









where  $\rho$ ,  $V$  and  $A$  are the mass density, volume and surface area of the panel, respectively; and  $\Delta p$  is the aerodynamic pressure which is usually calculated by the linear potential theory in the engineering analysis. For low Mach number subsonic flow, the self-induced aerodynamic pressure is approximately given by (Yao 2014)

$$\Delta p = \rho_{\infty} A_0 \left[ \frac{\partial^2 w}{\partial t^2} + 2U_{\infty} \frac{\partial^2 w}{\partial x \partial t} + U_{\infty}^2 \frac{\partial^2 w}{\partial x^2} \right] \quad (12)$$

where  $A_0 = \frac{-ab}{\pi \sqrt{i^2 b^2 + j^2 a^2}}$ ,  $\rho_{\infty}$  is the air density and  $U_{\infty}$  is the main flow velocity.

Substituting Eqs.(1), (2) and (3) into the governing equations of motion, the Galerkin method is used to obtain a coupled set of nonlinear ordinary differential equations

$$(M + M_{\Delta p}) \ddot{X} + C_{\Delta p} \dot{X} + (K_L + K_N + K_{\Delta p}) X = 0 \quad (13)$$

where  $X$  is the generalized coordinate vector,  $M_{\Delta p}$ ,  $C_{\Delta p}$  and  $K_{\Delta p}$  are the aerodynamic mass, damping and stiffness matrices.  $K_L$  is the linear stiffness matrix, and  $K_N$  is the nonlinear stiffness matrix. They are listed in **APPENDIX A**.

### 3. NUMERICAL SIMULATIONS AND DISCUSSIONS

For the sake of simplicity, we take  $m = 3$ ,  $n = 1$ , in the numerical simulations. With the velocity increasing, the vibration frequency of the panel in the subsonic flow decreases down to zero, which corresponds to the critical flow velocities for divergence of the panel (Dowell 1963, Païdoussis 2003). The variations of vibration frequency  $f$  versus the airspeed for different artificial spring stiffness are shown in Fig. 5.

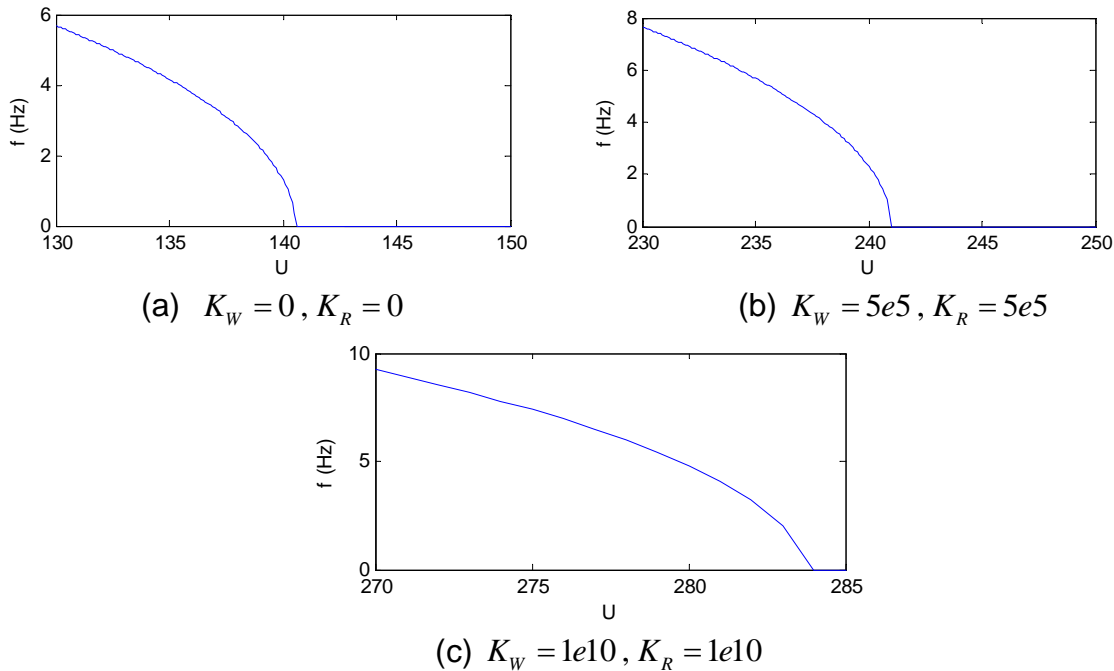


Fig.5 Variation of frequency  $f$  versus flow velocity

As is shown in Fig. 6, with the increase of the artificial spring stiffness, the critical flow velocity for divergence increases, which indicates that the aeroelastic stability is strengthened by the supporting stiffness at the boundary.

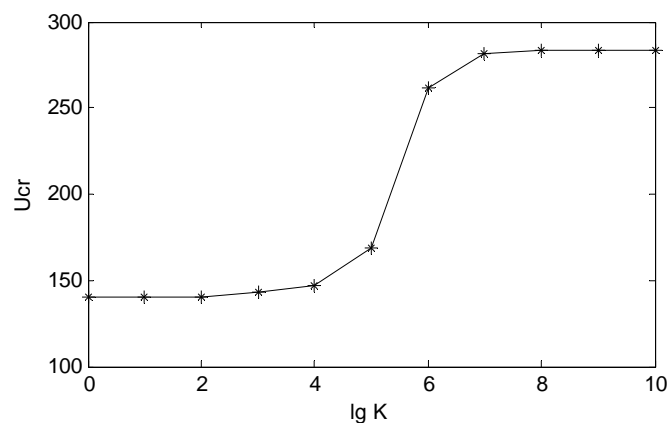


Fig.6 The critical flow velocities for divergence with respect to artificial spring stiffness

#### 4. CONCLUSIONS

The subsonic aero-elastic stability for a three-dimensional panel supported by an artificial spring boundary has been systematically investigated. The Rayleigh-Ritz method was employed to derive the frequency equations and modal functions of a rectangular panel with elastic constraints by taking the characteristic orthogonal polynomial series as the admissible functions and utilizing artificial springs to simulate the elastic constraints. A multi-degree-of-freedom nonlinear dynamical model has been established in terms of the incompressible aerodynamic model for the subsonic flow, the von Karman theory for the structure, and the Galerkin method for truncating the system into a series of ordinary differential equations. The critical flow velocity of the panel was calculated by analyzing the stability of the equilibrium point of the panel and the effects of artificial spring stiffness on the critical flow velocity are studied numerically.

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#### APPENDIX A

The matrices in Eq.(13) are written as

$$M = \begin{bmatrix} M_u & 0 & 0 \\ 0 & M_v & 0 \\ 0 & 0 & M_w \end{bmatrix} \quad (14)$$

where

$$M_u = \frac{1}{2} \rho h \int_0^b \int_0^a \phi \phi^T dx dy, \quad M_v = \frac{1}{2} \rho h \int_0^b \int_0^a \psi \psi^T dx dy \quad (15)$$

$$M_w = \frac{1}{2} \rho h \int_0^b \int_0^a \zeta \zeta^T dx dy + \frac{1}{8} \rho h^3 \int_0^b \int_0^a (\zeta_x \zeta_x^T + \zeta_y \zeta_y^T) dx dy \quad (16)$$

and

$$K_L = \begin{bmatrix} K_{uu} & K_{uv} & K_{uw} \\ K_{vu} & K_{vv} & K_{vw} \\ K_{wu} & K_{wv} & K_{ww} \end{bmatrix} \quad (17)$$

where

$$K_{uu} = \frac{1}{2} \frac{Eh}{1-\mu^2} \int_0^b \int_0^a \phi_x \phi_x^T dx dy + \frac{1}{2} Gh \int_0^b \int_0^a \phi_y \phi_y^T dx dy \quad (18)$$

$$K_{vv} = \frac{1}{2} \frac{Eh}{1-\mu^2} \int_0^b \int_0^a \psi_y \psi_y^T dx dy + \frac{1}{2} Gh \int_0^b \int_0^a \psi_x \psi_x^T dx dy \quad (19)$$

$$K_{ww} = \frac{1}{8} \frac{Eh^3}{1-\mu^2} \int_0^b \int_0^a (\zeta_{xx} \zeta_{xx}^T + \zeta_{yy} \zeta_{yy}^T) dx dy \\ + \frac{1}{4} \frac{E\mu h^3}{1-\mu^2} \int_0^b \int_0^a \zeta_{xx} \zeta_{yy}^T dx dy + \frac{1}{2} Gh^3 \int_0^b \int_0^a \zeta_{xy} \zeta_{xy}^T dx dy \quad (20)$$

$$K_{uw} = \frac{E\mu h}{1-\mu^2} \int_0^b \int_0^a \phi_x \psi_y^T dx dy + \frac{1}{2} Gh \int_0^b \int_0^a \phi_y \psi_x^T dx dy \quad (21)$$

$$K_{vu} = K_{uv}^T, K_{uw} = K_{wu}^T = 0, K_{vw} = K_{wv}^T = 0 \quad (22)$$

$$K_N = \begin{bmatrix} 0 & 0 & K'_{uw} \\ 0 & 0 & K'_{vw} \\ K'_{wu} & K'_{wv} & K'_{ww} \end{bmatrix} \quad (23)$$

in which

$$K'_{uw} = \frac{1}{2} \frac{Eh}{1-\mu^2} \int_0^b \int_0^a \phi_x \zeta_x^T g \zeta_x^T dx dy \\ + \frac{1}{2} \frac{E\mu h}{1-\mu^2} \int_0^b \int_0^a \phi_x \zeta_y g \zeta_y^T dx dy + \frac{1}{2} Gh \int_0^b \int_0^a \phi_y \zeta_x^T g \zeta_y^T dx dy \quad (24)$$

$$K'_{vw} = \frac{1}{2} \frac{Eh}{1-\mu^2} \int_0^b \int_0^a \psi_y \zeta_y^T g \zeta_y^T dx dy \\ + \frac{1}{2} \frac{E\mu h}{1-\mu^2} \int_0^b \int_0^a \psi_y \zeta_x^T g \zeta_x^T dx dy + \frac{1}{2} Gh \int_0^b \int_0^a \psi_x \zeta_x^T g \zeta_y^T dx dy \quad (25)$$

$$K'_{ww} = \frac{1}{8} \frac{Eh}{1-\mu^2} \int_0^b \int_0^a (\zeta_x \zeta_x^T g g^T \zeta_x \zeta_x^T + \zeta_y \zeta_y^T g g^T \zeta_y \zeta_y^T) dx dy \\ + \frac{1}{4} \frac{E\mu h}{1-\mu^2} \int_0^b \int_0^a (\zeta_x \zeta_x^T g g^T \zeta_y \zeta_y^T) dx dy + \frac{1}{2} Gh \int_0^b \int_0^a (\zeta_x \zeta_y^T g g^T \zeta_x \zeta_y^T) dx dy \quad (26)$$

$$K'_{wu} = K'^T_{uw}, K'_{vw} = K'^T_{vw} \quad (27)$$

$$M_{\Delta p} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \bar{M}_{\Delta p} \end{bmatrix}, C_{\Delta p} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \bar{C}_{\Delta p} \end{bmatrix}, K_{\Delta p} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \bar{K}_{\Delta p} \end{bmatrix} \quad (28)$$

where

$$\bar{M}_{\Delta p} = \rho_{\infty} A_0 \int_0^b \int_0^a \zeta \zeta^T dx dy, \bar{C}_{\Delta p} = 2U_{\infty} \rho_{\infty} A_0 \int_0^b \int_0^a \zeta \zeta_x^T dx dy, \bar{K}_{\Delta p} = \rho_{\infty} A_0 U_{\infty}^2 \int_0^b \int_0^a \zeta \zeta_{xx}^T dx dy \quad (29)$$

## REFERENCES

- Bhat, RB. (1985). Natural frequencies of rectangular plates using characteristic orthogonal polynomials in Rayleigh-Ritz method. *Journal of Sound and Vibration*, 102(4), 493-499.
- Dowell, E, Dugundji, J, & Perkin, Brian. (1963). Subsonic flutter of panels on continuous elastic foundations. *AIAA Journal*, 1(5), 1146-1154.
- Dowell, Earl H. (1974). *Aeroelasticity of plates and shells*: Springer.
- Ellen, CH. (1973). The stability of simply supported rectangular surfaces in uniform subsonic flow. *Journal of Applied Mechanics*, 40(1), 68-72.
- Yao, G. (2014). *Nonlinear vibration and control of composite laminated plates and shells in subsonic airflow*. (PHD), Harbin Institute of Technology.
- Gibbs IV, S Chad, Wang, Ivan, & Dowell, Earl H. (2014). Stability of Rectangular Plates in Subsonic Flow with Various Boundary Conditions. *Journal of Aircraft*, 1-13.
- Gislason, Thorsteinn. (1971). Experimental investigation of panel divergence at subsonic speeds. *AIAA journal*, 9(11), 2252-2258.
- Ishii, T. (1965). Aeroelastic instabilities of simply supported panels in subsonic flow. *AIAA Journal*, 65-752.
- Kornecki, A. (1974). Static and dynamic instability of panels and cylindrical shells in subsonic potential flow. *Journal of Sound and Vibration*, 32(2), 251-263.
- Kornecki, A, Dowell, EH, & O'Brien, J. (1976). On the aeroelastic instability of two-dimensional panels in uniform incompressible flow. *Journal of Sound and Vibration*, 47(2), 163-178.
- Li, Peng, Yang, Yiren, & Xu, Wei. (2012). Nonlinear dynamics analysis of a two-dimensional thin panel with an external forcing in incompressible subsonic flow. *Nonlinear Dynamics*, 67(4), 2483-2503.
- Païdoussis, MP. (2003). *Fluid-Structure Interactions: Slender Structures and Axial Flow*, Volume 2 (Vol. 2): ELSEVIER ACADEMIC PRESS, LONDON.
- Song, Zhi-Guang, & Li, Feng-Ming. (2014). Aerothermoelastic analysis of nonlinear composite laminated panel with aerodynamic heating in hypersonic flow. *Composites Part B: Engineering*, 56, 830-839.
- Weaver, DS, & Unny, TE. (1970). The hydroelastic stability of a flat plate. *Journal of Applied Mechanics*, 37, 823.