

## **Study on Local Fracture Energy Distribution in Reinforced Concrete Beams**

Shutong Yang

*Department of Civil Engineering, College of Engineering, Ocean University of China,  
Qingdao, P.R.China*  
[shutongyang2013@163.com](mailto:shutongyang2013@163.com)

### **ABSTRACT**

Fracture energy is a very important material parameter and can reflect the activity of crack propagation in concrete. The tri-linear model for the local fracture energy distribution can well explain the size effect in fracture energy indicating the boundary effects of concrete specimens. However, previous studies on the fracture energy are only aimed at plain concrete. The intention of this paper is to analyze the variation of local fracture energy near the steel bar. For three-point-bending notched reinforced concrete (RC) beams, an analytical model is proposed to establish the relationship between the applied load and the crack opening displacement or crack length during different loading stages. A critical fracture load is obtained during the fictitious crack propagation process in the model. It is also the first peak load during the variation of the applied load and proved to be much related to the cohesive crack-tip local fracture energy. Upon the comparison between the experimentally measured critical loads and the predicted critical loads under two extreme conditions, it is found that the aggregates interlocking and pull-out activities are much limited near the steel bar so that the values of local fracture energy are smaller than the size-independent fracture energy of concrete. As the notch length increases, the critical fracture load disappears in both the analytical model and the tested results.

### **1. INTRODUCTION**

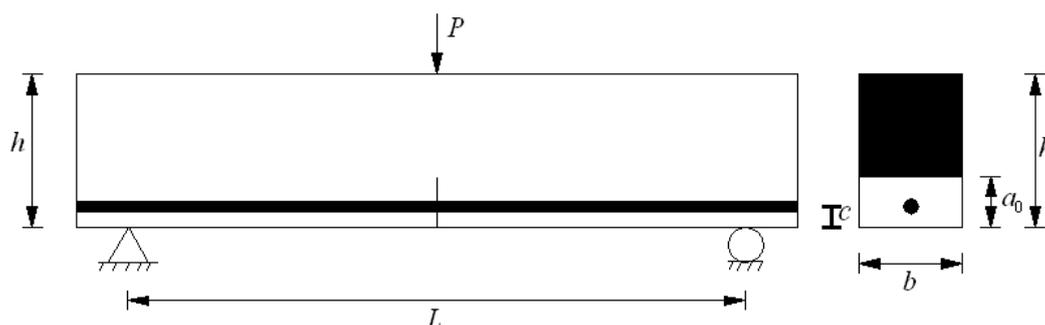
Fracture energy is a very important material parameter and can reflect the activity of crack propagation in concrete. However, it is found to be much dependent on the sizes and shapes of the specimens. Hu et al. (1992, 1995) pointed out that the size effect in concrete is mainly due to the non-constant distribution of local fracture energy along the ligament of specimen. Then a bi-linear model (Duan et al. 2003) is proposed for the distribution indicating the back free boundary effect. The author found that the local fracture energy distribution is also affected by the front free boundary effect (Yang et al. 2011). Thus, a tri-linear model (Yang et al. 2011) is presented and has been verified

experimentally by other researchers (Muralidhara et al. 2011; Saliba et al. 2012; Vydra et al. 2012). The size-independent specific fracture energy is determined by the tri-linear model by Karihaloo et al. (2013).

Although numerous researches have been carried out on fracture energy of concrete, only the fracture energy of plain concrete is considered. Since reinforced concrete structures are nowadays widely applied in civil engineering, the effect of steel bars on local fracture energy distribution deserves to be studied. The intention of this paper is to propose an analytical model to analyze the variation of local fracture energy near the steel bar.

## 2. ANALYTICAL MODEL

Fig. 1 shows a three-point-bending notched reinforced concrete beam. Herein,  $b$  and  $h$  are the width and depth of the beam, respectively. The span  $L$  of the beam is 4 times of the depth.  $c$  refers to the distance from the centroid of the steel bar to the bottom of the beam.  $a_0$  is the initial notch length.



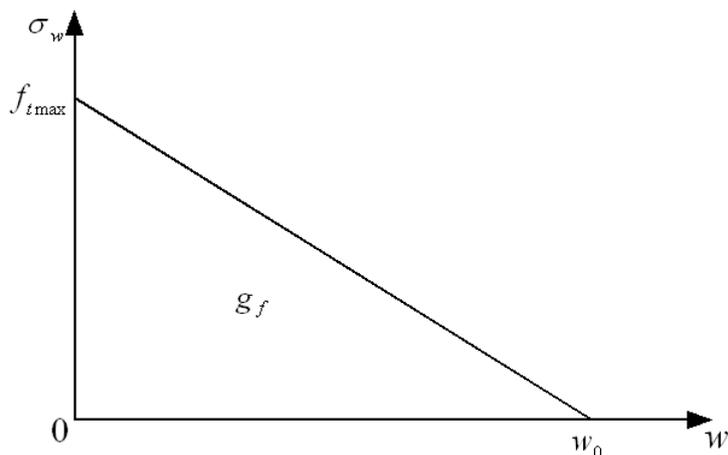
**Fig. 1** A three-point-bending notched reinforced concrete beam

When the crack at the mid-span starts to propagate, a micro-crack region appears ahead of the crack tip and can be simulated as a fictitious crack. The relationship between the cohesive stress  $\sigma_w$  and the crack opening displacement  $w$  in the fictitious crack can be simulated as a single-linear model as shown in Fig. 2. It reads

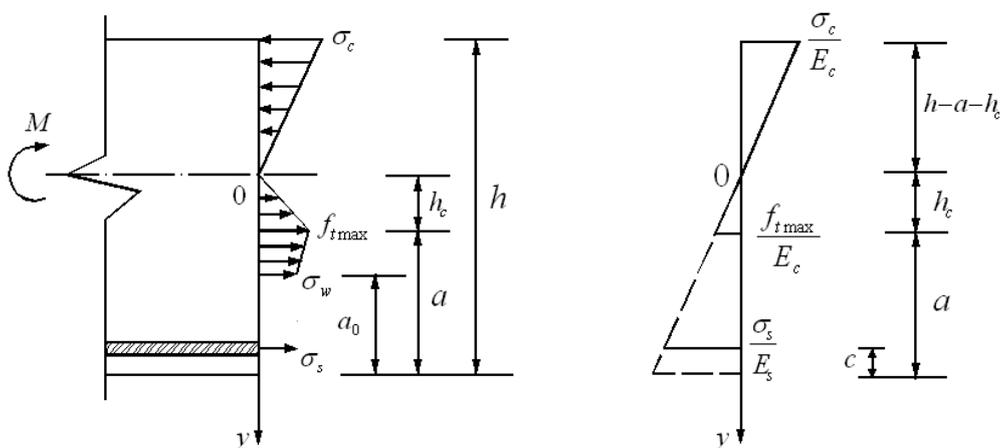
$$\sigma_w = f_{t\max} \left( 1 - \frac{w}{w_0} \right) \quad (1)$$

$f_{t\max}$  is the micro-critical tensile stress and the area under the single line refers to the local fracture energy  $g_f$  as indicated by Yang et al. (2011). Then the maximum crack opening displacement  $w_0$  can be expressed by

$$w_0 = \frac{2g_f}{f_{t\max}} \quad (2)$$



**Fig. 2** Single-linear model for  $\sigma_w$ - $w$  relationship



**Fig. 3** Distributions of stresses and strains in the cross-section at the mid-span

In the cross-section at the mid-span, it is assumed that the tensile strain of the steel bar and the strains in the un-cracked concrete distribute linearly along the beam depth as shown in Fig. 3. Then we have

$$\frac{1}{2} \sigma_c b (h - a - h_c) = \frac{1}{2} f_{t\max} b h_c + \frac{1}{2} (f_{t\max} + \sigma_w) b (a - a_0) + \sigma_s A_s \quad (3a)$$

$$\sigma_c = \frac{f_{t\max}(h-a-h_c)}{h_c} \quad (3b)$$

$$\sigma_s = \frac{f_{t\max} E_s (h_c + a - c)}{E_c h_c} \quad (3c)$$

$$\sigma_{wt} = f_{t\max} \left( 1 - \frac{w_t}{w_0} \right) \quad (3d)$$

An equilibrium equation related to  $a$ ,  $h_c$  and  $w_t$  (crack tip opening displacement) can be obtained by inserting Eqs. (3b), (3c) and (3d) into Eq. (3a), and denoted by

$$M_1(a, h_c, w_t) = 0 \quad (4)$$

Besides, the bending moment  $M$  can be expressed as a function of  $a$ ,  $h_c$  and  $w_t$  according to the force equilibrium condition in the critical cross-section.

Moreover, no slip is assumed between the steel bar and the concrete. The action of the steel bar can be simulated as a couple of tensile forces on the crack surfaces. Thus, the deformation compatibility equation by Tada et al. (1985) is still adopted as follows.

$$CMOD = \frac{24Ma}{h^2 b E_c} \left[ 0.76 - 2.28 \frac{a}{h} + 3.87 \left( \frac{a}{h} \right)^2 - 2.04 \left( \frac{a}{h} \right)^3 + 0.66 \left( \frac{h}{h-a} \right)^2 \right] \quad (5)$$

Herein,  $CMOD$  is the crack mouth opening displacement and can be given by

$$CMOD = \frac{a}{a-a_0} w_t \quad (6)$$

Substituting Eq. (5) with Eqs. (4) and (6), we have another equilibrium equation related to  $a$ ,  $h_c$  and  $w_t$  denoted by

$$M_2(a, h_c, w_t) = 0 \quad (7)$$

To obtain the critical value of  $M$ , the Lagrange Multiplier Method is adopted. A Lagrange function  $\Phi(a, h_c, w_t, \lambda_1, \lambda_2)$  should be constituted as follows.

$$\Phi(a, h_c, w_t, \lambda_1, \lambda_2) = M + \lambda_1 \times M_1(a, h_c, w_t) + \lambda_2 \times M_2(a, h_c, w_t) \quad (8)$$

By introducing the following conditions

$$\frac{\partial \Phi}{\partial a} = \frac{\partial \Phi}{\partial h_c} = \frac{\partial \Phi}{\partial w_t} = \frac{\partial \Phi}{\partial \lambda_1} = \frac{\partial \Phi}{\partial \lambda_2} = 0 \quad (9)$$

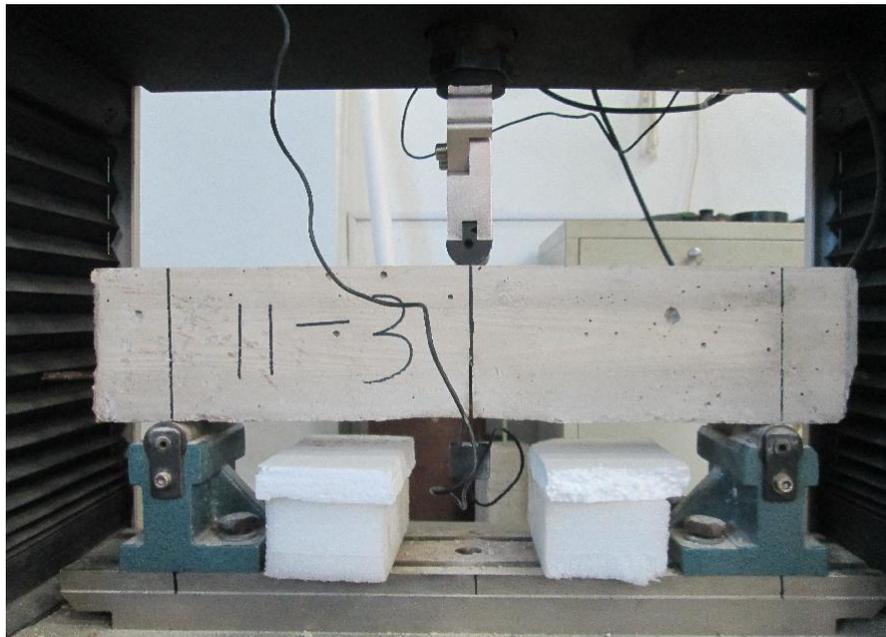
five equations can be established. By solving the equations, the critical crack length  $a_c$  and the critical bending moment  $M_{\max}$  can be obtained. Then the critical fracture load  $P_{\max}$  can be yielded as follows and it is found to be much dependent on the micro-critical tensile stress  $f_{t\max}$  and the cohesive crack-tip local fracture energy  $g_f$ .

$$P_{\max} = \frac{4M_{\max}}{L} \quad (10)$$

It should be noted that the  $P_{\max}$  is actually a critical load during the fictitious crack propagation process when the tensile stress of the steel bar is much lower than its yield strength. After the critical state, the applied load decreases but the action of the steel bar becomes more significant with the crack opening. When the tensile stress of the steel bar is high enough, the applied load increases again until the steel bar yields.

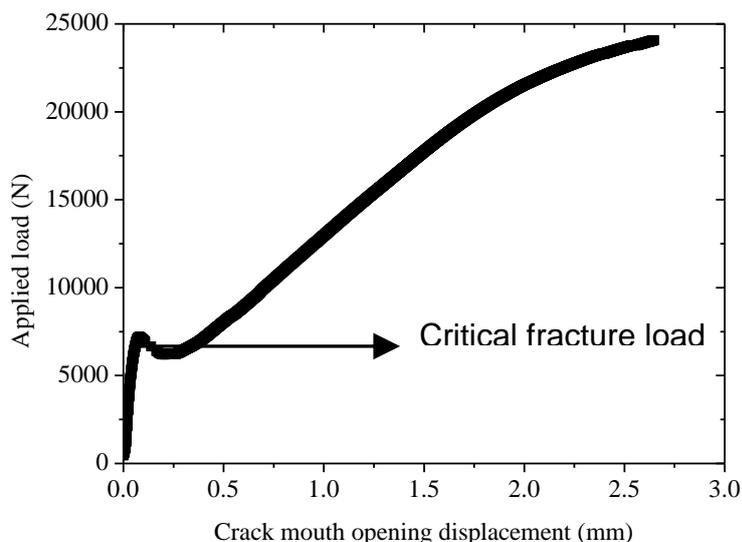
### 3. ANALYSIS OF LOCAL FRACTURE ENERGY NEAR THE STEEL BAR

A test on three-point-bending notched reinforced concrete beams was performed to study the local fracture energy distribution near the steel bar. The width, depth and span of the beams are 100mm, 200mm and 800mm, respectively. The thickness of the concrete cover is 20mm. The notch length varies from 30mm to 90mm. The steel bars have the diameters of 8mm and spirally ribbed surfaces. All the beams were tested in an Electrical Universal Testing System with maximum range of 100 kN after the curing of 28 days. The test setup is seen in Fig. 4.



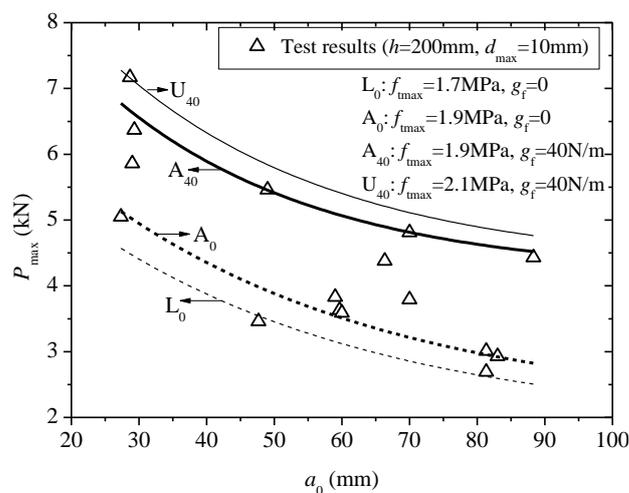
**Fig. 4** Test setup

A typical curve between the applied load and the crack mouth opening displacement *CMOD* is shown in Fig. 5. The critical load  $P_{\max}$  is also the first peak load in Fig. 5.



**Fig. 5** A typical load-displacement curve

Taking into account of the material variation properties, a band of variation should be considered for the  $f_{tmax}$  with the upper and lower limits of 2.1 MPa and 1.7 MPa, respectively. Moreover, the  $P_{max}$  is reached at the early stage of crack propagation and the quasi-stable growth is relatively small. Thus, the crack may go across several aggregates easily with no bridging stress and the cohesive crack-tip local fracture energy  $g_f$  is 0. As the aggregates interlocking and pull-out activities develop, the  $g_f$  becomes larger.  $g_f=40\text{N/m}$  is adopted as the maximum value in the region near the steel bar. Then the analytically predicted critical loads under the two extreme conditions and their comparisons with the experimentally measured critical loads are seen in Fig. 6.



**Fig. 6** Comparison between predicted and measured critical loads

All the experimental scattered points fall between the lines  $L_0$  and  $U_{40}$ . It means the adopted upper and lower limits of both the  $f_{tmax}$  and the  $g_f$  are rational. The line  $A_{40}$  represents the  $f_{tmax}$  is adopted as the average value and the  $g_f$  is adopted as the maximum. Most of the scattered points are under the line  $A_{40}$  and only 3 points just touch it. It demonstrates that the aggregates interlocking and pull-out activities are much limited near the steel bar so that the values of local fracture energy are apparently smaller than the size-independent fracture energy (around 140 N/m). The boundary effect by the steel bar is detected in the test. As the notch length increases, the crack-tip is far away from the steel bar and the  $g_f$  becomes larger. Then the critical fracture load can not be obtained in the analytical model and the first peak load disappears in the load-displacement curve as observed in the test.

#### **4. CONCLUSION**

An analytical model is proposed to study the local fracture energy distribution near the steel bar in reinforced concrete beams. A critical fracture load is obtained during the fictitious crack propagation process in the model. Then a test was performed on three-point-bending notched reinforced concrete beams. First peak load is observed in the load-displacement curve and it is actually the critical fracture load in the analytical model. Upon the comparison between the experimentally measured critical loads and the predicted critical loads under two extreme conditions, it is found that the aggregates interlocking and pull-out activities are much limited near the steel bar so that the values of local fracture energy are smaller than the size-independent fracture energy of concrete. As the notch length increases, the critical fracture load disappears in both the analytical model and the tested results.

#### **ACKNOWLEDGEMENT**

The authors would like to gratefully acknowledgement the National Natural Science Foundation of China (No. 51378481) and Basic Research for Application of Qingdao (No. 13-1-4-250-jch) which collectively funded this project.

#### **REFERENCES**

- Duan, K., Hu, X.Z. and Wittmann, F.H. (2003). "Boundary effect on concrete fracture and non-constant fracture energy distribution." *Engng. Fract. Mech.*, **70**, 2257-2268.
- Hu, X.Z. (1995), "Fracture process zone and strain softening in cementitious materials." *ETH building materials report No.1*, ETH, Switzerland, AEDIFICATIO, Freiburg.
- Hu, X.Z. and Wittmann, F.H. (1992), "Fracture energy and fracture process zone." *Mat. Struct.*, **25**, 319-326.
- Karihaloo, B.L., Ramachandra Murthy, A. and Iyer, Nagesh R. (2013), "Determination of

- size-independent specific fracture energy of concrete mixes by the tri-linear model.” *Cem. Concr. Res.*, **49**, 82-88.
- Muralidhara, S., Raghu Prasad, B.K., Karihaloo, B.L. and Singh, R.K. (2011), “Size-independent fracture energy in plain concrete beams using tri-linear model.” *Constr. Build. Mat.*, **25**, 3051-3058.
- Saliba, J., Loukili, A., Grondin, F. and Regoin, J.-P. (2012), “Experimental study of creep-damage coupling in concrete by acoustic emission technique.” *Mat. Struct.*, **45**, 1389-1401.
- Tada, H., Paris, P.C. and Irwin, G.R. (1985), *The stress analysis of cracks handbook*. St. Louis, MO: Paris Productions Incorporated.
- Vydra, V., Trtík, K. and Vodák, F. (2012), “Size independent fracture energy of concrete.” *Constr. Build. Mat.*, **26**, 357-361.
- Yang, S.T., Hu, X.Z. and Wu, Z.M. (2011), “Influence of local fracture energy distribution on maximum fracture load of three-point-bending notched concrete beams.” *Engng. Fract. Mech.*, **78**, 3289-3299.