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Three-dimensional singularity analysis for a magneto-electro-elastic wedge

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ABSTRACT

Three-dimensional asymptotic solutions are established for magneto-electro-elastic (MEE) singularities in MEE wedges. The solutions are obtained by combining an eigenfunction expansion approach with the power series solution method to solve three-dimensional equilibrium equations and Maxwell's equations in terms of mechanical displacement components and electric and magnetic potentials. The MEE material is assumed to be transversely isotropic and its polarization direction is not necessarily parallel to the thickness direction of wedge. The correctness of the proposed solutions is confirmed by comparing with the published result foraspecial case. The developed solutions are further employed to examine the effects of the direction of polarization, the configuration of wedge and the material components on the orders of the singularities in wedges.

1. INTRODUCTION

Magneto-electro-elastic (MEE) materials canexchange mechanical, electric and magnetic forms of energy among each other and have been widely used in electronic devices, including acoustic actuators, magneto-electro-mechanical transducers, electric field tunable microwave resonators, highly sensitive magnetic or electric current sensors and other smart structures. It is interesting and important to investigate geometrically-induced magneto-electro-elastic singularities in wedges because such singularities normally initiate cracking and worsen the functioning of structures.

Lots of studies on geometrically-induced stress singularities in elastic wedges have been carried out, and most of them were collected in the excellent review paper by Sinclair (2004). Williams (1952a, b) was at the forefront of investigating stress singularities at the vertex of an elastic wedge under extension or bending. Following his work, numerous studieshave been performed on elastic wedges, which arebased on

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plane elasticity theory (Bogy and Wang 1971,Dempsey and Sinclair 1979,Ting and Chou1981,Lin and Sung 1998), classical plate theory (Rao 1971, Ojikutu*et al*. 1984, Huang and Chang 2007), first-order shear deformation plate theory (Burton and Sinclair 1986, Huang 2003, McGee and Kim 2005), third-order plate theory (Huang 2002), and higher-order plate theory (Huang 2004), and three-dimensional elasticity theory (Hartranftand Sih 1969,Xie and Chaudhuri 1997).

In the last two decades, the geometrically-induced singularities in piezoelectric wedgeshave caught the attention of researchers. Most of investigations make the plane strain assumption or the generalized plane deformation assumption. Using the plane strain assumption, Xu and Rajapakse(2000)extended Lekhnitskii's complex potential functions for in-plane stresses and electric displacement components to examine the singularities at the vertex ofa piezoelectric wedge, while Shang and Kitamura (2005) utilized a modified version of the general solution of Wang and Zheng (1995). Hwu and Ikeda (2008) made the generalized plane strain assumption and proposed an extended Stroh formulation to investigatethe in-plane and out-of-plane electroelastic singularities in wedges. Chue and his co-workers (2002, 2003 and 2004) adopted the generalized plane deformation assumption and conducted a series of analytical studies of geometrically-inducedelectroelasticsingularities by using the extended Lekhnitskii formulation or the Mellin transform.By using various finite element approaches,Scherzer and Kuna (2004), Chen *et al*. (2006) and Chen and Ping (2007)examined in-plane singular electroelastic states at the vertex of a wedge. Based on three-dimensional piezoelasticity theory, Huang and Hu (2013)developed asymptotic solutions to comprehensively investigate the geometrically-inducedelectroelasticsingularities at the vertex of a wedge whose direction of polarization is arbitrary.

There are only few studies conducted for geometrically-induced singularities in MEE wedges. These published works considered wedges subjected to anti-plane deformation and in-plane electric and magnetic fieldswith assuming that all of the physical quantities were independent of the coordinates along the thickness. To determine the singularities at the vertex of a bi-material MEE wedge,Liu and Chue(2006)employed the Mellin transform,andSue *et al*. (2007)used the complex potential function witheigenfunction expansion method. Liu (2009)further extended the solution of Liu and Chue(2006) to examine the singularities at the apex of an MEE wedge-junction structure, with considering the air effect.

The main purpose of the work is based on three-dimensional magneto-electro-elasticity theory to develop an asymptotic solution to investigate geometrically-induced singularities in MEE wedges.Since the direction of polarization of an MEE wedge can be arbitrary in space, the in-plane components of displacement, electric and magnetic fields are generally coupled with the out-of-plane components, which much complicates the solution. An eigenfunction expansion approach with the power series solution technique and domain decomposition is adopted to establish the asymptotic solutionsby directly solvingthe three-dimensional equations of motion and Maxwell's equations in terms of mechanical displacement components and electric and magnetic potentials.The correctness of the proposed solutions is confirmed by comparing the order of MEE singularities with the published result for a wedge under anti-plane deformation and in-plane electric and magnetic fields. The proposedsolutions

are furtherapplied to study the effects of the direction of polarization, vertex angle, and boundary conditionson the singularities in MEEwedges.

2. CONSTRUCTION OF ASYMPTOTIC SOLUTION

Consider a rectilinearly anisotropicMEE wedge with vertex angle β,shown in Fig. 1, where the $\bar{x}$-$\bar{y}$-$\bar{z}$ coordinate system is used to describe the material anisotropy, and the x-y-z coordinate system is used to describe the geometry of the wedge.It is easy to solve for the MEE singularities at the vertex of the wedge in the cylindrical coordinate system (r,θ,z) given in Fig. 1. The material properties defined in the $\bar{x}$-$\bar{y}$-$\bar{z}$ coordinate system have to transform to the cylindrical coordinate system, and linear constitutive equations in the cylindrical coordinate systemcan be expressed as

$$\{\sigma\} = [c]\{\varepsilon\} - [e]^T \{E\} - [d]^T \{H\} \tag{1a}$$

$$\{D\} = [e]\{\varepsilon\} + [\eta]\{E\} + [g]\{H\} \tag{1b}$$

$$\{B\} = [d]\{\varepsilon\} + [g]\{E\} + [\mu]\{H\} \tag{1c}$$

where $\{\sigma\}$, $\{\varepsilon\}$, $\{D\}$, $\{B\}$, $\{E\}$ and $\{H\}$ are the stress, strain,electric displacement,magnetic flux, electric field and magnetic field vectors, respectively; $[c]$, $[e]$, $[d]$, $[\eta]$, $[\mu]$ and $[g]$ are the elastic stiffness constant, piezoelectric coefficient,piezomagnetic coefficient, dielectric constant, magnetic permeability and magnetoelectric coefficient matrices, respectively.

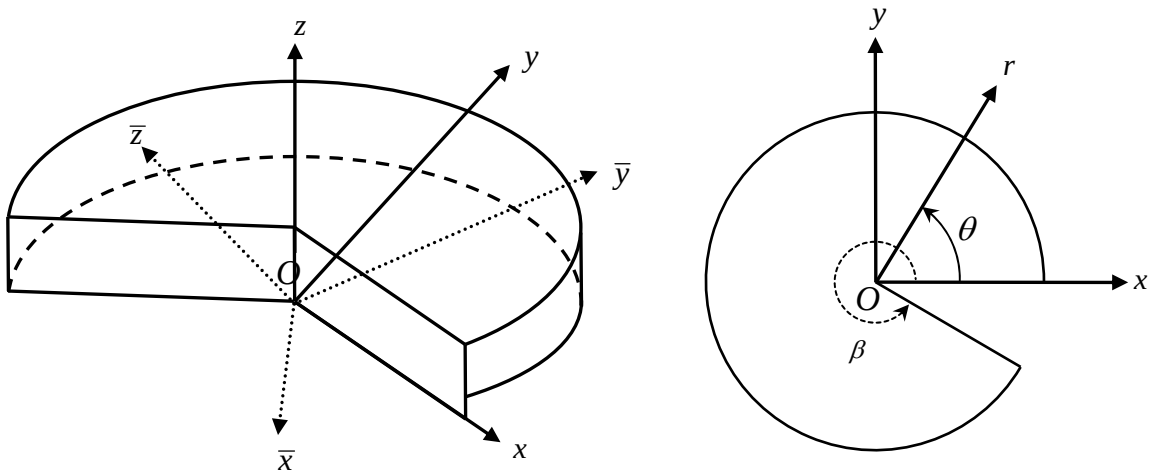

Fig. 1 Coordinate systems for a wedge

Following the eigenfunction expansion approach, which Hartranft and Sih(1969) used for 3D elastic wedges, the mechanical displacements, electric potential and magnetic potential are expressed as

$$u_r(r,\theta,z,t)=\left[\sum_{m=1}^{\infty}\sum_{n=0}^{\infty} r^{\lambda_m+n}\hat{U}_n^{(m)}(\theta,z)\right]e^{i\omega t},\ u_\theta(r,\theta,z,t)=\left[\sum_{m=1}^{\infty}\sum_{n=0}^{\infty} r^{\lambda_m+n}\hat{V}_n^{(m)}(\theta,z)\right]e^{i\omega t},$$

$$u_z(r,\theta,z,t)=\left[\sum_{m=1}^{\infty}\sum_{n=0}^{\infty} r^{\lambda_m+n}\hat{W}_n^{(m)}(\theta,z)\right]e^{i\omega t},\ \phi\ (r,\theta,z,t)=\left[\sum_{m=1}^{\infty}\sum_{n=0}^{\infty} r^{\lambda_m+n}\hat{\Phi}_n^{(m)}(\theta,z)\right]e^{i\omega t},$$

$$\psi\ (r,\theta,z,t)=\left[\sum_{m=1}^{\infty}\sum_{n=0}^{\infty} r^{\lambda_m+n}\hat{\Psi}_n^{(m)}(\theta,z)\right]e^{i\omega t}, \tag{2}$$

where $u_i\ (i=r,\theta,z)$ is the displacement component in the*i* direction, and electric potential and magnetic potential are related to electric and magnetic fields, respectively, by

$$E_r=-\frac{\partial\phi}{\partial r},\ E_\theta=-\frac{1}{r}\frac{\partial\phi}{\partial\theta},\ E_z=-\frac{\partial\phi}{\partial z},\ H_r=-\frac{\partial\psi}{\partial r},\ H_\theta=-\frac{1}{r}\frac{\partial\psi}{\partial\theta},\ \text{and}\ H_z=-\frac{\partial\psi}{\partial z}. \tag{3}$$

The characteristic values λ_m may be real or complex for MEE wedges. To ensure the finite displacements, electric field and magnetic field at $r=0$, the real part of $\lambda_m$is required to be positive.Notably, when the real part of λ_m (Re[λ_m]) is less than unity, the order of the singularities of stress, electric displacement and magnetic flux is Re[λ_m]-1.

Substituting Eqs. (2) into the equations of motion and Maxwell's equations withoutthe body forces, free electric charges and magnetic charges andin terms of mechanical displacement components, electric potential and magnetic potential, carefully rearranging and considering the terms with the lowest order of *r*yields,

$$\frac{\partial^2\hat{U}_0^{(m)}}{\partial\theta^2}+p_1\frac{\partial\hat{U}_0^{(m)}}{\partial\theta}+p_2\hat{U}_0^{(m)}+p_3\frac{\partial^2\hat{V}_0^{(m)}}{\partial\theta^2}+p_4\frac{\partial\hat{V}_0^{(m)}}{\partial\theta}+p_5\hat{V}_0^{(m)}+p_6\frac{\partial^2\hat{W}_0^{(m)}}{\partial\theta^2}+p_7\frac{\partial\hat{W}_0^{(m)}}{\partial\theta}+p_8\hat{W}_0^{(m)}$$
$$+p_9\frac{\partial^2\hat{\Phi}_0^{(m)}}{\partial\theta^2}+p_{10}\frac{\partial\hat{\Phi}_0^{(m)}}{\partial\theta}+p_{11}\hat{\Phi}_0^{(m)}+p_{12}\frac{\partial^2\hat{\Psi}_0^{(m)}}{\partial\theta^2}+p_{13}\frac{\partial\hat{\Psi}_0^{(m)}}{\partial\theta}+p_{14}\hat{\Psi}_0^{(m)}=0, \tag{4a}$$

$$\frac{\partial^2\hat{V}_0^{(m)}}{\partial\theta^2}+q_1\frac{\partial\hat{V}_0^{(m)}}{\partial\theta}+q_2\hat{V}_0^{(m)}+q_3\frac{\partial^2\hat{U}_0^{(m)}}{\partial\theta^2}+q_4\frac{\partial\hat{U}_0^{(m)}}{\partial\theta}+q_5\hat{U}_0^{(m)}+q_6\frac{\partial^2\hat{W}_0^{(m)}}{\partial\theta^2}+q_7\frac{\partial\hat{W}_0^{(m)}}{\partial\theta}+q_8\hat{W}_0^{(m)}$$
$$+q_9\frac{\partial^2\hat{\Phi}_0^{(m)}}{\partial\theta^2}+q_{10}\frac{\partial\hat{\Phi}_0^{(m)}}{\partial\theta}+q_{11}\hat{\Phi}_0^{(m)}+q_{12}\frac{\partial^2\hat{\Psi}_0^{(m)}}{\partial\theta^2}+q_{13}\frac{\partial\hat{\Psi}_0^{(m)}}{\partial\theta}+q_{14}\hat{\Psi}_0^{(m)}=0, \tag{4b}$$

$$\frac{\partial^2 \hat{W}_0^{(m)}}{\partial \theta^2} + r_1 \frac{\partial \hat{W}_0^{(m)}}{\partial \theta} + r_2 \hat{W}_0^{(m)} + r_3 \frac{\partial^2 \hat{U}_0^{(m)}}{\partial \theta^2} + r_4 \frac{\partial \hat{U}_0^{(m)}}{\partial \theta} + r_5 \hat{U}_0^{(m)} + r_6 \frac{\partial^2 \hat{V}_0^{(m)}}{\partial \theta^2} + r_7 \frac{\partial \hat{V}_0^{(m)}}{\partial \theta} + r_8 \hat{V}_0^{(m)}$$
$$+ r_9 \frac{\partial^2 \hat{\Phi}_0^{(m)}}{\partial \theta^2} + r_{10} \frac{\partial \hat{\Phi}_0^{(m)}}{\partial \theta} + r_{11} \hat{\Phi}_0^{(m)} + r_{12} \frac{\partial^2 \hat{\Psi}_0^{(m)}}{\partial \theta^2} + r_{13} \frac{\partial \hat{\Psi}_0^{(m)}}{\partial \theta} + r_{14} \hat{\Psi}_0^{(m)} = 0, \quad (4c)$$

$$\frac{\partial^2 \hat{\Phi}_0^{(m)}}{\partial \theta^2} + s_1 \frac{\partial \hat{\Phi}_0^{(m)}}{\partial \theta} + s_2 \hat{\Phi}_0^{(m)} + s_3 \frac{\partial^2 \hat{U}_0^{(m)}}{\partial \theta^2} + s_4 \frac{\partial \hat{U}_0^{(m)}}{\partial \theta} + s_5 \hat{U}_0^{(m)} + s_6 \frac{\partial^2 \hat{V}_0^{(m)}}{\partial \theta^2} + s_7 \frac{\partial \hat{V}_0^{(m)}}{\partial \theta} + s_8 \hat{V}_0^{(m)}$$
$$+ s_9 \frac{\partial^2 \hat{W}_0^{(m)}}{\partial \theta^2} + s_{10} \frac{\partial \hat{W}_0^{(m)}}{\partial \theta} + s_{11} \hat{W}_0^{(m)} + s_{12} \frac{\partial^2 \hat{\Psi}_0^{(m)}}{\partial \theta^2} + s_{13} \frac{\partial \hat{\Psi}_0^{(m)}}{\partial \theta} + s_{14} \hat{\Psi}_0^{(m)} = 0, \quad (4d)$$

$$\frac{\partial^2 \hat{\Psi}_0^{(m)}}{\partial \theta^2} + t_1 \frac{\partial \hat{\Psi}_0^{(m)}}{\partial \theta} + t_2 \hat{\Psi}_0^{(m)} + t_3 \frac{\partial^2 \hat{U}_0^{(m)}}{\partial \theta^2} + t_4 \frac{\partial \hat{U}_0^{(m)}}{\partial \theta} + t_5 \hat{U}_0^{(m)} + t_6 \frac{\partial^2 \hat{V}_0^{(m)}}{\partial \theta^2} + t_7 \frac{\partial \hat{V}_0^{(m)}}{\partial \theta} + t_8 \hat{V}_0^{(m)}$$
$$+ t_9 \frac{\partial^2 \hat{W}_0^{(m)}}{\partial \theta^2} + t_{10} \frac{\partial \hat{W}_0^{(m)}}{\partial \theta} + t_{11} \hat{W}_0^{(m)} + t_{12} \frac{\partial^2 \hat{\Phi}_0^{(m)}}{\partial \theta^2} + t_{13} \frac{\partial \hat{\Phi}_0^{(m)}}{\partial \theta} + t_{14} \hat{\Phi}_0^{(m)} = 0, \quad (4e)$$

where p_i, q_i, r_i, s_i and t_i (i= 1~14) are known functions of θ.

Equations (4) are a set of ordinary differential equations with variable coefficients that depend only on θ, and the three displacement components, electric potentialand magnetic potential are generally coupled.The exact closed-form solutions toEqs. (4) are intractable, if they exist. The power series method can be directly adopted to develop a general solution for ordinary differential equations with variable coefficients. Very high-order terms must be considered to obtain an accurate solution and this requirement can cause numerical difficulties. To overcome these difficulties, a domain decomposition technique is used in conjunction with the power series method to establish a general solution of Eqs. (4).

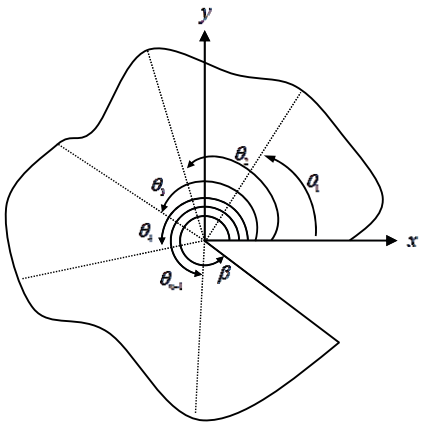

Fig. 2 Sub-domains for $\theta \in [0, \beta]$

The range of θ under consideration is first divided into a number of sub-domains (see Fig. 2). A series solution toEqs. (4) is established in each sub-domain. Consequently, a general solution cover the whole θ domain is constructed from these series solutions in the sub-domains by imposing the continuity conditions between each pair of adjacent sub-domains. This process isa very convenient means of constructing solutions that can be used to analyze multi-material wedges, whicharealso considered in this work.

To establish the power series solution for sub-domain *i*where $\theta_{i-1} \le \theta \le \theta_i$, the variable coefficients in Eqs. (4) are expanded in terms of the power series of θ with respect to the middle point of the sub-domain, $\bar{\theta}_i$:

$$p_j(\theta) = \sum_{k=0}^{K} \left(\chi_j\right)_k^{(i)} \left(\theta - \bar{\theta}_i\right)^k, q_j(\theta) = \sum_{k=0}^{K} \left(\kappa_j\right)_k^{(i)} \left(\theta - \bar{\theta}_i\right)^k, r_j(\theta) = \sum_{k=0}^{K} \left(\vartheta_j\right)_k^{(i)} \left(\theta - \bar{\theta}_i\right)^k,$$

$$s_j(\theta) = \sum_{k=0}^{K} \left(\xi_j\right)_k^{(i)} \left(\theta - \bar{\theta}_i\right)^k, t_j(\theta) = \sum_{k=0}^{K} \left(\zeta_j\right)_k^{(i)} \left(\theta - \bar{\theta}_i\right)^k. \quad (5)$$

Similarly, the solutions toEqs. (4) in the sub-domain are expressed as

$$\hat{U}_{0i}^{(m)} = \sum_{j=0}^{J} \hat{A}_j^{(i)} \left(\theta - \bar{\theta}_i\right)^j, \ \hat{V}_{0i}^{(m)} = \sum_{j=0}^{J} \hat{B}_j^{(i)} \left(\theta - \bar{\theta}_i\right)^j, \ \hat{W}_{0i}^{(m)} = \sum_{j=0}^{J} \hat{C}_j^{(i)} \left(\theta - \bar{\theta}_i\right)^j,$$

$$\hat{\Phi}_{0i}^{(m)} = \sum_{j=0}^{J} \hat{D}_j^{(i)} \left(\theta - \bar{\theta}_i\right)^j, \ \hat{\Psi}_{0i}^{(m)} = \sum_{j=0}^{J} \hat{E}_j^{(i)} \left(\theta - \bar{\theta}_i\right)^j. \quad (6)$$

Substituting Eqs. (5)and (6) into Eqs. (4) yields the recurrence relations among the coefficients in Eqs.(6), andthe coefficients $\hat{A}_j^{(i)}$, $\hat{B}_j^{(i)}$, $\hat{C}_j^{(i)}$, $\hat{D}_j^{(i)}$ and $\hat{E}_j^{(i)}$ with $j \ge 2$ aredeterminedfrom the recurrence relations if $\hat{A}_0^{(i)}$, $\hat{A}_1^{(i)}$, $\hat{B}_0^{(i)}$, $\hat{B}_1^{(i)}$, $\hat{C}_0^{(i)}$, $\hat{C}_1^{(i)}$, $\hat{D}_0^{(i)}$, $\hat{D}_1^{(i)}$, $\hat{E}_0^{(i)}$ and $\hat{E}_1^{(i)}$ are known. Accordingly, the series solutions in sub-domain*i* are obtained with ten coefficients $\hat{A}_0^{(i)}$, $\hat{A}_1^{(i)}$, $\hat{B}_0^{(i)}$, $\hat{B}_1^{(i)}$, $\hat{C}_0^{(i)}$, $\hat{C}_1^{(i)}$, $\hat{D}_0^{(i)}$, $\hat{D}_1^{(i)}$, $\hat{E}_0^{(i)}$ and $\hat{E}_1^{(i)}$ to be determined.

When the range of θ is decomposed into *n* sub-domains, a total of 10*n* coefficients must be determined in all of the sub-domain solutions that are constructed using the above procedure. These solutions must satisfy the continuity conditions between pairs of adjacent sub-domains. These include continuities of tractions, mechanical displacements, electric displacements, electric potential and magnetic potential. These continuity conditions yield 10(*n*-1) algebraic equations. Homogenous boundary conditions at $\theta = 0$ and $\theta = \beta$ must be satisfied, yielding another ten equations. As a result, 10*n* coefficients are to be determined from 10*n* homogenous algebraic equations. A nontrivial solution for the coefficients yields a10*n*×10*n* matrix with a determinant of zero. The roots of the zero determinant (λ_m), which can be complex

numbers, are obtained herein using the numerical approach of Müller (1956); they are ordered as $\mathrm{Re}[\lambda_i] \leq \mathrm{Re}[\lambda_{i+1}]$ (i=1, 2, 3,…).

3. VERIFICATION OF SOLUTION

To validate the proposed solution, convergence studies for $\mathrm{Re}[\lambda_1]$ (real part of λ_1) are conducted by increasing the number of sub-domains or increasing the number of polynomial terms in each sub-domain, and the convergent solution is compared with the published result. The wedge under consideration is made of$BaTiO_3$-$CoFe_2O_4$ particulate composite, which is a typical MEE material. According to the macroscopic mixture rule, the composite material properties of this MEE material are related to the material properties of $BaTiO_3$ and $CoFe_2O_4$ by (Song and Sih 2003)

$$\bar{\kappa}_{ij} = \bar{\kappa}_{ij}^{B} V_I + \bar{\kappa}_{ij}^{F} \left(1 - V_I\right), \tag{7}$$

where $\bar{\kappa}_{ij}^{B}$ and $\bar{\kappa}_{ij}^{F}$ denote the material properties of $BaTiO_3$ and $CoFe_2O_4$ (Table 1), respectively, and V_I is the volume fraction of the inclusion $BaTiO_3$. The material properties in Table 1 were taken from Wang *et al*. (2011).

The homogeneous boundary conditions can be specified as follows: (1) mechanical boundary conditions, $\sigma_{r\theta} = \sigma_{\theta\theta} = \sigma_{z\theta} = 0$ (traction free) or $u_r = u_\theta = u_z = 0$ (clamped); (2) magneto-electrical boundary conditions, $D_\theta = B_\theta = 0$ (magneto-electrically open) or $\phi = \psi = 0$ (magneto-electrically closed).For simplicity, four letters are utilized to specify the boundary conditions along the two radial side faces of a wedge with vertex angle β. The first and third letters indicate mechanical boundary conditionsat $\theta = 0$ and $\theta = \beta$, respectively; C and F specify clamped and traction-free, respectively. Similarly,the second and fourth letters represent the magneto-electric boundary conditions at $\theta = 0$ and $\theta = \beta$, respectively; C and O denote magneto-electrically closed and open boundary conditions, respectively.

To compare with the published result, the convergence study is conducted for an MEE wedge with $\beta = 360^{\circ}$ and having polarization direction along the z axis. For such a wedge, the out-of-plane deformation and in-plane electric and magnetic fields are independent of in-plane deformation and out-of-plane electric and magnetic fields. The wedge is made of $BaTiO_3$ and $CoFe_2O_4$ with $V_I = 50\%$ for $0^{\circ} \leq \theta < 180^{\circ}$ and with $V_I = 20\%$ for $180^{\circ} < \theta \leq 360^{\circ}$ and has FOFO boundary conditions. Notably, for comparison, $\bar{\mu}_{11} = 100 \times 10^{-6} Ns^2 / C^2$ for $CoFe_2O_4$, as in the work of Liu and Chue (2006), was used for the results in Table 2, which shows the results obtained using different numbers of sub-domains and polynomial terms for each sub-domain. Table 2 demonstrates that the convergent result, which is consistent with the result of Liu and Chue (2006), could be obtained by increasing the number of sub-domains or the number of polynomial terms for each sub-domain.

4. NUMERICAL RESULTS

This section presents numerical results of Re[λ_1],taking into account the effects of material components, boundary conditions, polarization direction and geometry on the strength of the singularities of stress, electric displacement and magnetic flux. When λ_1 shown in a figure are all real, λ_1, rather than Re[λ_1], is used to label vertical axis of the

Table 1 Material properties

Parameters	$BaTiO_3$ #	$CoFe_2O_4$ #
$\bar{c}_{11}$(GPa)	166	286.0
$\bar{c}_{12}$ (GPa)	77	173.0
$\bar{c}_{13}$ (GPa)	78	170.5
$\bar{c}_{33}$ (GPa)	162	269.5
$\bar{c}_{44}$ (GPa)	43	45.3
$\bar{e}_{15}$ (C/m^2)	11.6	0.
$\bar{e}_{31}$ (C/m^2)	-4.4	0.
$\bar{e}_{33}$ (C/m^2)	18.6	0.
$\bar{d}_{15}$ (N/Am)	0.	550.0
$\bar{d}_{31}$ (N/Am)	0.	580.3
$\bar{d}_{33}$ (N/Am)	0.	699.7
$\bar{\eta}_{11}$ (C^2/Nm^2)	112.0×10^{-10}	0.8×10^{-10}
$\bar{\eta}_{33}$ (C^2/Nm^2)	126.0×10^{-10}	0.93×10^{-10}
$\bar{\mu}_{11}$ (Ns^2/C^2)	5.0×10^{-6}	590.0×10^{-6}
$\bar{\mu}_{33}$ (Ns^2/C^2)	10.0×10^{-6}	157.0×10^{-6}
$\bar{g}_{11}$ (Ns/VC)	0.	0.
$\bar{g}_{33}$ (Ns/VC)	0.	0.

Note: #: data from Wang *et al*. (2011).

figure. The numerical results herein were obtained by dividing the whole domain of θ into eight sub-domains and using series solutions that comprise ten terms for each sub-domain. The $\bar{x}$-$\bar{y}$-$\bar{z}$ coordinate system is formed by rotatingthe *x*-*y*-*z*coordinate system counter-clockwise about the y-axis through an angle γ_y, so the angle between the polarization direction of an MEE wedge and its thickness direction is γ_y.

Table 2 Convergence of Re[λ_1] for an MEE wedge

Number of Sub-domains	Terms					Liu and Chue (2006)
	6	8	10	12	15	
3	0.4978	0.4417	0.4998	0.4750	0.4999	0.5000
4	0.4963	0.4999	0.4984	0.4999	0.4999	
6	0.4993	0.4999	0.5000	0.4999	0.5000	
8	0.4999	0.4999	0.5000	0.5000	0.5000	

4.1Wedges made of a single MEE material

Figure 3showsRe[λ_1] for $BaTiO_3$-$CoFe_2O_4$ wedges with $V_f = 20\%$, $\beta = 270^o$ and different boundary conditions varying with γ_y, while Fig. 4 displays the results for wedges with $\beta = 360^o$. As expected, the results are symmetric with respect to γ_y =90^o. Changing the polarization direction may cause λ_1 changing from real numbers to complex numbers and vice versa. For example, λ_1 for FCFC wedges are complex for $44^o \le \gamma_y \le 136^o$. Gradually changing γ_y form 0^o to 90^o leads the increase or decrease of Re[λ_1], depending on β and boundary conditions. Varying γ_y results in larger changes of Re[λ_1] for FCFC and COCO wedges with $\beta = 270^o$ than those for wedges with $\beta = 360^o$, and the change can reach 9%.

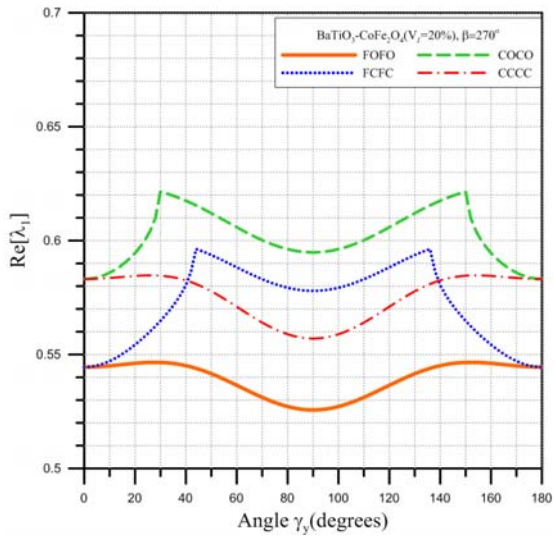

Fig. 3 Variation of Re[λ_1] with polarization direction for MEE wedges with $\beta = 270^o$

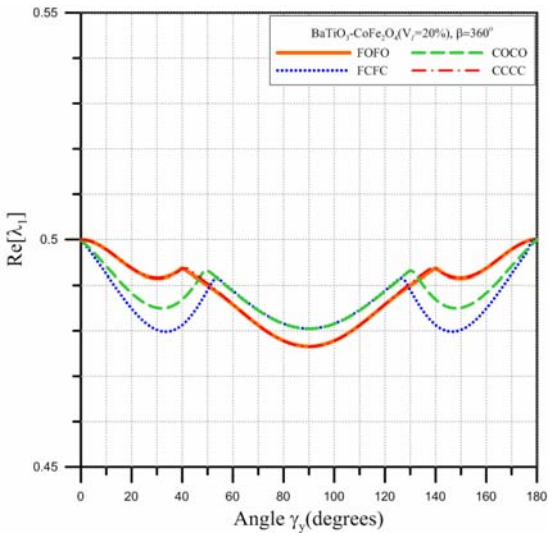

Fig. 4 Variation of Re[λ_1] with polarization direction for MEE wedges with $\beta = 360^o$

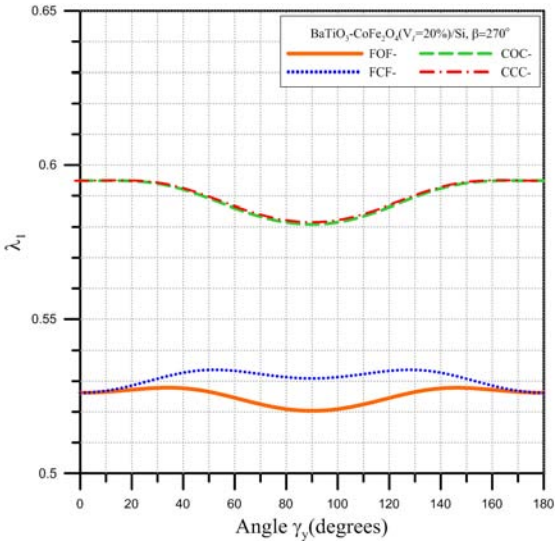

Fig. 5 Variation of Re[λ_1] with polarization direction for MEE/Si wedges with $\beta = 270^o$

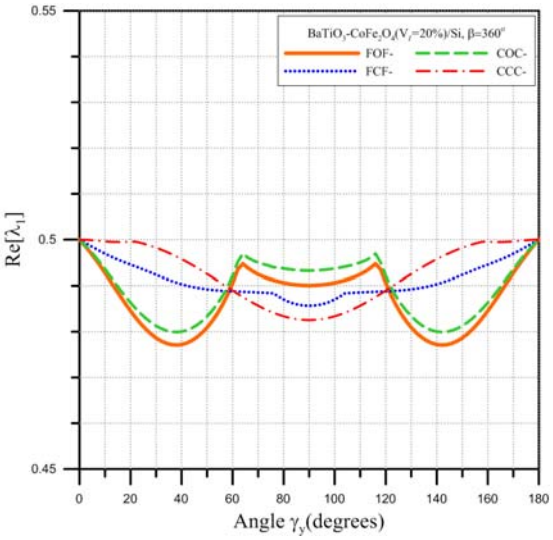

Fig. 6 Variation of Re[λ_1] with polarization direction for MEE/Si wedges with $\beta = 360^o$

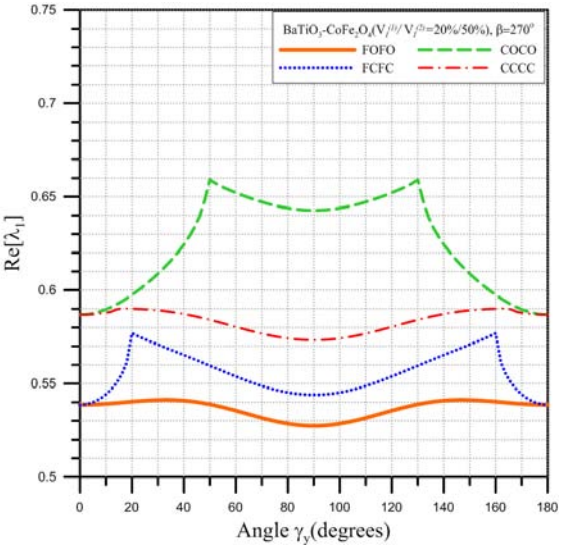

Fig. 7 Variation of Re[λ_1] with polarization direction for MEE/MEE wedges with $\beta = 270^o$

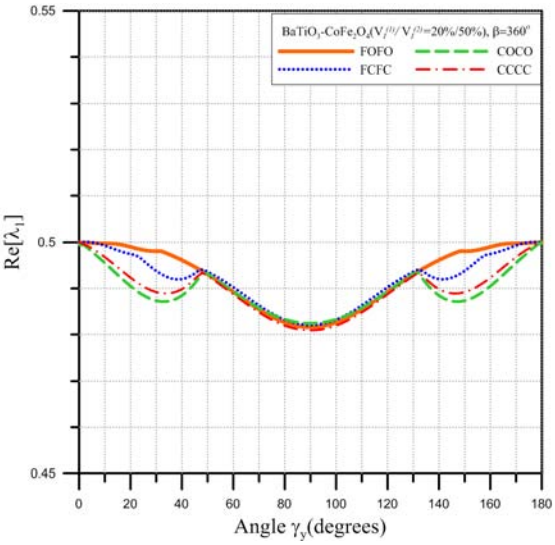

Fig. 8 Variation of Re[λ_1] with polarization direction for MEE/MEE wedges with $\beta = 360^o$

4.2Bi-materialWedges

This section concerns bi-material wedges with the same geometry as those considered in the preceding section. Figures 5 and 6 show Re[λ_1] varying with γ_y forBaTiO$_3$-CoFe$_2$O$_4$/Si wedges, in which Siis in the region of $\beta - 180^o < \theta \le \beta$, while Figs. 7 and 8 illustrate the results for wedges made of BaTiO$_3$-CoFe$_2$O$_4$ with $V_f = 50\%$ in $0^o \le \theta < 180^o$ and BaTiO$_3$-CoFe$_2$O$_4$ with $V_f = 20\%$ in $180^o < \theta \le \beta$. Notably, when specifying the boundary conditions in Figs. 5 and 6, "-" denotesthe absence of any magneto-electricalboundary conditions at $\theta = \beta$.

Figure 5 shows that λ_1 for wedges with $\beta = 270^o$ are all real, and different polarization directions cause small changes of λ_1 with less than 3%. It is interesting to observe that λ_1 for wedges with COC- boundary conditions are very close to those for wedges with CCC- boundary conditions. Free-free boundary conditions result in more severe singularities than clamped-clamped boundary conditions. However, these observations are not valid to the results for wedges with $\beta = 360^o$ in Fig. 6. The orientation of polarization may change the order of the singularity by approximately 5%. Comparing Figs. 5 and 6 with Figs. 3 and 4, respectively, discovers that bi-material wedges with $\beta = 270^o$ and free-free mechanical boundary conditions have stronger singularities than wedges made of a single MEE material, while wedges with CCCC boundary conditions show an opposite trend, which is also observed for wedges with $\beta = 360^o$ and CCCC boundary conditions.

Figure 7 demonstrates thatchanging the direction of polarization can change Re[λ_1] by up to approximately 10% for wedges with $\beta = 270^o$ and COCO boundary conditions, while the changes of Re[λ_1] are less than 3% for wedges with FOFO and CCCC boundary conditions. Figure 8 discloses that varying the direction of polarizationonlychange Re[λ_1] of wedges with $\beta = 360^o$ and different boundary conditions by less than 4%.Comparing Fig. 8 with Fig. 4 finds that bi-material MEE wedges with $\beta = 360^o$ yield less severe singularities than do wedges made of a single MEE material.

5. CONCLUSIONS

This study found an asymptotic solution to anMEE wedge to investigate geometrically-induced magneto-electro-elastic singularities at the vertex of the wedge based on three-dimensional magneto-electro-elasticity theory in a cylindrical coordinate system.The piezoelectric material has an arbitrary direction of polarization. The solution was obtained using an eigenfunctionexpansion approach in conjunction with a power series technique to the three-dimensional equations of motion and Maxwell's equations, which are five coupled partial differential equations in terms of the displacement components and electric and magnetic potentials.The solutions was validated by performing convergence studies of Re[λ_1] for a BaTiO$_3$-CoFe$_2$O$_4$ wedge with polarization in the thickness direction, and comparing the convergent Re[λ_1] with the resultpublished for a wedge under anti-plane deformation and in-plane electric and magnetic fields.

The proposed solution was employed to examinesingularities in wedges that comprise a single magneto-electro-elastic material ($BaTiO_3$-$CoFe_2O_4$), boundedMEE/isotropic elastic materials ($BaTiO_3$-$CoFe_2O_4$/Si), or MEE/MEE materials ($BaTiO_3$-$CoFe_2O_4$($V_f = 50\%$)/$BaTiO_3$-$CoFe_2O_4$($V_f = 20\%$)).The value of Re[λ_1], which is directly related to the order of the singularity, is displayed for different wedge angles, combinations of boundary conditions, and directions of polarization. As expected, the polarization direction may significantly affect Re[λ_1].

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