The 2014 World Congress on
Advances in Civil, Environmental, and Materials Research (ACEM14)
Busan, Korea, August 24-28, 2014

Optimization of Composite Laminates Stacking Sequence for Buckling using Adaptive Genetic Algorithm

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ABSTRACT

Genetic Algorithms (GA) are commonly used for optimization of composite laminates. However, one of its main pitfalls is its failure to consistently converge to a global optimum solution. The purpose of this work is to investigate different adaptations to the GA meta-heuristic search technique for use in optimization of fiber reinforced composite laminate stacking sequence. An adaptive selection technique is studied which incorporates the Laplace crossover operator, and uniform and power mutation operators. The optimization example focusses on buckling of a laminate plate.

1. INTRODUCTION

Composite materials due to their high specific stiffness are widely used in many applications in a variety of industries, and as a consequence it's growth rate in the past 50 years has greatly outpaced other structural materials such as steel and aluminum (Agarwal et al., 2006). The mechanical properties of fiber reinforced composite (FRC) laminates are affected by the orientation and relative position of its constituent laminae, also known as the stacking sequence. This stacking sequence has a strong effect on the bending stiffness of the laminate (Gürdal et al., 1999).

The problem of composite laminate stacking sequence optimization has been studied by a number of investigators. This optimization problem is particularly complex, as it consists of a very vast search space of discrete variables with many possible constraints. The characteristics of this optimization problem make it highly suitable for using genetic algorithms (GA), and many efforts have been made in using this technique for the optimization of composite laminates (Xiao et al., 2013). One of the main drawbacks of the GA is its failure to consistently converge to a global optimum solution (Gürdal et al., 1999).

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2. BACKGROUND

The genetic algorithm as an optimization method is inspired by the principles of the biological evolutionary process (Rao, 2009). This optimization method involves evaluating a group of possible solutions to a given problem referred to as “individuals”. Each of these sets of possible solutions is called a population. The GA consists of evaluating these possible solutions with an “objective function”, and passing on the information of the best individuals through so called inheritance operators. The concept behind this algorithm is similar to that of evolution, where individuals compete to achieve a defined goal. In it, new “generations” based on the best individuals will adapt until they reach an optimal design for a particular problem. In the case of stacking sequence optimization, these individuals each represent a different stacking sequence where its mechanical properties are of interest. The general GA process is depicted in Figure 1.

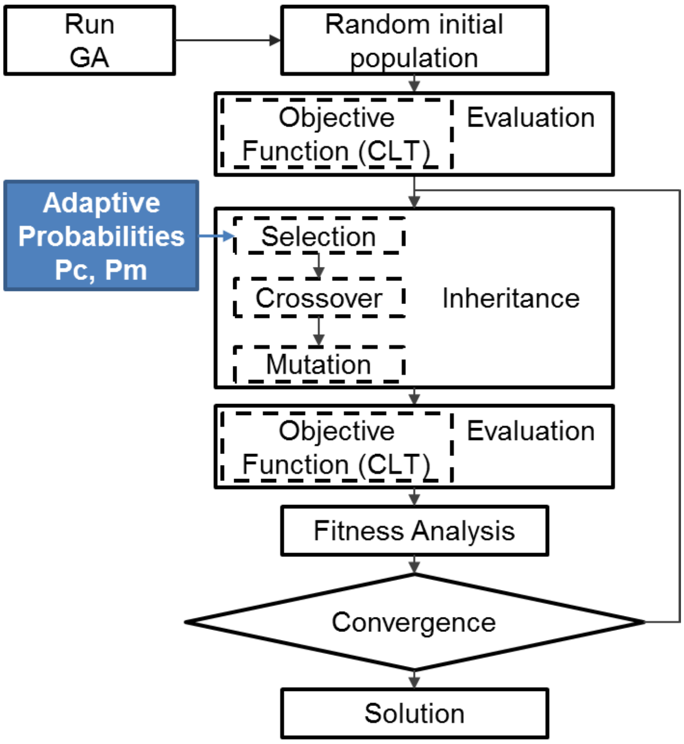

Figure 1 - Genetic Algorithm diagram.

The aforementioned inheritance operators are used to generate new “individuals” until an optimal solution is found according to the given convergence criteria. These operators consist in basic terms of:

Selection- from the current population and with information from the results of the objective function evaluation (fitness evaluation), individuals are chosen to be crossed-over and mutated.

Crossover- two individuals are combined to form a new individual with characteristics from its parent individuals. In this case, two different stacking sequences are mixed to create a new one.

Mutation- a change of random nature is induced in one of the information bits of the individual. In this specific application, that would be the orientation of a lamina is randomly changed when an individual is set for mutation.

3. ADAPTIVE GENETIC ALGORITHM (AGA)

In an attempt to improve the performance and results of genetic algorithms used in composite laminate stacking sequence optimization, the probabilities of both crossover (Pc) and mutation (Pm) should self-adapt to the current state of the algorithm solution (Xiao et al., 2013). The adaptation affects the GA inheritance as shown in Figure 1. The self-adaptation should maintain a healthy level of diversity and keep the genetic information of higher performing individuals. Such an algorithm will be referred to as an Adaptive Genetic Algorithm (AGA), which can be achieved using the following expressions (Srinivas and Patnaik, 1994):

$$P_{ci} = \begin{cases} \dfrac{F_{max} - F_i}{F_{max} - F} P_{c0}, F_i > \bar{F} \\ P_{c0}, F_i \leq \bar{F} \end{cases} \quad (1)$$

$$P_{mi} = \begin{cases} \dfrac{F_{max} - F_i}{F_{max} - F} P_{m0}, F_i > \bar{F} \\ P_{m0}, F_i \leq \bar{F} \end{cases} \quad (2)$$

These calculated probabilities, depend on the current state of the simulations, and they determine the probability of each individual being used for crossover or mutation.

In a preliminary study it was observed that using a Laplace crossover operator (Deep and Thakur, 2007a) resulted in more consistent global optimum solutions from the AGA. This particular method will be used for studying the composite laminate buckling problem, and it will be referred to as AGA-LX. The AGA-LX method was implemented using MATLAB (The MathWorks, Inc., 2014).

4. BUCKLING

As originally studied, this problem consists in maximizing the buckling load factor of a laminated composite (Haftka and Walsh, 1992). This problem has been the subject of a number of studies (Karakaya and Soykasap, 2009; Soremekun et al., 2001). For a simply supported laminate plate, the buckling load factor is given by (Soremekun et al., 2001):

$$\lambda_b(m,n) = \pi^2 \left[\frac{m^4 D_{11} + 2(D_{12} + D_{66})(rmn)^2 + (rn)^4 D_{22}}{(am)^2 N_x + (ran)^2 N_y} \right] \quad \textbf{(3)}$$

where, the laminate has m and n half waves, D_{ij} are the elements of the bending stiffness matrix, *a* is the dimension of the plate along the x direction, r is the plate aspect ratio, and N_x and N_y are the loads applied to the plate. Both M and N are assumed to have values of 1 or 2 (Erdal and Sonmez, 2005).

4.1. AGA with Laplace crossover operator

An array of different AGA-LX configurations was tested with the defined buckling factor optimization problem. This array consists of 12 different configurations of the AGA. Ten iterations of each of these configurations are executed to gather a set of data. Again, ten such sets of data are gathered and summarized in 2.

The AGA is tested using the Laplace crossover operator with a simple mutation operator, and an alternative configuration uses said Laplace crossover with a power mutation operator (Deep and Thakur, 2007b). For the crossover operator, values of: -1, 0, +1 are used for the location parameter a. For the power mutation operator, values of 0.25, 0.50 and 1.00 are tested for the p index of the distribution. The various parameters for AGA configurations are shown in Table 1.

Table 1 – Parameters for AGA configurations.

Parameter	**Configuration #**											
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Mutation	U	U	U	P	P	P	P	P	P	P	P	P
p	-	-	-	0.25	0.25	0.25	0.50	0.50	0.50	1.00	1.00	1.00
a	-1	0	1	-1	0	1	-1	0	1	-1	0	1

Tests where run for each of the 12 configurations to identify the parameters that show best results with the proposed method, 10 iterations were ran per each configuration. Figure 22 shows the results of the tests where the average number of misses and average number of optimum solutions found per 10 iterations of each configuration are plotted.

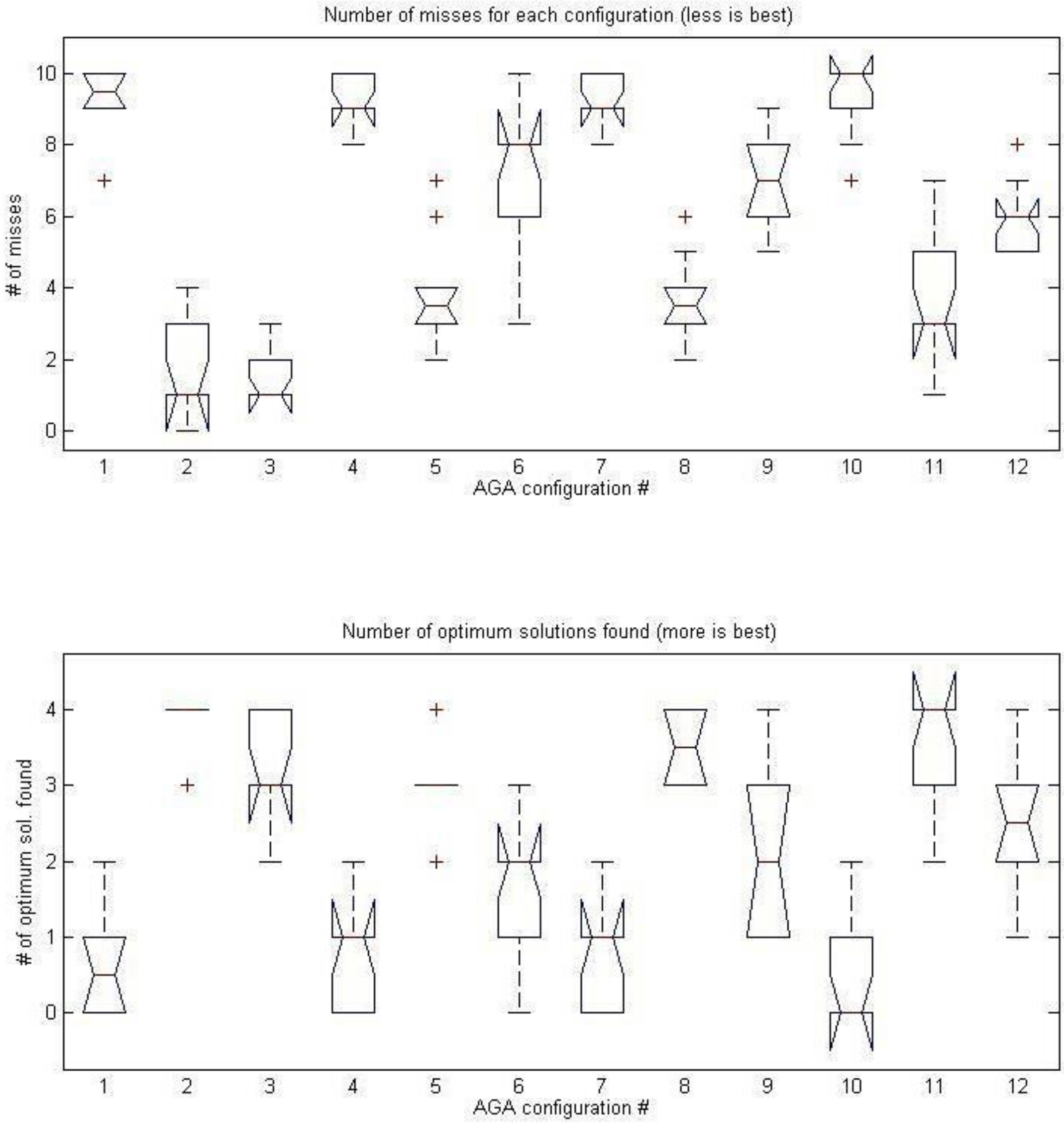

Figure 2 - Summary of buckling problem data.

We can see that configurations 2 and 3 stand out in both of the metrics of interest. Subsequently, AGA-LX configurations 2 and 3 (Laplace crossover operator with uniform mutation and a = 0 and 1, respectively) are selected to perform further comparison of these methods against those found in literature.

4.2. Method

The buckling problem was solved using the AGA-LX method for two different governing stacking sequence criteria. In the first criterion, the parameters used for the AGA matched those used by (Soremekun et al., 2001) and (Karakaya and Soykasap, 2009), for ease of comparison. Lamina orientations are based on pairs of /0°$_2$/, /±45°/ and /90°$_2$/ plies. In the second criterion, the AGA was run with additional orientations in an attempt to find higher values for the discussed buckling factor optimization. The proposed method was employed without using lamina pairs to impose the constraint of specially orthotropic laminates. In addition, orientations of ±30° and ±60° were included in the possible orientations.

4.3. Results

The two selected configurations (2 and 3) of the AGA are compared against those obtained by (Erdal and Sonmez, 2005; Karakaya and Soykasap, 2009; Soremekun et al., 2001). In accordance with the referenced literature, the value of the global optimum buckling load factor is found to be λ_{cb} = 3.973013726664593e+03.

Using criterion 1, the proposed AGA results are less efficient than those in the literature (Soremekun et al., 2001) under the same criteria. However if we take into account the number of quasi-optimal solutions being found by the AGA, certain configurations (combination of parameters) become more efficient. These results are shown in Table 2, where P is the size of the population, R is number of runs with at least one global optimal divided by the total number of runs ("apparent reliability") (Soremekun et al., 2001), A_{No} is the average number of optimal solutions found per iteration of the GA, and C_o is the cost per optimum found. Columns marked with * take into account the quasi-optimal solutions.

One the most significant observation is that two quasi-optimal solutions are reported as optimal solutions in (Karakaya and Soykasap, 2009). While there may be practical applications for these solutions, it is important to distinguish between the quasi-optimal and optimal solutions. In addition, the proposed AGA finds 10 more such quasi-optimal solutions.

Using criterion 2, after 40 iterations of the algorithm, two stacking sequences were found with higher buckling factors: $[90/60/90_2/\text{-}45/90_3/\text{-}60/90_2/60/45/90/60/90/\text{-}60/90_4/60/\text{-}60/45/\text{-}60/90_3/60/90/\text{-}45/\text{-}60]_s$ and $[90_2/60/90/\text{-}60/90_6/\text{-}45/60_2/90/\text{-}60/30/90/60/\text{-}30/\text{-}60/60/\text{-}60/90_2/\text{-}60_2/60/90/45/\text{-}60/60]_s$, with corresponding λ_{cb} values of 4.02511e+03 and 4.01125e+03, respectively. Both of these reported stacking sequences are balanced, so that $A_{16} = A_{26} = 0$. However, their values for bending-twisting coupling D_{16} are 84.34502812213 and -42.21255385328, respectively.

5. DISCUSSION

Two additional global optimum solutions ($[90_{10}/\pm45/90_2/\pm45_7/90_2/\pm45]_s$ and $[90_4/\pm45/90_2/\pm45/90_{10}/\pm45/90_4/\pm45/90_4]$s) were reported in (Karakaya and Soykasap, 2009). However, upon further inspection we observe that said stacking sequences are not precisely optimal stacking sequences. For these stacking sequences the buckling load factor is: λ_{cb} = 3.972996045360481e+03. The significance of quasi-optimal solutions in real life applications using evolutionary algorithms has been discussed (Ono et al., 2007), however its implications have not been fully discussed for the purposes of composite laminate stacking sequence optimization.

Table 2 - Results of (Soremekun 2001) compared to results of proposed AGA.

				# of optima found over 50 runs										
	P	R	R*	1	2	3	4	5	6	q-opt.	A_{No}	A_{No} *	C_o	Co*
EL	15	0.78		38	1	0	0	0	0		0.8		2,812.50	
EL	45	1		46	4	0	0	0	0		1.08		6,250.00	
EL	75	1		43	6	1	0	0	0		1.16		9,698.28	
ME1	15	0.8		16	24	0	0	0	0		1.28		1,757.81	
ME1	45	1		0	6	15	29	0	0		3.46		1,950.87	
ME1	45	1		0	11	11	27	1	0		3.36		2,008.93	
ME1	75	1		0	2	4	40	4	0		3.92		2,869.90	
ME2	15	0.84		14	16	12	0	0	0		1.64		1,371.95	
ME2	45	1		2	6	14	28	0	0		3.36		2,008.93	
ME2	75	1		0	0	2	46	2	0		4		2,812.50	
VE	15	0.36		13	5	0	0	0	0		0.46		4,891.30	
VE	45	0.88		11	13	16	4	0	0		2.02		3,341.58	
VE	75	1		7	15	13	13	1	1		2.78		4,046.76	
LX2	15	0.20	0.38	4	2	2	0	2	0	9	0.2	7.44	11,250.00	302.42
LX2	45	0.42	0.68	11	6	2	0	2	0	13	0.5	6.3	13,500.00	1,071.43
LX2	75	0.80	0.96	12	10	9	0	9	0	8	0.98	6.16	11,479.59	1,826.30
LX3	15	0.30	0.54	8		2	0	5	0	12	0.28	5.66	8,035.71	397.53
LX3	45	0.68	0.78	23	2	2	0	6	0	6	0.58	5	11,637.93	1,350.00
LX3	75	0.80	0.90	24	3	5	0	8	0	5	0.68	4.72	16,544.12	2,383.47

It should be remarked that these quasi-optimal solutions come within 99.99955% of the fitness of the optimum solution, therefore becoming of interest for practical applications. Nonetheless, the nomenclature difference is important, as it is possible that comparison between methods be affected in performance indicators such as number of optimal solutions found per iteration.

When the code was run without imposing laminate pairs, and with the addition of ±30° and ±60° laminate orientations, the AGA found higher values for λ_{cb}. However it is very important to note that while there is no shear-extension coupling (A_{16}, A_{26} = 0) in the found stacking sequences, as they're balanced, there is some degree of bending-twisting coupling. It is observed that the magnitude of the ratio of shear-extension D_{16}/D_{11} is 0.01, therefore making this solution worth mentioning even though the assumptions that lead to Eq. (3) are no longer truly valid. Since no closed form solution is available for non-orthotropic laminates (Bettebghor and Bartoli, 2012), the contributions of the small D_{16} and D_{26} elements need to be further studied.

6. CONCLUSIONS

The proposed Adaptive Genetic Algorithm with Laplace crossover operator (AGA-LX) search algorithm is capable of finding optimal and quasi-optimal solutions with a computational expense comparable to that of previously proposed algorithms. The result given by the AGA-LX algorithm could be a quasi-optimal solution, which in case of application problems may very well be a valid solution.

Two quasi-optimal solutions are reported as optimal solutions in (Karakaya and Soykasap, 2009). Despite the usability of these solutions, it is important to distinguish between the quasi-optimal and optimal solutions. In addition, the proposed AGA-LX finds 10 more such quasi-optimal solutions. Further discussion is necessary in the desirability of such quasi-optimal solutions.

Higher optimal values could be achieved with additional orientations and if laminates are not forced to be in pairs. However, the effects of small bending-twisting coupling terms in the bending stiffness matrix should be further studied in order to open the possibility of a much broader search space when optimizing this kind of stacking sequence problems.

ACKNOWLEDGEMENTS

D. Gutiérrez-Delgadillo gratefully acknowledges the financial support from the Mexican National Council of Science and Technology (CONACYT), scholarship No. 311386.

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