cases derived from all incident wind directions should be identified and a limited number accounting for governing conditions may be selected for design. A general outline from the Boundary Layer Wind Tunnel Laboratory in the University of Western Ontario (2007) specified 24 nominal critical load cases for all wind directions in which each critical load case was defined that the largest load effects possibly occurred when the load in one principal load direction was at its peak together with nominal loads in the other two principal directions. Boggs and Lepage (2006) suggested 10 to 20 or so load cases in which critical combinations were either defined as the principal component experiencing its peak values with other two companion values, or the maximum vector resultant values.

Nonetheless, as far as the authors are aware, most if not all common practices for obtaining critical wind load cases and the appropriate number of load cases partially relies on subjective judgments. It is also important to realize that the use of existing combination methods may result in inconsistent and subjective assessments of critical wind load effects on buildings. Hence the research objective is to develop an explicit framework for systematically obtaining wind tunnel derived load cases for structural design of tall buildings.

This paper presents a computer-based optimization approach to determine peak resultant load cases of wind-excited tall buildings with 3D correlated wind loads measured in wind tunnel tests. Multivariate normal distribution is assumed for random wind-induced structural load components in each incident wind direction, where the equivalence surface of probability can be interpreted as an ellipsoid. An optimization-based framework is proposed for searching for a convex polyhedral hull which serves as a design envelope enclosing all the ellipsoids of wind loads corresponding to all incident wind directions. Individual combined load cases could be expressed in terms of the coordinates of the vertexes of the optimized polyhedral design envelope. The Pareto front is also integrated with this new methodology allowing for predicting the appropriate total number of load cases. As an illustrative example, a 30-story building is used to demonstrate the proposed optimization-based framework. The accuracy of the proposed method for systematic prediction of wind load cases of the building is examined and compared with the load cases obtained by the current wind tunnel practice.

2. ANALYSIS OF WIND-INDUCED RESPONSE AND EQUIVALENT STATIC WIND LOADS IN HFBB TESTS

In HFBB tests, the time-variant base moment components in two translational and one twisting directions are measured and considered as the aerodynamic loads acting on the building. For a multistory building modeled as three degrees of freedom with a lumped mass at each floor level, the equation of motion can be conveniently written in matrix notations as

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=[F] \tag{1}$$

where $[M]$, $[C]$, $[K]$, $\{x\}$, $[F]$ are the mass matrix, matrix of damping coefficient, stiffness matrix, the displacements and external forces, respectively. Normal modes can be used to transform the system of coupled differential equations into a set of uncoupled differential equations, and then the n uncoupled equations in a transformed system of generalized coordinates appear as

$$m_j\ddot{\xi}_j + c_j\dot{\xi}_j + k_j\xi_j = f_j \tag{2}$$

where m_j, c_j, k_j, are the generalized mass, damping, stiffness for the jth mode; ξ_j and f_j are the generalized coordinate and force for the jth mode. Assuming that the structure has linear mode shape in two swaying directions and constant mode shape in torsion direction, the generalized force could be expressed by base moments measured in wind tunnel tests as

$$\begin{aligned} f_j &= \sum [\phi_{xj}(z_i) f_x(z_i) + \phi_{yj}(z_i) f_y(z_i) + \phi_{\theta j}(z_i) f_\theta(z_i)] \\ &= \sum [\phi_{xj}(z_h) \frac{z}{h} f_x(z_i) + \phi_{yj}(z_h) \frac{z}{h} f_y(z_i) + \phi_{\theta j}(z_h) f_\theta(z_i)] \\ &= c_{xj} M_{yy} - c_{yj} M_{xx} + c_{\theta j} M_{\theta\theta} \end{aligned} \tag{3}$$

in which ϕ_j is the jth mode shape vector; c_{xj}, c_{yj} and $c_{\theta j}$ are constants; M_{xx}, M_{yy}, $M_{\theta\theta}$ are measured time histories of base moments. The time-variant responses ξ_j derived from each uncoupled equation are then superimposed for determining total responses of the structural system by

$$x = \Phi_1 \xi_1 + \Phi_2 \xi_2 + \cdots + \Phi_n \xi_n = \sum_{j=1}^{n} \Phi_j \xi_j \tag{4}$$

where $\Phi_j=[\phi_{xj}\ \ \phi_{yj}\ \ \phi_{zj}]^T$. The equivalent static wind load can be expressed as

$$F_{eq} = [K]\{x\} \tag{5}$$

3. DEVELOPMENT OF AN ELLIPSOIDAL THRESHOLD FOR WIND LOAD COMBINATIONS IN EACH INCIDENT WIND DIRECTION

Once the time-variant equivalent static wind loads are obtained, the base moment responses can be further utilized to estimate extreme wind load cases. The ellipsoidal threshold in each incident wind direction is firstly developed similar to that proposed by AIJ Recommendations (2004) to derive elliptical threshold. The assumption that two translational and one torsional base moment responses follow multivariate normal distribution is made, where the joint probability density function can be expressed as

$$f(X) = \frac{1}{(2\pi)^{1.5} |\Sigma|^{1/2}} \exp(-\frac{1}{2}(X-\mu)^T \Sigma^{-1} (X-\mu)) \tag{6}$$

where X, μ and Σ denote the vector of variables, mean values and covariance matrix of variables that are defined as

$$X = [M_x\ M_y\ M_z]^T;$$

$$\mu = [E(M_x)\ E(M_y)\ E(M_z)]^T;$$

$$\Sigma = \begin{bmatrix} E\{[M_x - E(M_x)][M_x - E(M_x)]\} & E\{[M_x - E(M_x)][M_y - E(M_y)]\} & E\{[M_x - E(M_x)][M_z - E(M_z)]\} \\ E\{[M_y - E(M_y)][M_x - E(M_x)]\} & E\{[M_y - E(M_y)][M_y - E(M_y)]\} & E\{[M_y - E(M_y)][M_z - E(M_z)]\} \\ E\{[M_z - E(M_z)][M_x - E(M_x)]\} & E\{[M_z - E(M_z)][M_y - E(M_y)]\} & E\{[M_z - E(M_z)][M_z - E(M_z)]\} \end{bmatrix}. \tag{7}$$

Mean values as well as covariance matrix are determined by the statistical analysis method from time history samples in each incident wind direction. Then the distribution forms ellipsoidal isopleths with sloped major axes due to the consideration of correlations among base moment responses. The length of the semi-principal axes of ellipsoidal threshold is correlated with the value of the peak factor *g* defined as

$$M_{i,\max} = \overline{M}_i \pm g\sigma_{M_i} \tag{8}$$

For a given value of the peak factor, the magnitude of the ellipsoidal threshold is determined. Extreme wind load combinations for three-dimensional base moment responses are then depicted as the surface of the ellipsoid. The confidence level of peak resultant loads that is equivalent to the volume of the ellipsoid is derived from the cumulative distribution function of the multivariate normal distribution shown as

$$F(X) = \iiint f(X)dX = \iiint \frac{1}{(2\pi)^{1.5}|\Sigma|^{1/2}} \exp(-\frac{1}{2}(X-\mu)^T \Sigma^{-1}(X-\mu))dX \tag{9}$$

By substituting Eq. (8) into the Eq. (9), the confidence level of the ellipsoidal threshold can be simplified and given explicitly in terms of the peak factor, *g*, as

$$F(X) = 2\Phi(g) - 1 - \frac{\sqrt{2}g}{\sqrt{\pi}} e^{-\frac{g^2}{2}} \tag{10}$$

where Φ is the cumulative probability for the standard normal distribution.

4. OPTIMIZATION FRAMEWORK FOR DEVELOPMENT OF DESIGN ENVELOPE

Once all the equal-probability ellipsoidal thresholds corresponding to wind approaching from all azimuths are established, they are integrated to form a statistical envelope boundary representing all critical wind load combinations for the building. In theory, every point on the envelope surface can be regarded as a peak resultant load case; but it is not practical to consider all possible points on the envelope for structural design. Consequently, the method is of interest in certain packing problems that a finite number of representative discrete critical load cases are to be determined from infinite number of load cases. In searching for the optimal design envelope encompassing critical wind load cases as its vertexes, one major goal of this paper is to develop a numerical optimization technique for defining a polyhedral envelope which encloses the ellipsoidal thresholds corresponding to all incident wind directions while attaining the minimum deviation from the original statistical envelope boundary as much as possible.

4.1 Objective functions

In the proposed optimization framework of obtaining the polyhedral design envelope, the objective function is delineated as minimizing the volume of the polyhedral envelope since the polyhedron with the smallest volume after an outer approximation indicates that the difference between the polyhedral and the integrated ellipsoidal thresholds has been reduced to minimum in terms of volume. Furthermore, the objective function also serves as an important index to reflect that the shape distortion of the optimal design envelope from the integrated ellipsoidal threshold has been minimized.

Consider a polyhedron with *m* triangular surfaces and *n* vertexes, design variables are defined as coordinates of each vertex ((x_1,y_1,z_1), (x_2,y_2,z_2), …, (x_n,y_n,z_n)). For a convex polyhedron with the irregular shape, a simple way to calculate its volume is to split it into several tetrahedrons and sum their volumes up. For one tetrahedron with three vertexes *i, j, k* and an interior point *O* of a triangular surface of the polyhedron, the volume of the tetrahedron can be calculated in terms of design variables by (Slaught and Lennes 1919)

$$V_{O-ijk} = \frac{1}{6}|\vec{r}_{Oi}||\vec{r}_{Oj}||\vec{r}_{Ok}|(1-\cos^2\langle\vec{r}_{Oi},\vec{r}_{Oj}\rangle - \cos^2\langle\vec{r}_{Oi},\vec{r}_{Ok}\rangle - \cos^2\langle\vec{r}_{Oj},\vec{r}_{Ok}\rangle + 2\cos\langle\vec{r}_{Oi},\vec{r}_{Oj}\rangle\cos\langle\vec{r}_{Oi},\vec{r}_{Ok}\rangle\cos\langle\vec{r}_{Oj},\vec{r}_{Ok}\rangle)^{0.5} \quad (11)$$

where $\vec{r}_{Oi},\vec{r}_{Oj},\vec{r}_{Ok}$ area vectors defined respectively as

$$\vec{r}_{Oi} = (x_i - x_O, y_i - y_O, z_i - z_O);\ \vec{r}_{Oj} = (x_j - x_O, y_j - y_O, z_j - z_O);\ \vec{r}_{Ok} = (x_k - x_O, y_k - y_O, z_k - z_O) \quad (12)$$

By summing up the volumes of all the collective tetrahedrons, the total volume of the polyhedral design envelop with *n* vertexes and *m* triangular surfaces can be given and minimized as

$$\min \quad V_{polyhedron} = \sum_m V_{O-ijk} \qquad (i, j, k \in S_h; h = 1,2,...m) \quad (13)$$

4.2 Constraint functions

<u>Side constraints</u> Constraints are defined as restrictions that must be satisfied to ensure the feasibility of a design requirement. To restrict undue conservatism in determining a critical load case, the coordinates of each vertex of the polyhedral design envelope must be limited within the statistical minimum and maximum values of measured base moment responses as

$$\begin{aligned} M_{x,\min} &\le x_i \le M_{x,\max} \\ M_{y,\min} &\le y_i \le M_{y,\max} \qquad (i = 1,2,...,n) \\ M_{z,\min} &\le z_i \le M_{z,\max} \end{aligned} \quad (14)$$

<u>Constraints for outer approximation</u> The outer approximation of the polyhedron based on integrated ellipsoidal thresholds is employed in this optimization framework for the sake of minimizing the risk of underestimating peak resultant loads. It is necessary that all the ellipsoidal base moment thresholds should be enclosed within the polyhedral design envelope. Mathematically such a restriction can be given implicitly for a HFBB test with *d* number of incident wind directions as follows

$$v(Polyhedron) \supseteq v(Ellipsoid_p) \qquad (p = 1,2,...,d) \tag{15}$$

where *v* is denoted as the set of points. In order to explicitly formulate Eq. (15), a linear transformation is needed to be first applied to transform each ellipsoid to a unit sphere centered at its origin. Eigenvalue analysis is carried out to facilitate this transformation by

$$Cov(\Sigma_p) \cdot \begin{bmatrix} \phi_{11} & \phi_{21} & \phi_{31} \\ \phi_{12} & \phi_{22} & \phi_{32} \\ \phi_{13} & \phi_{23} & \phi_{33} \end{bmatrix}_p = \begin{bmatrix} \phi_{11} & \phi_{21} & \phi_{31} \\ \phi_{12} & \phi_{22} & \phi_{32} \\ \phi_{13} & \phi_{23} & \phi_{33} \end{bmatrix}_p \cdot \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}_p \qquad (p = 1,2,...,d) \tag{16}$$

where ϕ and λ are the eigenvector and eigenvalue, respectively, derived from the eigenvalue analysis of the covariance matrix of base moment responses. The outer approximation can be implemented by ensuring that all surfaces of the polyhedral envelope and the transformed spheres should not intersect to each other in the transformed coordinate system. Then, it can be mathematically stated that the distance from the center of each sphere to each surface of the polyhedron should be larger than the radius of the sphere as

$$\frac{\left|A_{0,h} \cdot 0 + B_{0,h} \cdot 0 + C_{0,h} \cdot 0 + D_{0,h}\right|}{\sqrt{A_{0,h}^{\ 2} + B_{0,h}^{\ 2} + C_{0,h}^{\ 2}}} \geq 1 \Rightarrow \frac{A_{0,h}^{\ 2} + B_{0,h}^{\ 2} + C_{0,h}^{\ 2}}{D_{0,h}^{\ 2}} - 1 \leq 0 \qquad (h = 1,2,...,m) \tag{17}$$

where $A_{0,h}$, $B_{0,h}$, $C_{0,h}$ and $D_{0,h}$ are parameters in the function of surface *h* of the transformed polyhedron. For surface *h* with three vertexes *i*, *j*, *k* of the polyhedron, $A_{0,h}$, $B_{0,h}$, $C_{0,h}$ and $D_{0,h}$ can be given in terms of the coordinate design variables of the three vertexes as

$$\begin{aligned}
A_{0,h} = {} & \sigma_{M_{x,p}} \phi_{11,p} g \sqrt{\lambda_{1,p}} [(y_j - y_i)(z_k - z_i) - (y_k - y_i)(z_j - z_i)] + \\
& \sigma_{M_{y,p}} \phi_{12,p} g \sqrt{\lambda_{2,p}} [(z_j - z_i)(x_k - x_i) - (z_k - z_i)(x_j - x_i)] + \\
& \sigma_{M_{z,p}} \phi_{13,p} g \sqrt{\lambda_{3,p}} [(x_j - x_i)(y_k - y_i) - (x_k - x_i)(y_j - y_i)] \\
B_{0,h} = {} & \sigma_{M_{x,p}} \phi_{21,p} g \sqrt{\lambda_{1,p}} [(y_j - y_i)(z_k - z_i) - (y_k - y_i)(z_j - z_i)] + \\
& \sigma_{M_{y,p}} \phi_{22,p} g \sqrt{\lambda_{2,p}} [(z_j - z_i)(x_k - x_i) - (z_k - z_i)(x_j - x_i)] + \\
& \sigma_{M_{z,p}} \phi_{23,p} g \sqrt{\lambda_{3,p}} [(x_j - x_i)(y_k - y_i) - (x_k - x_i)(y_j - y_i)] \\
C_{0,h} = {} & \sigma_{M_{x,p}} \phi_{31,p} g \sqrt{\lambda_{1,p}} [(y_j - y_i)(z_k - z_i) - (y_k - y_i)(z_j - z_i)] + \\
& \sigma_{M_{y,p}} \phi_{32,p} g \sqrt{\lambda_{2,p}} [(z_j - z_i)(x_k - x_i) - (z_k - z_i)(x_j - x_i)] + \\
& \sigma_{M_{z,p}} \phi_{33,p} g \sqrt{\lambda_{3,p}} [(x_j - x_i)(y_k - y_i) - (x_k - x_i)(y_j - y_i)]
\end{aligned} \tag{18}$$

$$
\begin{aligned}
D_{0,h} = 1 - \sigma_{M_{x,p}} g\sqrt{\lambda_{1,p}}(\phi_{11,p}+\phi_{21,p}+\phi_{31,p})[(y_j-y_i)(z_k-z_i)-(y_k-y_i)(z_j-z_i)] \\
-\sigma_{M_{y,p}} g\sqrt{\lambda_{2,p}}(\phi_{12,p}+\phi_{22,p}+\phi_{32,p})[(z_j-z_i)(x_k-x_i)-(z_k-z_i)(x_j-x_i)] \\
-\sigma_{M_{z,p}} g\sqrt{\lambda_{3,p}}(\phi_{13,p}+\phi_{23,p}+\phi_{33,p})[(x_j-x_i)(y_k-y_i)-(x_k-x_i)(y_j-y_i)] \\
(p=1,2,\ldots,d; i,j,k \in S_h; h=1,2,\ldots,m)
\end{aligned}
$$

where σ is the RMS of time history samples in the incident wind direction *p*; *g* is the peak factor.

Constraints for convexity The third type of constraint is to keep the polyhedral design envelope convex so that every vertex as a representative of one critical load case can be selected for practical use. To ensure convexity, every internal angle between two adjacent surfaces of the polyhedron must be within the range of 0 to 180 degrees as

$$
0^\circ < \langle \vec{n}_{S_h}, \vec{n}_{S_t} \rangle < 180^\circ \qquad (S_h \cap S_t = l; h,t = 1,2,\ldots m) \tag{19}
$$

where $\vec{n}_{S_h}$ and $\vec{n}_{S_t}$ denote the inner normal vectors of surface for every two adjacent surfaces S_h and S_t of the polyhedron.

4.3 Optimization algorithms

The minimization of the volume of the convex polyhedral design envelope enclosing all ellipsoidal wind load thresholds corresponding to all incident wind directions can be summarized as follows

Minimize $V_{polyhedron} = \sum_m V_{O-ijk}$

Subject to

$$
\begin{aligned}
&\nu(Polyhedron) \supseteq \nu(Ellipsoid_p) && (p=1,2,\ldots,d) \\
&0^\circ < \langle \vec{n}_{S_h}, \vec{n}_{S_t} \rangle < 180^\circ && (S_h \cap S_t = l; h,t=1,2,\ldots m) \\
&M_{x,\min} \le x_i \le M_{x,\max} && \\
&M_{y,\min} \le y_i \le M_{y,\max} && (i=1,2,\ldots,n) \\
&M_{z,\min} \le z_i \le M_{z,\max} &&
\end{aligned}
$$

Once the optimization problem is formulated with the objective function and design constraints being explicitly expressed in terms of design variables, the optimization solution can then be sought by the sequential quadratic programming (SQP) method. The SQP method is used to model nonlinear programming problems at a given approximate solution x^k and then to use that solution to the subproblem to construct a better approximation x^{k+1} (Boggs and Tolle 1996).The optimal search direction is given in term of the Hessian matrix of the Lagrangian function, which is updated by a quasi-Newton method in the optimization process. The searching for the optimization solution is repeated a number of times until the minimum volume of the convex polyhedral design envelope is attained while satisfying all the specified design constraints.

5. ILLUSTRATIVE EXAMPLE

5.1 The 30-storey building and the wind tunnel test

A study of a 30-story commercial building in Hong Kong was carried out to illustrate the effectiveness of this proposed optimization-based framework for obtaining wind load combinations. A 1:300 scale rigid model subjected to approaching wind profiles of the 50 year return period for 36 attack wind angles at 10° intervals for the 360° azimuth was examined in the HFBB test to obtain the aerodynamic base moments of the structure shown in Fig. 1. Once the finite element model was set up, an eigenvalue analysis was carried out to obtain dynamic properties of the building. The first natural frequencies for three fundamental modes are 0.306Hz, 0.368Hz and 0.737Hz respectively. Upon dynamic characteristics being determined, dynamic analysis of the structure in time domain was subsequently conducted to obtain base moment responses in two translational and one torsional directions corresponding to the time duration of 3600s.

The ellipsoidal threshold for each incident wind direction based on multivariate normal distribution can be derived from time history samples. The value of the peak factor is taken as 3.5, the confidence level is thus to be 99.35% according to Eq. (10). The proposed optimization-based framework was carried out in search for the minimum volume polyhedral design envelope with a given number of load cases that encloses all ellipsoidal wind load thresholds corresponding to all wind load directions.

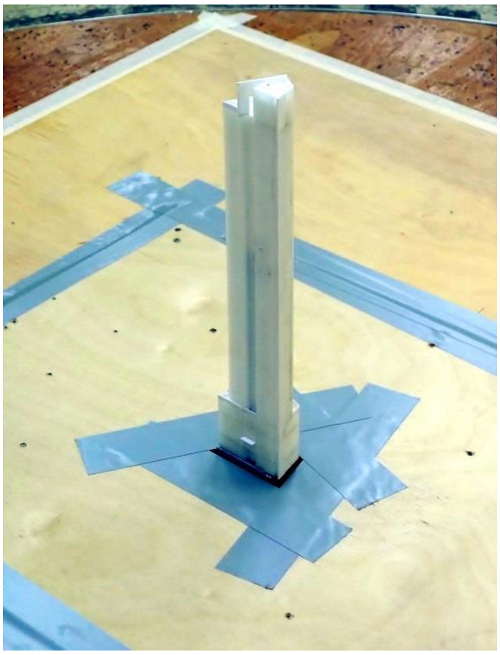

Fig. 1 HFBB tests for a 30-story building

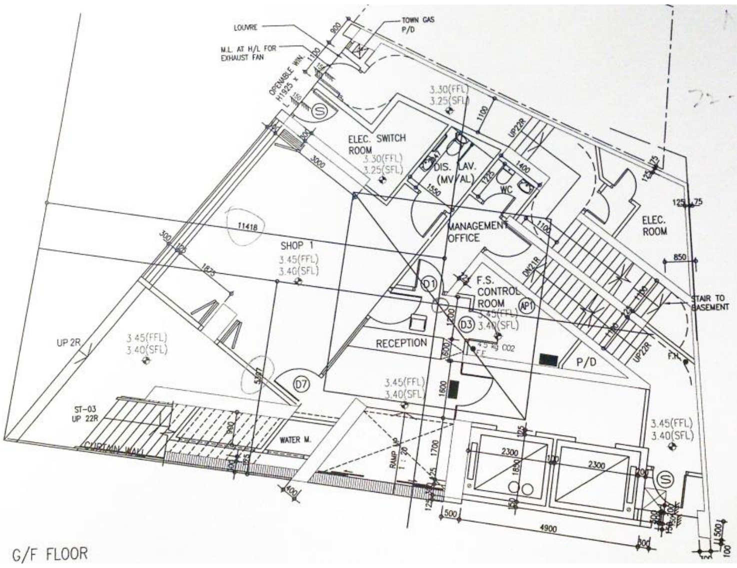

Fig. 2 Floor plan for a 30-story building

5.2 Results and discussion

Fig. 3 and Fig. 4 present the optimized polyhedral envelopes that enclose all ellipsoids with 16 and 24 critical wind load cases respectively for structural design. The volume of each polyhedron converges to the minimum value with the aid of the SQP optimization algorithm. The number of load cases is specified in a range from 8 to 28 and the corresponding volumes of the polyhedrons are computed. The result of the different trade-offs between the minimum volume polyhedrons and the number of load cases is given in the Pareto front as shown in Fig. 5. It is evident that a higher number of load cases is generally ended up with a smaller volume of the design envelope. It appears that the minimum volume of polyhedron reaches convergence when the number of load cases approach 24.

The comparison of 24 load cases derived from the proposed optimization methodology and the current wind tunnel practice is given in Table. 1. These two approaches result in the volumes of design envelopes at $1.16\times10^{7}(\mathrm{MN}\cdot\mathrm{m})^{3}$ and $1.40\times10^{7}(\mathrm{MN}\cdot\mathrm{m})^{3}$ respectively, indicating a 17% reduction has been achieved by the proposed optimization method. Since the volume is considered as an index to examine the departure of the approximated polyhedron from ellipsoidal thresholds, the smaller volume implies that the optimized design envelope provides more accurate prediction of maximum combined wind load cases. The comparison of critical load cases of two sets also indicates significant improvement of minimizing the risk of overestimating the extreme wind load effects.

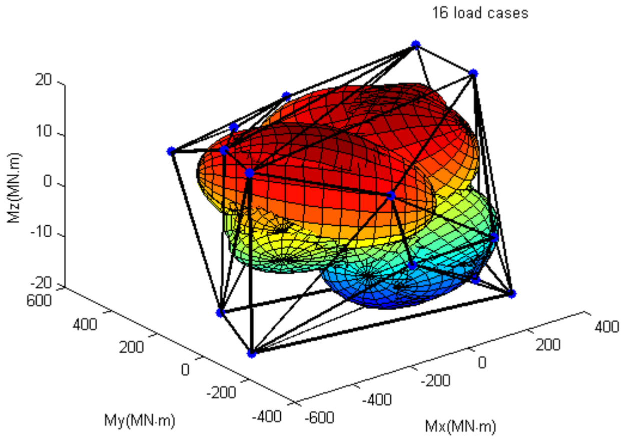

Fig. 3 Optimized design envelope with 16 load cases

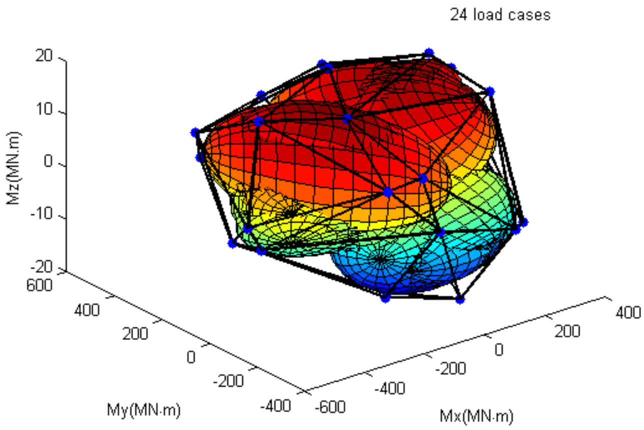

Fig. 4 Optimized design envelope with 24 load cases

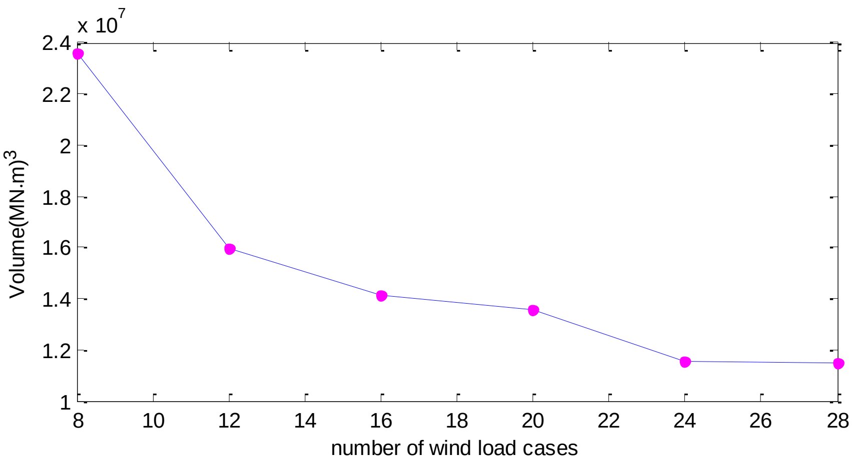

Fig. 5 The Pareto front for volumes of envelopes

Table. 1 Comparison of 24 load cases

Load Cases	Traditional methods			Optimization-based approach		
	M_x(MN•m)	M_y(MN•m)	M_z(MN•m)	M_x(MN•m)	M_y(MN•m)	M_z(MN•m)
1	200.0	-90.0	17.1	-195.1	-68.5	17.1
2	200.0	250.0	17.1	260.7	162.1	16.0
3	-330.0	250.0	17.1	-25.4	221.8	17.1
4	-330.0	-90.0	17.1	-387.1	59.0	17.1
5	376.6	-120.0	10.0	288.1	-59.1	13.3
6	376.6	180.0	10.0	376.6	213.4	10.0
7	220.0	450.0	10.0	136.2	450.0	9.7
8	-180.0	450.0	10.0	-69.3	450.0	7.4
9	-515.2	180.0	3.0	-402.1	302.7	9.6
10	-515.2	-80.0	3.0	-515.2	-53.8	1.8
11	-300.0	-330.6	10.0	-265.1	-330.5	10.4
12	120.0	-330.6	10.0	-151.6	-330.6	10.9
13	376.6	-120.0	-8.0	376.5	-88.6	-12.5
14	376.6	180.0	-8.0	376.6	275.3	7.7
15	220.0	450.0	0.0	78.3	450.0	-1.8
16	-180.0	450.0	0.0	-352.1	349.4	3.1
17	-515.2	180.0	-3.0	-515.2	8.5	-2.6
18	-515.2	-80.0	-3.0	-515.2	-113.9	-1.2
19	-300.0	-330.6	-3.0	-92.1	-330.6	-0.4
20	120.0	-330.6	-3.0	185.5	-298.6	-5.7
21	200.0	-200.0	-18.5	63.5	-219.7	-18.5
22	200.0	0.0	-18.5	185.6	26.9	-18.4
23	-90.0	0.0	-18.5	-62.6	-69.1	-18.5
24	-90.0	-200.0	-18.5	-157.2	-181.5	-15.1

6. CONCLUSIONS

This paper presents a systematic combination scheme for obtaining critical wind load cases for tall building design with the less overestimation of peak resultant loads from time-variant aerodynamic wind loads measured in HFBB tests. The multivariate normal distribution was applied in determining extreme wind load combinations in each incident wind direction. Subsequently, the optimization-based framework was developed to seek the minimum volume of the polyhedral design envelope subject to the satisfaction of design constraints for ensuring the outer approximation and convexity of the optimal design envelope. Design variables were the coordinates of each vertex on the polyhedron. The Pareto front is used to determine the best trade-off solution between the minimum volume of polyhedral design envelope and the minimum number of wind load cases. The new optimization method provides a powerful tool for accurately predicting extreme wind load combinations for structural design of tall buildings.

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