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Finite Element Model Updating using Acceleration and Angular Velocity

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ABSTRACT

The finite element (FE) model updating is a commonly used tool in civil engineering, enabling damage detection, design verification, and load capacity identification. In the FE modelupdating, acceleration responses are generally employed to determine modal properties of a structure, which are subsequently used to update the numerical model. While the acceleration-based model updating has been successful in finding better approximations of the physical systems, the boundary conditions are considered yet to be difficult to accurately estimate as the acceleration responses only correspond to the translational degree-of-freedoms (DOF).Recent advancements in the sensor technology have enabled low-cost, high-precision gyroscopes that can be adopted in the FE modelupdating to provide angular information of a structure. Especially, as the angular DOF is closely related to the rotational stiffness, gyroscopes in the FE model updating would give information of boundary conditions. This study proposes a FE modelupdating strategy based on data fusion of acceleration and angular velocity. The use of acceleration and angular velocity gives richer information than the sole use of acceleration, allowing the enhanced performance particularly in determining the boundary conditions. In this paper, a numerical simulation is presented to demonstrate the proposed FE modelupdating approach using the data fusion. A beam modelled using the Euler-Bernoulli beamelement is consideredwith various boundary conditions (i.e., simply supported, fixed supports, and in between). Optimization variables of the FE model updating are selected to be the elastic modulus and the rotational stiffness of two supports. The proposed FE modelupdating based on acceleration and angular velocityis shown to be able to more accurately identifythe boundary condition than the sole use of acceleration. Consequently, this paper verifies that data fusion of acceleration and angular velocity enhances the performance of the FE modelupdating.

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1. INTRODUCTION

Civil engineering structures are exposed toa wide variety ofloadings, such as earthquake, ocean wave, typhoon, hurricane, wind, and overloaded traffic.The damage of the structure caused by these diverse loadingscan produceloss of lives and physical destruction as a failure of the structure.Social overhead capital facilities are closely related to the national social economic and public safety, which are particularly important in this problem.

In the numerical analysis of structures, a number of finite element (FE) model updating methods in structural dynamics have been proposed (Ewins 1984; Mottershead and Friswell 1993; Friswell and Mottershead 1995; Maia and Silva 1997; Link 1999).The main objective of the model updating is to improve the correlation between the measured data and the analytical model.Thisaccurate FE modelhas been used in damage detectionof structures (Fritzen, Jennewein, and Kiefer 1998; Teughels, Maeck, and Roeck 2002; Jaishi and Ren 2006).FE model is used as a reference of structural health monitoring(Brownjohn*et al*. 2003; Jaishi and Ren 2005).It is also contribute to verify design verification and load capacity identification of civil engineering structures.

In the FE model updating, acceleration responses are generally employed to determine modal properties of a structure, which are subsequently used to update the numerical model. While the acceleration-based model updating has been successful in finding better approximations of the physical systems, the boundary conditions are considered yet to be difficult to accurately estimate as the acceleration responses only correspond to the translational DOF.The boundary conditions at supports which are involved in modal properties of the structure, thereforethe boundary condition of the structureis significant considerations to make accurate FE model.

In modern analysis, the data fusion strategy is evolving with the different types of sensors (Park, Sim and Jung 2013; Sung, Park, Nagayama, and Jung 2014). Recent advancements in the sensor technology have enabled low-cost, high-precision gyroscopes that can be adopted in the FE modelupdating to provide angular information of a structure.Especially, as the angular DOF is closely related to the rotational stiffness, gyroscopes in the FE model updating would give information of boundary conditions.

This study proposes a FE modelupdating strategy based on data fusion of acceleration and angular velocity. The boundary conditions of structure are important factor to make accurate FE model. In most FE model updating, acceleration responses are generally employed to determine modal properties of a structure, which strategy is difficult to accurately estimate the boundary condition. The angular DOF is closely related to the rotational stiffness at the supports, gyroscopes in the FE model updating would give information of boundary conditions. Consequently, the use of acceleration and angular velocity gives richer information than the sole use of acceleration, allowing the enhanced performance particularly in determining the boundary conditions.

2. FEMODEL UPDATING USING ACCELEARTION AND ANGULAR VELOCITY

The main objective of this paper is to present FE model updating technique using acceleration and angular velocity for civil engineering structures.Generally, the modal properties (i.e., natural frequencies and mode shapes) are used as an index of objective function in the FE model updating.When the modal properties are measured from the response of civil engineering structure, optimization variables of the FE model updating can be determined from the comparison between measured behavior of a structure and analytical prediction.Here, the objective functionof the proposed FE model updating is formulated in terms of natural frequencies, acceleration modeshapes, and angular modeshapes, which can be posed as a minimization problem to find optimization variables of the FE model updating. The detailed parts of the objective function are, respectively, shown below.

$$f_{err} = \frac{f_e - f_t}{f_e} \tag{0}$$

$$M_{acc} = MAC(\phi_{e\_acc}, \phi_{t\_acc}) \tag{0}$$

$$M_{gyro} = MAC(\phi_{e\_gyro}, \phi_{t\_gyro}) \tag{0}$$

where f_t and f_e are measured and analytical natural frequency,respectively, ϕ_{t_acc} and ϕ_{e_acc} are measured and analytical mode shape of acceleration,respectively, ϕ_{t_gyro} and ϕ_{e_gyro} aremeasured and analytical mode shape of angular velocity,respectively.The Modal Assurance Criterion (MAC) is a mathematical indicator that is most sensitive to large differences and relatively insensitive to small differences in the mode shapes.The MAC isa good mathematical indicator and a degree of consistency between mode shapesby convertingranges from 0 to 1, whichis defined by (Allemang and Brown 1982) as:

$$MAC(\phi_A, \phi_B) = \frac{\left|\phi_A{}^T \cdot \phi_B\right|^2}{\left(\phi_A{}^T \cdot \phi_A\right)\left(\phi_B{}^T \cdot \phi_B\right)} \tag{0}$$

where ϕ_A is analytical mode shape that has been paired with the measured mode shape ϕ_B. The full objective function of FE model updating based on data fusion, which considered natural frequencies, acceleration modeshapes, and angular modeshapes is shown below.

$$J = \sum_{i=1}^{N_m} \left\{ w_{1,i}\left(f_{err,i}\right)^2 + w_{2,i}\left(\frac{\left(1-\sqrt{M_{acc,i}}\right)^2}{M_{acc,i}}\right) + w_{3,i}\left(\frac{\left(1-\sqrt{M_{gyro,i}}\right)^2}{M_{gyro,i}}\right)\right\} \tag{0}$$

where N_m is number of measured modes. $w_{1,i}$, $w_{2,i}$, and $w_{3,i}$ are weighting factors for i th natural frequencies, acceleration mode shapes, and angular mode shapes, respectively. $f_{err,i}$ isdifferences for i th natural frequencies, $M_{acc,i}$ is MAC between the i th measured acceleration mode shape and i th analytical acceleration mode shape. $M_{gyro,i}$ is MAC between the i th measured angular velocity mode shape and i th analytical angular velocity mode shape.The Nelder-Mead Simplex Method (Lagarias 1998) of numerical analysis method is used for solving this minimization problem, to find optimum optimization variables.

The acceleration-based FE model updating is corresponding to the translational DOF, while the proposed FE model updating strategy considersboth of translational DOF and angular DOF. Therefore, the use of acceleration and angular velocity gives richer information than the sole use of acceleration, allowing the enhanced performance particularly in determining the boundary conditions.In this paper, the two cases of numerical simulation are presented to verify the proposed FE model updating approach using the data fusion.Firstly, the optimization variables (Elastic modulus and rotational stiffness) will be determined using acceleration based FE model updating. Secondly, the optimization variables will be determined using data fusion based FE model updating. The properties of the numerical model and other conditions are constant in each case.

3. NUMERICAL VERIFICATION

3.1 Numerical Simulation

In the numerical verification, a numerical simulation is presented to demonstrate the proposed FE modelupdating approach using the data fusion.A beam modelled using the Euler-Bernoulli beamelement is considered in here,the form and properties of beam are, respectively, shown below.

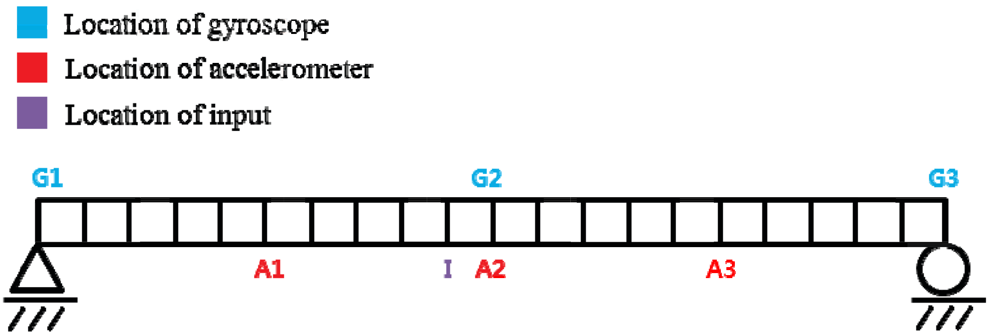

Fig. 1 Beam model in numerical simulation

Table 1 Properties of experimental beam model

Item	Properties
Number of elements	20 ea
Length of element	0.1 m
Width of beam	0.08 m

Height of beam	$0.01\ \mathrm{m}$
Material of beam	Steel
Steel mass density of unit volume	$7850\mathrm{kg}/m^3$

The simulated beam with a length of $2\ \mathrm{m}$ and a material is steel.The mass and stiffness matrix can be calculated by following expression

$$m=\frac{\rho A l}{420}\begin{bmatrix}156 & 22l & 54 & -13l\\ 22l & 4l^2 & 13l & -3l^2\\ 54 & 13l & 156 & -22l\\ -13l & -3l^2 & -22l & 4l^2\end{bmatrix} \tag{0}$$

$$k=\frac{EI}{l}\begin{bmatrix}\frac{12}{l^2} & -\frac{6}{l} & -\frac{12}{l^2} & -\frac{6}{l}\\ -\frac{6}{l} & 4 & \frac{6}{l} & 2\\ -\frac{12}{l^2} & \frac{6}{l} & \frac{12}{l^2} & \frac{6}{l}\\ -\frac{6}{l} & 2 & \frac{6}{l} & 4\end{bmatrix} \tag{0}$$

where ρ is steel mass density of unit volume, A iscross sectional area, l is length of element, and I is moment of inertia for bending. After making mass and stiffness matrix, the acceleration and angular velocity responses of a structure can be simulated by using MATLAB Simulink.In this numerical simulation, the responses of acceleration are measured at the A1, A2, and A3; the responses of angular velocity are measured at the G1, G2, and G3; the location of input is I as shown in Fig. 1.Random signal is used for input which is created by Band-Limited White Noise command in the Simulink of MATLAB. This input signal is newly coined in each simulation.Then, the measured natural frequencies and mode shapes are determined by Eigensystem Realization Algorithm (Juang and Pappa 1985).Optimization variables of the FE model updating are selected to be the elastic modulus and the rotational stiffness of two supports.The analytical prediction of natural frequencies and mode shapes can be calculated by solving eigenvalue problem.Here, the objective function is posed as a minimization problem to find optimization variables of the FE model updating as compared to measured modal properties and analytical modal properties.

To verify the proposed FE model updating strategy based on data fusion of acceleration and angular velocityin determining the boundary condition, a beam structure modelled using the Euler-Bernoulli beamelement should considervarious boundary conditions (i.e., simply supported, fixed supports, and in between).Theoretically, aninfinite value of the rotational stiffness at the support makes the fixed boundary condition. In this paper, we established the Rotational Freedom Indicator(RFI) for analyzing boundary condition of system. The RFIranged from 0 to 1 reflects the boundary conditions of experimental beam in acertain rotational stiffness at the supports as shown below.

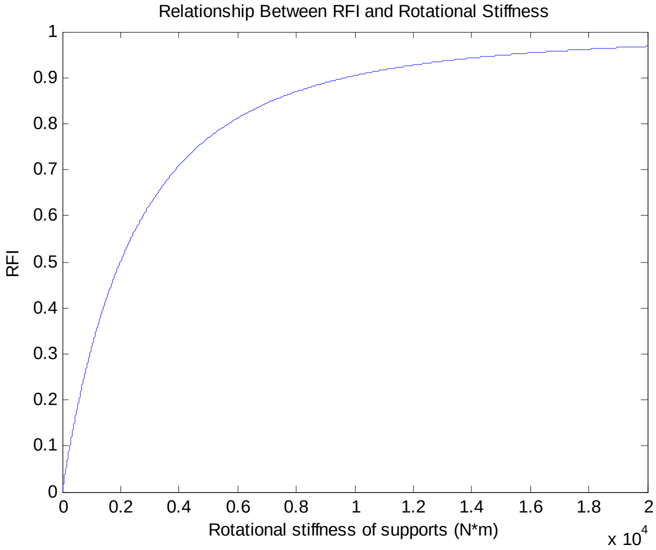

Fig. 2 Relationship between rotation stiffness and RFI

Table 2 Real value of rotational stiffness according to the RFI

RFI	Rotational stiffness (N*m)
0 (Simply Supported)	0
0.1	260
0.2	580
0.3	950
0.4	1410
0.5	1990
0.6	2770
0.7	3880
0.8	5700
0.9	9710
1 (Fixed Supported)	∞

The value of RFI is close to 0, which reflects that experimental beam is almost simply supported beam. Still the RFIis close to 1, this experimental beam is almost fixed supported beam.TheRFIis determined as comparedtomodal properties of experimental beam and fixed supported beam,which can be calculated by using Eq. (5), and (8).

$$RFI = 1 - \frac{J_C}{J_0} \qquad (0)$$

where J_0 isvalue of objective function usingsimply supported beam and fixed supported beam, J_c isvalue of objective function using experimental beam in acertain value of rotational stiffness and fixed supported beam.

3.2 Results

The RFI and corresponding errors of elastic modulusare shown below.

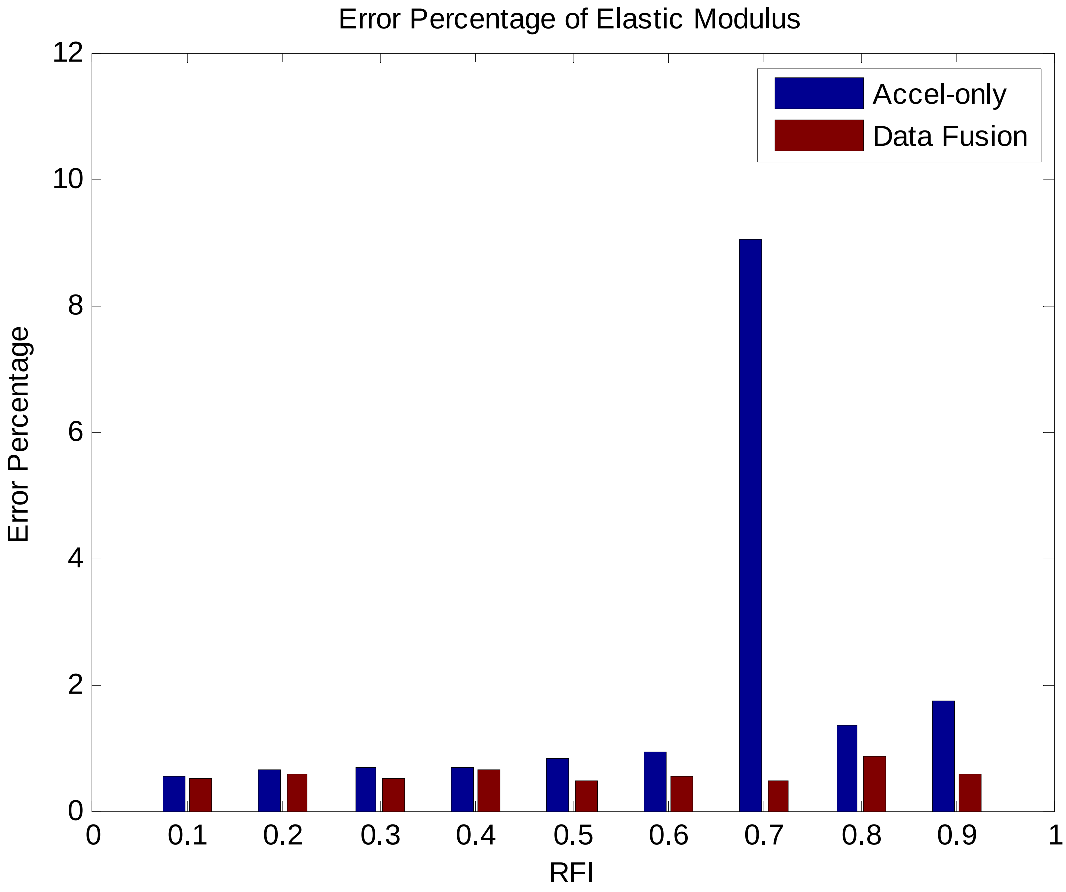

Fig. 3 Error percentage of elastic modulus

The real value of the elastic modulus is 200GPa in this numerical simulation.Theresults of each RFI are calculated by the average number of hundred enforcements, to increase the credibility of the results.The FE model updating based on accelerationand data fusion both strategies have been successful in finding better approximations of the elastic modulus, except the acceleration based FE model updating in case of RFI = 0.7 in Fig .3. The RFI and corresponding errors of rotational stiffness are shown below.

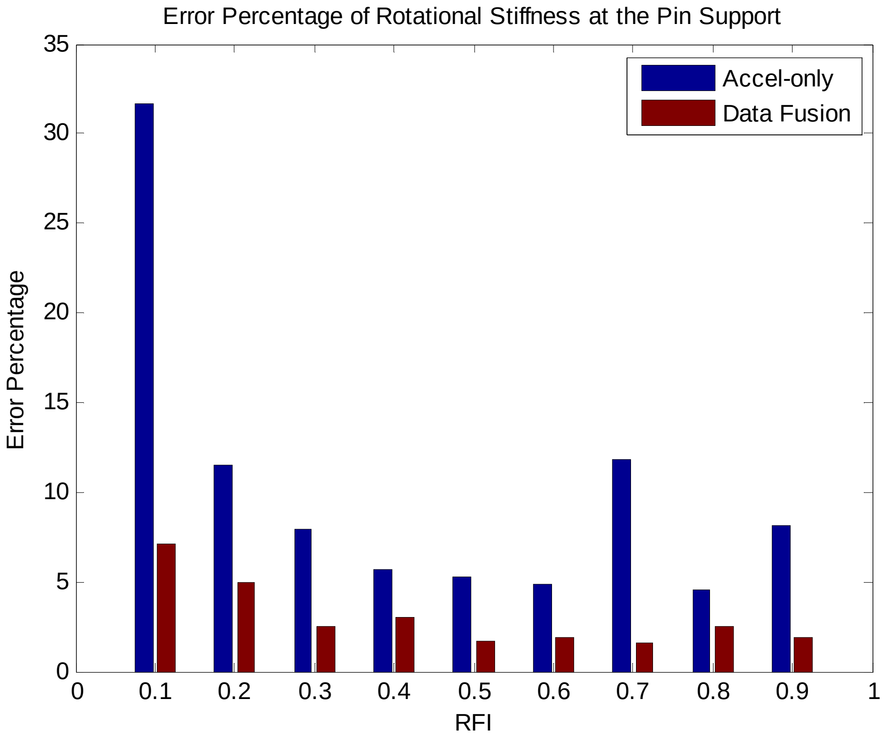

Fig. 4 Error percentage of rotational stiffness at the pin support

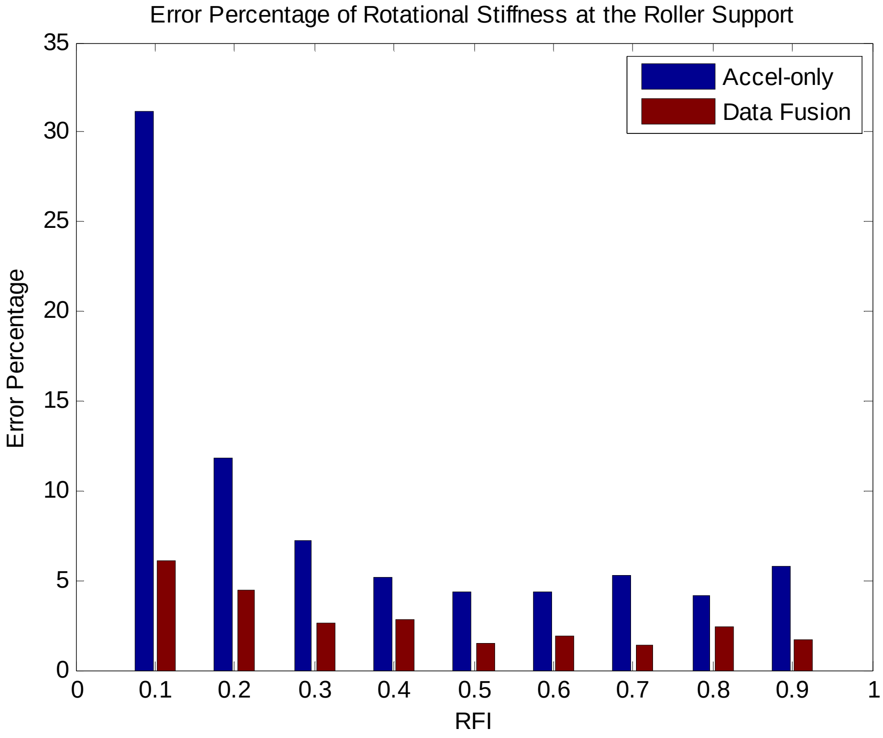

Fig. 5 Error percentage of rotational stiffness at the roller support

The FE model updating based on acceleration is not being accurate to get the rotational stiffness at the each supports, while the proposed FE modelupdating strategy based on data fusion of acceleration and angular velocity has been successful in finding better approximations of the rotational stiffness at the each supports in Fig.4 and Fig.5.In the acceleration-based model updating,consideringboundary condition yet to be difficult to accurately estimate as the acceleration responses only correspond to the translational DOF.The use of acceleration and angular velocity gives richer information than the sole use of acceleration, allowing the enhanced performance particularly in determining the boundary conditions by considering angular DOF.

The convergence historiesof the objective function in different strategies of FE model updating are shown below.

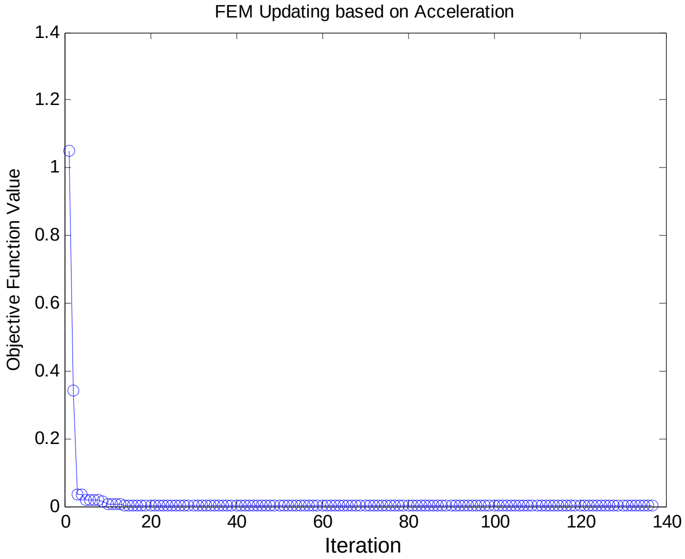

Fig.6Convergence history of FE model updating based on acceleration

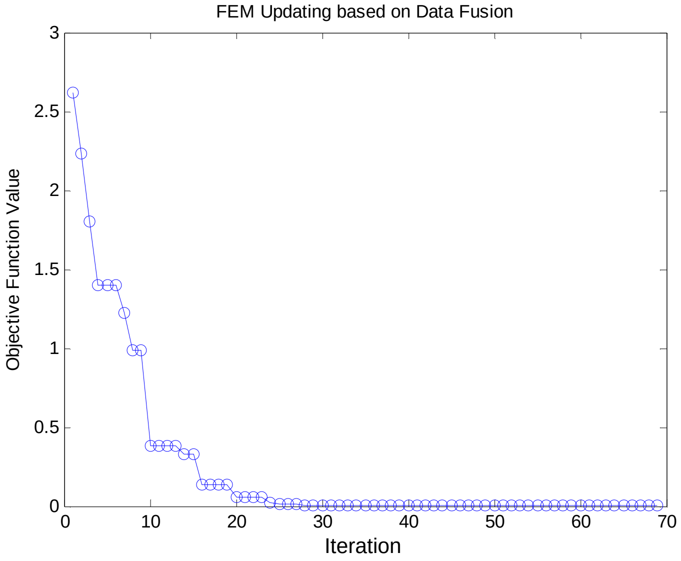

Fig. 7 Convergence history of FE model updating based on data fusion

In case of FE model updating based on acceleration, after 100thgeneration, the final objective function values of all model updating have converged to similar values around 0 in Fig 6. In case of FE model updating based on data fusion, after 60thgeneration, the final objective function values of all model updating have converged to similar values around 0 in Fig 7.Both of FE model updating strategies have converged tominimum value of objective function value, which showsthat all of the model updating isperforming well.

4. CONCLUSIONS

This paper presented a FE modelupdating strategy based on data fusion of acceleration and angular velocity. The angular DOF is closely related to the rotational stiffness at the supports, gyroscopes in the FE model updating would give information of boundary conditions.Therefore,the use of acceleration and angular velocity gives richer information than the sole use of acceleration, allowing the enhanced performance particularly in determining the boundary conditions.The objective function is constituted by combination of natural frequencies, acceleration modeshapes, and angular modeshapes in the data fusion strategy.A numerical simulation is presented to demonstrate the proposed FE model updating approach using the data fusion. A beam modelled using the Euler-Bernoulli beamelement is consideredwith various boundary conditions (i.e., simply supported, fixed supports, and in between). Optimization variables of the FE model updating are selected to be the elastic modulus and the rotational stiffness of two supports. The proposed FE model updating based on acceleration and angular velocityis shown to be able to more accurately identifythe boundary condition than the sole use of acceleration. Consequently, this paper verifies that data fusion of acceleration and angular velocity enhances the performance of the FE model updating.

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