







## COMMUNICATING WITH THE OTHER WORLD

Equations are our tools in our structural engineering analysis and structural reliability work. However, after all the work in our investigation is done, we must ask ourselves, even if the client does not ask us – How confident are we with our opinions? To bridge this potential communication gap, we can use the confidence scale in Table 3 to assign values based to a great extent on professional experience. This table is based on the American Society of Civil Engineers publication called *Degrees of Belief*. (Vick 2002)

**Table 3 Confidence Scale**

<b>Confidence</b>	<b>Probability</b>	<b>Single Number Probability</b>
Almost Certain	90 to 99.5% sure	90%
Very Likely or Very Probable	75 to 90% sure	80%
Likely or Probable	60 to 75% sure	70%
Medium Chance	40 to 60% sure	50%

## DEMAND AND CAPACITY

When the demand exceeds the capacity for a limit state we call this, in structural reliability language, the *Failure of the Limit State*. In structural reliability theory, the Safety Factor (F) is defined as the Capacity (C) of the limit state divided by the Demand (D) on the limit state. That is  $F = (C/D)$  and a value of F less than one defines failure.

In a real world forensic study, we never exactly know the capacity or the demand because of limited information. Therefore, C, D and F are in structural reliability language called *Random Variables*. The end result is that failure is a random event. Our structural engineering building code and standard committees have recognized this for almost half a century in design and structural reliability theory has for decades been a part of the foundation of building codes and standards.

We also use a parameter called the Safety, or Reliability, Index to enable us to incorporate the different consequences of failure and the probability of failure accounting for the uncertainty in both the demand and the capacity.

The quantification of either the capacity or the demand corresponding to a limit state in a forensic study always starts with the structural engineer making a best estimate of the expected value of the capacity or demand. Typically, these original estimates are improved upon using results from structural analysis equations and results from experimental tests on structural members conducted at universities or similar laboratories.

ASCE 7-10 provides us with valuable input because many structural members we study in a forensic investigation require us to do structural member testing because test data does not exist on its performance. The following are quotations from ASCE 7-10, Section 1.3.1.3:

*Assumptions of stiffness, strength, damping, and other properties of components and connections incorporated in the analysis shall be based on approved test data or referenced Standards.*

*Testing used to substantiate the performance capability of structural and nonstructural components and their connections under load shall accurately represent the materials, configuration, construction, loading intensity, and boundary conditions anticipated in the structure...Evaluation of test results shall be made on the basis of the values obtained from not less than 3 tests, provided that the deviation of any value obtained from any single test does not vary from the average value for all tests by more than 15%. If such deviation from the average value for any test exceeds 15%, then additional tests shall be performed until the deviation of any test from the average value does not exceed 15% or a minimum of 6 tests have been performed.*

Also, ASCE 7-10 uses the resistance factor to quantify uncertainty. Even though this equation needs to be presented in a slightly different form in some forensic engineering work, it is important to quote here:

*Similarly, resistance factors that are consistent with the above load factors are well approximated for most materials by*

$$\phi = (\mu_R / R_n) \exp[-\alpha_R \beta V_R]$$

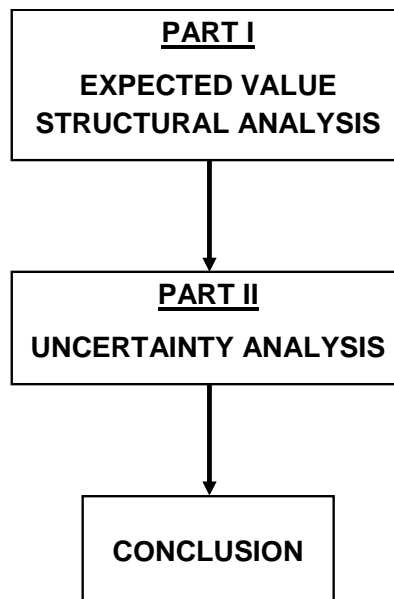
## **THE ART OF STRUCTURAL ENGINEERING AND THE BAYESIAN VIEW OF UNCERTAINTY**

Due to the cost of testing and analysis, most forensic structural engineering studies cannot afford the unlimited testing and analysis. Most of the time, no testing is allowed to the forensic engineers due to the limit of budget. Even with this circumstance, forensic engineers can do calculations and analysis based on their educational background, working experience and technical advances made by others. Without the formal mathematics of Bayesian methods of analysis we all as experts offer our opinions and have done so for decades. However, with the mathematical equations of the Bayesian approach we can improve upon the accuracy and credibility of our forensic study opinion. It enables us to use this experience and learning in a well-developed scientific way with very limited test data.

In other perspective, if the forensic engineers just rely on the test data without using their educational background, working experience and technical advances made by others, it is loss of valuable source of information the engineers can use. The faulty

selection of limited test locations without using prior knowledge can also lead engineers to incorrect decisions.

In a forensic engineering study, we must perform the three basic parts shown in Figure 1. Figure 2 presents a more detailed view of the Expected Value Analysis and Uncertainty Analysis parts of a forensic structural engineering study. In performing the strengthening or report part of Figure 2, we used the Expected Capacity and Demand to develop a capacity reduction factor.



**Figure 1 Parts of a Forensic Study**

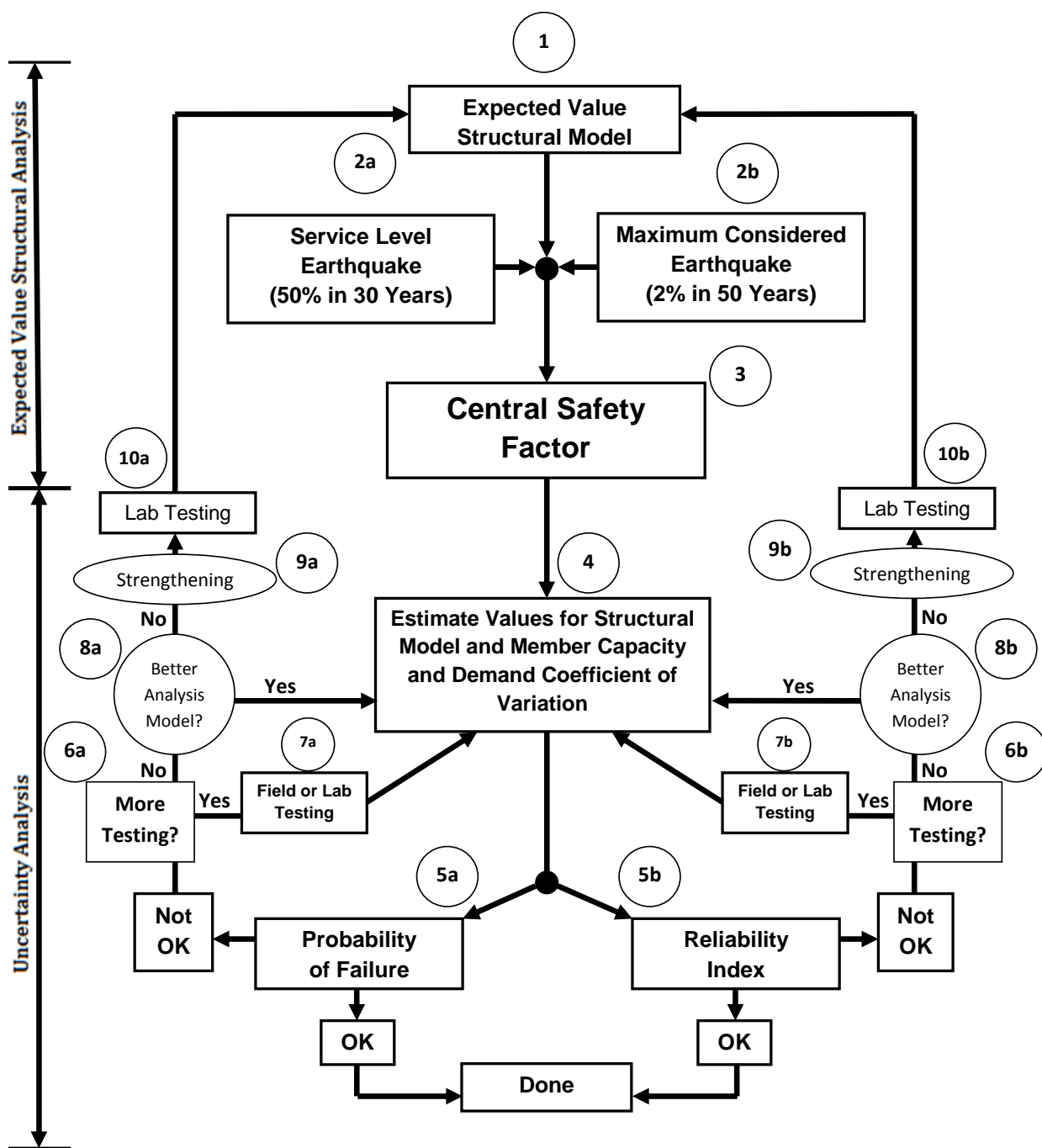


Figure 2 The Structural Analysis Procedure

## TRANSPARENCY AND BAYESIAN ANALYSIS

The building code is like a box of Duncan Hines brownie mix in many ways. If one follows the directions on the back of the brownie box, then one gets OK brownies. Similarly, if one follows the code provisions, then one gets an OK but not great building design. To cook the brownie, and to design the structural members in a building using the directions / provisions, can be done without any or perhaps only superficial understanding of what is behind the directions / provisions. Stated differently, there is no transparency and the intent is satisfaction for the “typical” person / building.

We must not be “Duncan Hines cooks” in forensic engineering. But this requires us to make decisions that are based on professional experience or as the fore-fathers called it, the “Art of Structural Engineering.” Consider the following quotation from the classic 1961 book by J. A. Blume titled Design of Multistory Reinforced Concrete Buildings for Earthquake Motions (Blume, 1961):

*“Considerable knowledge has been gained in the last three decades about the phenomenon of ground motions, the characteristics of structures, and their behavior in earthquakes. Despite this progress the complexities are still so great that earthquake-resistant design is not yet capable of complete and rigorous execution solely by means of mathematical analysis, design codes or rules. It is an art as well as a science, and requires experience and judgment on the part of the engineer.”*

The mathematics of Bayesian Updating is not simple to follow and understand without considerable effort. Following example will show how to use Bayesian Updating by take prior estimates of the Mean and Coefficient of Variation of Capacity and Demand based only on experience and then to update these estimates with the benefit of Destructive/Non-Destructive field testing, different levels of structural analysis, and laboratory testing of structural members.

### **EXAMPLE: Classic Case of Normal Base Variable with Unknown Mean and Known Standard Deviation**

The Base Random Variable  $X$  is the Minimum Compressive Strength of Concrete,  $f'_c$ , and the Standard Deviation of  $X$  is assumed to be known and equal to  $\sigma_x = 990$  psi. Three ( $n = 3$ ) test results are available, which are:  $f'_c = 5,471$  psi, 7,823 psi, and 5,326 psi. Note that for this example, the true value of the mean of  $f'_c$  (used to simulate the test result data values of  $f'_c$ , i.e. 5,471 psi, 7,823 psi, and 5,326 psi) is  $\bar{X} = 5,500$  psi, but it is assumed to be unknown.

Step 1:  $Y$  is the Nested Random Variable representing the unknown Expected Value of the Base Random Variable  $X$ . The Prior Expected Value of  $Y$ , i.e.  $\bar{Y}_p$ , is assumed by the structural engineer to be 5,000 psi.



**Steps 2 and 3:** The Standard Deviation of X,  $\sigma_x$ , is assumed to be known and equal to 990 psi.

**Step 4:** Prior Expected Value of X, i.e.  $\bar{Y}_p$ .

$$\begin{aligned}\bar{Y}_p &= \text{Prior value of } \bar{Y} \text{ (i.e. the Expected Value of Mean of } f_c'). \\ &= 5,000 \text{ psi}\end{aligned}$$

**Step 5:** Confidence in  $\bar{Y}_p$ .

In this example, we will assume a Coefficient of Variation of X. Prior estimated value of the Coefficient of Variation of the Nested Random Variable Y,  $\rho_{yp}$ , is 80%. Note this is a very large Coefficient of Variation and is only selected in this example to show convergence with essentially no confidence in the prior estimate of the Expected Value of X. Therefore,

$$\begin{aligned}\sigma_{yp} &= \text{Prior value of the Standard Deviation of the Nested Random Variable Y,} \\ &\text{i.e. the Standard Deviation of the random variable Expected Value of } f_c'. \\ &= \rho_{yp} \bar{Y}_p = 4,000 \text{ psi}\end{aligned}$$

**Step 6:** Test Data

$$\begin{aligned}\bar{Y}_t &= \text{Sample Mean} = [X^{(1)} + X^{(2)} + \dots + X^{(n)}] / n \\ &= \frac{1}{3} [5471 + 7823 + 5326] = 6,207 \text{ psi}\end{aligned}$$

**Step 7:** Updated Expected Value of X, i.e.  $\bar{Y}_u$ .

$$\begin{aligned}\bar{Y}_u &= \text{Posterior, updated, Expected Value of the Mean of } f_c' \\ &= \left[ \frac{\bar{Y}_t \sigma_{yp}^2 + \bar{Y}_p (\sigma_x^2 / n)}{\sigma_{yp}^2 + (\sigma_x^2 / n)} \right] = \left[ \frac{1}{1 + [(\sigma_x^2 / \sigma_{yp}^2) / n]} \right] \bar{Y}_t + \left[ \frac{1}{1 + [n(\sigma_{yp}^2 / \sigma_x^2)]} \right] \bar{Y}_p \\ &= C_1 \bar{Y}_t + C_2 \bar{Y}_p\end{aligned}$$

where

$$\begin{aligned}C_1 &= \left[ \frac{1}{1 + (R/n)} \right], \quad C_2 = \left[ \frac{1}{1 + (n/R)} \right] \text{ and } R = (\sigma_x / \sigma_{yp})^2 \\ R &= (990 / 4,000)^2 = 0.0613\end{aligned}$$

The value of R is small because  $\rho_{yp} = 0.8$  which has a very large uncertainty.

$$\begin{aligned}C_1 &= \left[ \frac{1}{1 + (0.0613/3)} \right] = 0.98 \\ C_2 &= \left[ \frac{1}{1 + 3/0.0613} \right] = 0.02\end{aligned}$$

Therefore,

$$\begin{aligned}\bar{Y}_u &= 0.98\bar{Y}_t + 0.02\bar{Y}_p \\ &= 0.98(6,207) + 0.02(5,000) \\ &= 6,183 \text{ psi}\end{aligned}$$

Notice how the test data is very heavily weighted (i.e. 0.98) because the uncertainty of the Prior of the Mean of Y is very large.

**Step 8:** Updated Variance and Standard Deviation of Y

$\sigma_{yu}^2$  = Posterior, updated, Variance of the Expected Value of  $f_c'$

$$\sigma_{yu}^2 = \left[ \frac{\sigma_{yp}^2 (\sigma_x^2 / n)}{\sigma_{yp}^2 + (\sigma_x^2 / n)} \right] = \frac{(\sigma_x^2 / n)}{\left[ 1 + \left[ (\sigma_x^2 / \sigma_{yp}^2) / n \right] \right]} = \left[ \frac{1}{n + (\sigma_x^2 / \sigma_{yp}^2)} \right] \sigma_x^2 = C_3^2 \sigma_x^2$$

where

$$C_3 = \sqrt{1 / (n + R)} = \sqrt{1 / (3 + 0.0613)} = \sqrt{0.327} = 0.57$$

Therefore,

$$\sigma_{yu} = 0.57\sigma_x = 0.57(990) = 566 \text{ psi}$$

**Steps 9 and 10:** Predictive Expected Value, Standard Deviation and Coefficient of Variation of X

Predictive Expected Value of X,  $\bar{X}_u = \bar{Y}_u = 6,183 \text{ psi}$

Predictive Standard Deviation of X,  $\sigma_{xu} = \sqrt{\sigma_x^2 + \sigma_{yu}^2} = \sqrt{990^2 + 566^2} = 1,140 \text{ psi}$

Predictive Coefficient of Variation of X,  $\rho_{xu} = \sigma_{xu} / \bar{Y}_u = 1140 / 6183 = 0.184$

## CONCLUSION

Bayesian updating method provides the users the chance of using their professional knowledge and the experience of others as documented in the published literature not just simply relying on Destructive/Non-Destructive field testing and laboratory testing of structural members. This method can prevent or minimize forensic engineers' incorrect decisions based on the faulty selection of limited test locations or members and allows the forensic engineer to share the benefits from information updating.

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