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Improvement of the decentralized random decrement technique in wireless sensor networks

*Chengyin Liu[1)], Jun Teng[2)] and Jian Liu[3)]

[1),2)] *School of Civil and Environmental Engineering, Harbin Institute of Technology Shenzhen Graduate School, Shenzhen 518055, China*
[3)] *Guangdong Highway Construction Co. Ltd., Guangzhou, 510600, China*
[1)] *chengyin.liu*08@gmail.com

ABSTRACT

Random Decrement Technique (RDT) based decentralized computing approaches implemented in wireless sensor networks (WSNs) have shown advantages for modal parameters and data aggregation identification. However, previous studies of RDT-based approaches from ambient vibration data have usually assumed that the input excitation is a broad-band stochastic process modeled by stationary white or filtered white noise. In addition, the choice of the triggering condition in RDT is intimately related to data communication. In the present paper, a theoretical justification of the random decrement technique is presented for nonstationary white noise excitations. Local extremum triggering condition is chosen and implemented for the purpose of minimum data communication in RDT based distributed computing strategy. The performance of improved RDT was assessed in terms of (1) accuracy of the estimated modal properties and (2) efficiency in the wireless data communication. Numerical simulation confirms the validity of the proposed method for identification of modal parameters from nonstationary ambient response data and better efficiency in decentralized data aggregation.
Keywords: Random Decrement Technique decentralized computing, wireless sensor networks, nonstationary.

1. INTRODUCTION

Wireless sensor networks are one of the most promising emerging technologies, providing the opportunity for real-time monitoring of civil infrastructures that are prone to various disasters. Wireless sensor network technology has the capability to capture quickly, process, and transmit of critical high resolution data for real-time monitoring. It also has the advantage of dense deploying sensors that require low maintenance, which is ideal for operational conditions. However, the real applications of wireless sensor nodes are still constrained in the amount of power consumption and memory available. Hence, decentralized computing strategies that employ local data processing have been adopted increasingly in the WSNs due to its efficiency for data communication and feature extraction.

Gao (2006) proposed a coordinated computing strategy for damage detection that retains local spatial information, while concurrently reducing data communication in the network. As part of the data processing scheme, communication between sensor nodes takes place within each local sensor community to provide condensed information, such as correlation functions, modal properties, or damage information. This information can then be shared between the cluster-heads in the WSSN or brought to the centralized base station. Nagayama (2007) implemented a coordinated computing strategy in a WSSN employing the Imote2 sensor platform for damage detection. An output-only identification approach, the Natural Excitation Technique (NExT) (James 1993), was employed in conjunction with Eigensystem Realization Algorithm (ERA) (Juang 1985). The network topology employed in this approach consists of three types of sensor nodes: (a) gateways, (b) cluster head, and (c) leaf node. The gateway node is directly linked to the base station, controlling operation of the sensor network and interfacing users with the WSSN. Once vibration data is measured, the cluster-head transmits reference data to the leaf nodes in the local sensor community, where correlation functions are estimated and sent back to the cluster-head. Because the correlation function is in general much smaller than the raw sensor data, the total amount of transferred data can be significantly reduced, as compared to the centralized data aggregation approach.

Recently, Sim (2011) proposed a Random Decrement Technique (RDT) based decentralized computing method which has been shown to be quite efficient from the data communication perspective. The decentralized implementation of NExT (Nagayama 2007) requires the complete time history data from the cluster-head in a group be transferred to the leaf nodes to calculate the correlation functions. In contrast, RDT only requires the trigger crossings be sent to the leaf nodes, which is typically of a much smaller size than the raw sensor data. The output of the RDT is the random decrement (RD) function, which can be used as input to system identification methods such as ERA, Ibrahim Time Domain (ITD) (Ibrahim 1973, Yang 2004), Frequency Domain Decomposition (FDD) (Rodrigues 2004).

Although the RDT well serves as a decentralized computing for estimating modal parameters from ambient vibration data, it is based on an assumption that the input excitation is a broad-band stochastic process modeled by stationary white or filtered white noise and does not yet have sound mathematical basis for general cases (Vandiver 1982). In fact, most ambient excitation, such as earthquakes, encountered in many engineering problems is nonstationary in nature. Hence, it is desirable to develop applicable decentralized computing methods of modal identification in WSSN for nonstationary ambient vibration.

In addition, when introduced by Cole in 1968 the RD technique was intended to extract the "signature" of one particular vibration mode for a single narrow-band filtered random response measurement. The time records to be averaged are started when the response reaches a prespecified level. Such a condition is referred to as a triggering condition. In the decentralized implementation of RDT (Sim 2011), a positive-point condition is used for triggering. Because data communication in RDT based decentralized computing strategy is closely related to the triggering condition, the transferred data could be further reduced if proper triggering condition is chosen.

In the present paper, a theoretical justification of the random decrement technique is presented for nonstationary white noise excitations. Local extremum triggering condition is chosen and implemented for the purpose of minimum data communication in RDT based distributed computing strategy. The implementation of decentralized NExT and standard RDT reported in (Nagayama 2007, Sim 2011) is used as baseline for comparison. It is shown that the nonstationary RDT/ITD method evaluated on a truss structure is of the same accuracy as the NExT/ITD method for modal identification. Meanwhile, the data communication in the new RDT method with local extremum triggering condition is significantly reduced compared to the standard RDT with positive-point triggering condition. Therefore, the efficacy of the proposed RDT method is demonstrated in terms of required data communication and accuracy of identified modal parameters.

2. RANDOM DECREMENT TECHNIQUE

2.1 Standard RDT

Consider stationary, Gaussian random processes $X(t)$ and $Y(t)$. The auto- and cross-random decrement (RD) functions of $X(t)$ and $Y(t)$ can be defined as the expected value $X(t)$ of $Y(t)$ and given a trigger condition $T_{X(t)}$:

$$D_{XX}(\tau) = E\left[X(t+\tau) \mid \mathrm{T}_{X(t)}\right] \tag{1}$$

$$D_{YX}(\tau) = E\left[Y(t+\tau) \mid \mathrm{T}_{X(t)}\right] \tag{2}$$

where $E[\bullet]$ denotes expectation. Eq. (1) and Eq. (2) also can be written as:

$$D_{XX}(\tau) = \frac{R_{XX}(\tau)}{\sigma^2{}_X} \bullet \tilde{a} - \frac{R'_{XX}(\tau)}{\sigma^2{}_{\dot{X}}} \bullet \tilde{b} \tag{3}$$

$$D_{YX}(\tau) = \frac{R_{YX}(\tau)}{\sigma^2{}_X} \bullet \tilde{a} - \frac{R'_{YX}(\tau)}{\sigma^2{}_{\dot{X}}} \bullet \tilde{b} \tag{4}$$

where $R_{XX}(\tau) = E\left[X(t)X(t+\tau)\right]$ and $R_{YX}(\tau) = E\left[Y(t)X(t+\tau)\right]$ are the auto-correlation and cross-correlation functions, respectively, $\tilde{a}$ and $\tilde{b}$ can be defined in terms of the probability density functions $P_X(x)$ and $p_{\dot{X}}(\dot{x})$ respectively, as:

$$\tilde{a} = \frac{\int_{a_1}^{a_2} x p_X(x)dx}{\int_{a_1}^{a_2} P_X(x)dx} \quad \text{and} \quad \tilde{b} = \frac{\int_{b_1}^{b_2} \dot{x} p_{\dot{X}}(\dot{x})d\dot{x}}{\int_{b_1}^{b_2} p_{\dot{X}}(\dot{x})d\dot{x}} \tag{5}$$

Now, consider the equation of motion for a multi-degree-of-freedom system:

$$\boldsymbol{M}\ddot{\boldsymbol{X}}(t) + \boldsymbol{C}\dot{\boldsymbol{X}}(t) + \boldsymbol{K}\boldsymbol{X}(t) = f(t) \tag{6}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$, and $\boldsymbol{K}$ are the mass, damping, and stiffness matrices, respectively. The externally applied force is assumed to be a zero-mean, stationary, Gaussian random process, and the mass, damping, and stiffness are assumed to be deterministic. James (1993) showed that the correlation function between responses and a reference response X_k is the homogeneous solution of the equation of the motion:

$$\boldsymbol{M}\ddot{\boldsymbol{R}}_{XX_k}(\tau) + \boldsymbol{C}\dot{\boldsymbol{R}}_{XX_k}(\tau) + \boldsymbol{K}\boldsymbol{R}_{XX_k}(\tau) = \boldsymbol{0} \tag{7}$$

Substituting Eq. (8) into Eq. (10), and multiplying by $\frac{\sigma_x^2}{\tilde{a}}$ yields

$$\boldsymbol{M}\ddot{\boldsymbol{D}}_{XX_k}(\tau) + \boldsymbol{C}\dot{\boldsymbol{D}}_{XX_k}(\tau) + \boldsymbol{K}\boldsymbol{D}_{XX_k}(\tau) = \boldsymbol{0} \tag{8}$$

where $\boldsymbol{D}_{XX_k}(\tau)$ is the vector of RD functions, with the scalar response process X_k being referenced for the trigger condition. Thus, the RD functions $\boldsymbol{D}_{XX_k}(\tau)$ are seen to satisfy the homogeneous equation of motion.

The RD functions can be estimated from data as follows:

$$\hat{D}_{X_j X_k}(\tau) = \frac{1}{N}\sum_{i=1}^{n} x_j(t_i+\tau) \mid \mathrm{T}_{X_k(t_i)} \tag{9}$$

where $\hat{D}_{X_j X_k}(\tau)$ is the RD function obtained from $x_j(t)$ with respect to the reference $x_k(t)$, N is the total number of trigger events, $\mathrm{T}_{X_k(t_i)}$ is the specified trigger condition, t_i and is the ith time obtained from the trigger event $\mathrm{T}_{X_k(t_i)}$. When $j=k$, $\hat{D}_{X_j X_j}(\tau)$ is an auto-RD function.

2.2 Modified RDT for nonstationary processes

The RD function obtained in Eq. (9) was effective only for stationary processes. In the following, we extend the random decrement theory to deal with zero-mean nonstationary processes. Nie (2009) constructed a new RD function based on the Brown movement (many practical random excitations can be assumed as Brown movement in good approximation).

The probability density functions for Brown movement $B(t)$ is

$$p\left\{B(t+s)-B(t)\le x\right\} = \frac{1}{\sqrt{2\pi s}}\int_{-\infty}^{\tau} e^{-\frac{t^2}{2s}}dt \tag{10}$$

in which, $B(t)$ can be expressed as:

$$B(t) = \int_0^{\tau} w(\tau)d\tau \tag{11}$$

and $w(\tau)$ is a stationary Gaussian white noise.

For any Brown moment random response $X(t)$, $X_i = X(t_i)$, we have $B_i = B(t_i)$. Then

$$B_i = \sum_{j=1}^{i} w\left(\overline{t_j}\right)\left(t_j - t_{j-1}\right)^{1/2} \quad i=1,\ldots,n \quad \overline{t_j} \in \left[t_j, t_{j-1}\right] \tag{12}$$

Since B_i has equal time intervals, $t_i - t_{i-1} = \Delta t$, Δt is a constant. Eq. (12) can be simplified as

$$B_i \approx \sum_{j=1}^{i} w\left(\overline{t_j}\right) \quad \left(i = 1, \dots, n\right) \tag{13}$$

or

$$w_i = B_i - B_{i-1} \quad \left(i = 1, \dots, n\right) \tag{14}$$

where w_i satisfies the Gaussian distribution, hence from Eq. (12) can be used to construct a new RD functions.

$$\begin{aligned} D_{X_i - X_{i-1}, X_i - X_{i-1}}(t) &= E\left[X(t_i + t) - X(t_{i-1} + t) \mid \mathrm{T}_{X(t)} \right] \\ D_{X_i - X_{i-1}, Y_i - Y_{i-1}}(t) &= E\left[X(t_i + t) - X(t_{i-1} + t) \mid T_{Y(t)} \right] \end{aligned} \tag{15}$$

The auto- and cross-random decrement (RD) functions of $X(t)$ and $Y(t)$ can be defined as the expected value $X(t)$ of $Y(t)$ and given a trigger condition $T_{X(t)}$:

$$\begin{aligned} \hat{D}_{X_i - X_{i-1}, X_i - X_{i-1}}(\tau) &= \frac{1}{N} \sum_{i=1}^{N} \left(x(t_i + \tau) - x(t_{i-1} + \tau) \mid T_{X(t)} \right) \\ \hat{D}_{Y_i - Y_{i-1}, X_i - X_{i-1}}(\tau) &= \frac{1}{N} \sum_{i=1}^{N} \left(y(t_i + \tau) - y(t_{i-1} + \tau) \mid T_{X(t)} \right) \end{aligned} \tag{16}$$

If the externally applied force $f(t)$ is a Brown movement, then

$$f(t_i + \tau) - f(t_{i-1} + \tau) = w\left(t_i + \tau\right) \tag{17}$$

when $N \to \infty, \frac{1}{N} \sum_{i=1}^{N} w(t_i + \tau) \approx 0$, $\hat{D}_{X_i - X_{i-1}, X_i - X_{i-1}}(\tau)$ is analogous to Eq. (15). Then the nonstationary responses transform into stationary ones.

If the externally applied force is assumed to be a zero-mean, stationary, Gaussian random process, then $f(t_i+\tau)-f(t_{i-1}+\tau)$ is zero-mean too, which only contains higher order terms compared to $f(t)$. Therefore, the new RD functions may have wider application than standard RD functions in identifying the modal parameters of a structural system subjected to general ambient excitation.

2.3 Triggering Condition

Well known triggering conditions used in RDT are:

$$T^{L}_{X(t)}=\{X(t)=a\} \tag{18}$$

$$T^{P}_{X(t)}=\{a_1\le X(t)<a_2, -\infty\le \dot{X}(t)<\infty\} \tag{19}$$

$$T^{E}_{X(t)}=\{a_1\le X(t)<a_2, \dot{X}(t)=0\} \tag{20}$$

The triggering conditions in Eq. (18) - (20) are referred to as level crossing, positive-point, and local extremum, respectively.

In previous study (Sim 2001), the positive-point triggering condition is used. Data communication required by the decentralized RDT implementation is closely related to the number of triggering points. The purpose of this work is to present a RDT approach for distributed WSSN, which produces the accuracy for nonstationary ambient excitation at the same computational time compared to the standard RD technique, while decreases the data transfer in a minimum level. The local extremum triggering condition is in general known to have less data transfer than the positive-point triggering condition. Therefore, in this study, the local extremum triggering condition is chosen.

3. RDT-based decentralized data aggregation

RDT can significantly enhance the efficiency of data aggregation in the distributed computing environment in WSSNs. Nagayama (2007) proposed a decentralized NExT implementation, taking advantage of each node's computing capability to reduce data communication (see Fig. 1). Node 1 sends a measured time history record as a reference signal to each node. Correlation functions are calculated in all nodes in the community and subsequently collected at Node 1. Assuming the community has ns nodes, each sensor node measures data and transmits to Node 1 that provides reference information for correlation function estimation. For time history records of length N and n_d averages, the amount of transmitted data is at most N × n_d + N/2 × (n_s − 1). As the numbers of nodes or averages increase, the efficiency of the decentralized NExT implementation becomes clearer.

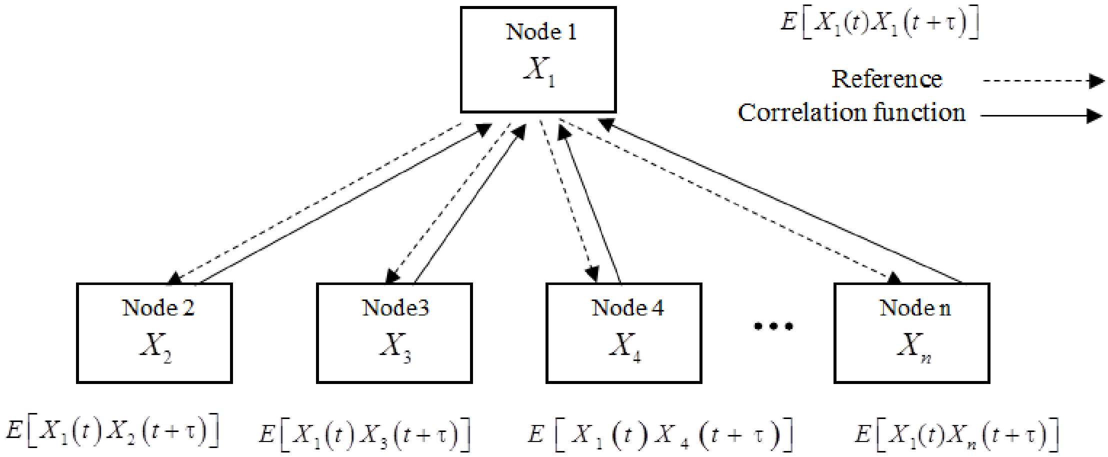

Fig. 1 Decentralized NExT implementation (Nagayama, 2007)

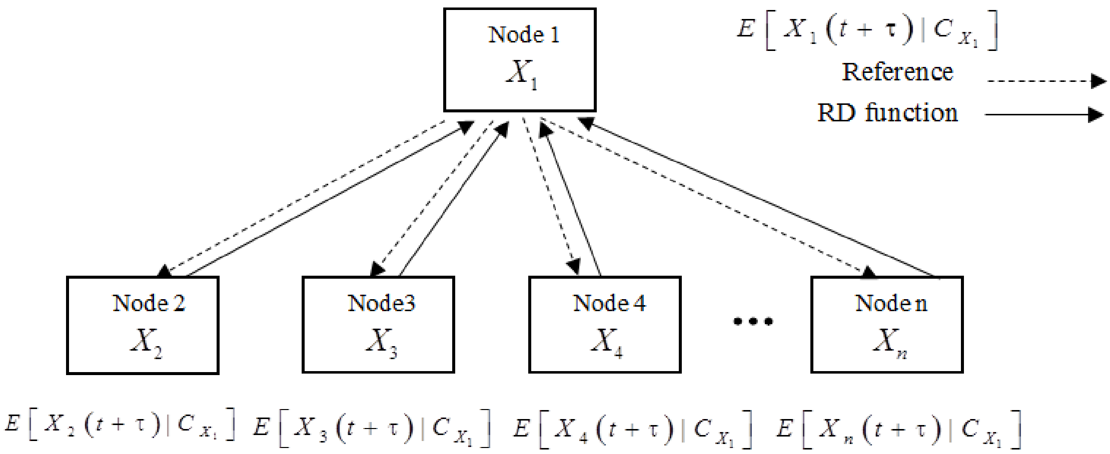

Fig. 2 Decentralized RDT implementation (Sim, 2011)

Sim (2011) proposed a decentralized RDT implementation shown in Fig. 2 can considerably reduce data communication requirement. In this approach, Node 1 sends the trigger information to all nodes in the community. Once the reference is received, each node calculates the RD functions that are subsequently collected at Node 1. For the positive-point triggering condition found in Eq. (19), the expected number of triggering points is (Asmussen 1997):

$$E\left[n(a_1,a_2)\right]=(N_x-N_\tau)\bullet\int_{a_1}^{a_2}p_X(x)dx \tag{21}$$

where n (a1, a2) is the number of triggering points between a1 and a2,1t is the sampling rate, $p_X(x)$ the probability density function of $X(t)$, and N_x and N_τ are the number points in $X(t)$ and the RD function, respectively.

Data communication required by the decentralized RDT implementation is closely related to the number of triggering points. In this study, for the local extremum triggering condition, the expected number of triggering points is:

$$E\left[n(a_1,a_2)\right]=(N_x-N_\tau)\sum_i p_X(x\,|\,a_1\le X_i(t)<a_2,\dot{X}_i(t)=0) \tag{22}$$

From Eq. (22), it can be seen that the local extremum triggering condition can further reduce data communication in RDT-based decentralized data aggregation. At last, the total number of points to be wirelessly transferred in the decentralized RDT implementation is:

$$(N_x-N_\tau)\sum_i p_X(x\,|\,a_1\le X_i(t)<a_2,\dot{X}_i(t)=0)+N_\tau(n_s-1) \tag{23}$$

4. Numerical validation

The efficacy of the proposed RDT-based decentralized data aggregation is numerically investigated. The performance in the distributed computing environment is checked in two criteria: (1) estimation accuracy and (2) data communication requirements. The NExT-based decentralized data aggregation is employed as a reference.

Consider the simply supported truss model in Fig. 3. The truss consists of 53 elements, for which the area moments of inertia are $I_x=2.6\times10^{-9}m^4$ and $I_y=I_z=1.3\times10^{-9}m^4$, the elastic modulus is $1.13\times10^{11}pa$, and the mass density is $7.85\times10^5kg/m^3$. A Gaussian white noise of 0.1 Power spectral density is applied vertically as shown in Fig. 3. Vertical accelerations at all lower nodes are obtained, and

10% RMS noise is added to all measurements. For subsequent reference, the transfer functions between the input excitation and the measured accelerations at points N1 and N2 are obtained from the analytical model.

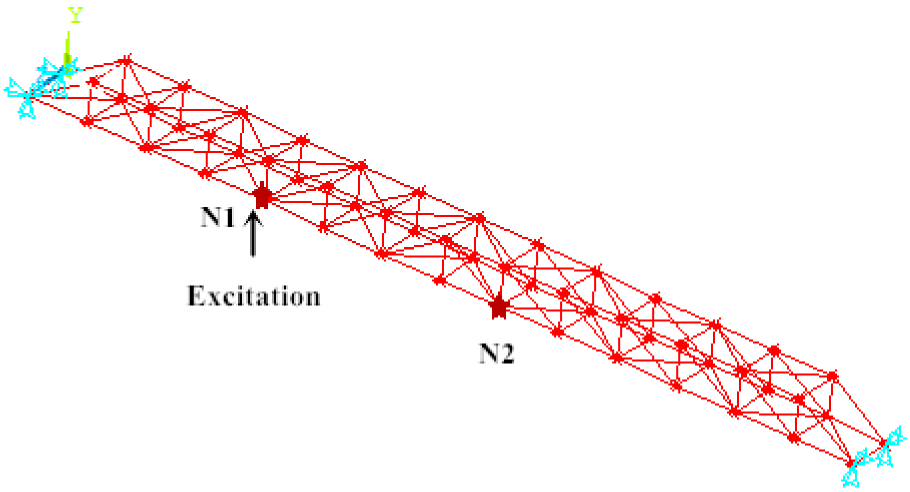

Fig. 3 Truss model

Four groups of sensor topology implemented on the truss are investigated. With 26 nodes in total, each local community has four, six, and eight nodes, and two, four, and six of which are the overlapping nodes, respectively. Once acceleration responses are obtained, RD and correlation functions are decentrally estimated in each local group as previously described. Subsequently, ITD is employed to estimate local modal properties, from which global modal properties are obtained. Sim (2011) introduced a G value, shown in Eq. (15), to access the efficiency of RDT in terms of accuracy and data communication obtained by the RDT-based decentralized data aggregation. For this purpose, the identical acceleration records are used in the numerical simulation for both positive-point triggering crossing and local extremum triggering crossing.

$$G = \frac{MAC}{E \bullet S} \tag{24}$$

$$E = \sum_{m} \frac{\left| f_{\Omega}^{m} - f_{\Omega,ref}^{m} \right|}{\left| f_{\Omega,ref}^{m} \right|} \tag{25}$$

$$MAC = \frac{\left| \varphi_{\Omega,ref}^{m}{}^{T} \bullet \varphi_{\Omega}^{m} \right|}{\left(\varphi_{\Omega,ref}^{m}{}^{T} \bullet \varphi_{\Omega,ref}^{m} \right)\left(\varphi_{\Omega}^{mT} \bullet \varphi_{\Omega}^{m} \right)} \tag{26}$$

S is the total number of trigger crossings, φ_{Ω}^{m} and $\varphi_{\Omega,ref}^{m}$ are the m th estimated and exact global mode shapes, respectively, and f_{Ω}^{m} and $f_{\Omega,ref}^{m}$ are the m th estimated and exact natural frequencies, respectively. The modal assurance criterion (Allemang 1982) is utilized in Eq. (25). The G value can be utilized to determine appropriate local community with optimal overlap nodes that produces accurate modal identification while minimizes data transmitting.

Table 1 Transferred data for NExT and RDT

Sensor Group	NExT Total	RDT							
		Positive-point triggering condition				Local extremum triggering condition			
		Trigger crossing	Total	G	RDT/ NExT(%)	Trigger crossing	Total	G	RDT/ NExT(%)
1 (4,2)	141312	8905	27337	0.042	19.35	3391	21823	0.097	15.44
2 (6,2)	76800	4446	19806	0.085	25.79	1699	17059	0.174	22.21
3 (8,2)	55296	2926	17262	0.126	31.22	1141	15477	0.253	27.99
4 (8,4)	130406	4446	24926	0.097	19.11	1699	22179	0.206	17.01

When applying RDT, the trigger crossing information in the cluster-head is required as the reference in the leaf nodes. The number of trigger crossings in each of the cluster-heads for the four local groups are indicated in Table 1, along with the amount data required for NExT. The maximum total transmitted data over the radio link in the case of new RDT with local extremum triggering condition is only 28% of NExT. In addition, compared to the positive-point triggering condition, it can be found that the local extremum triggering condition is capable of further reducing the transmitted data. Thus, the efficiency in the data communication can be significantly improved. The sensor topology in group 3 with eight local nodes of which two are overlapping nodes is optimally chosen for RDT to accurately estimate modal properties while maintaining a low number of trigger crossings by maximizing the G value.

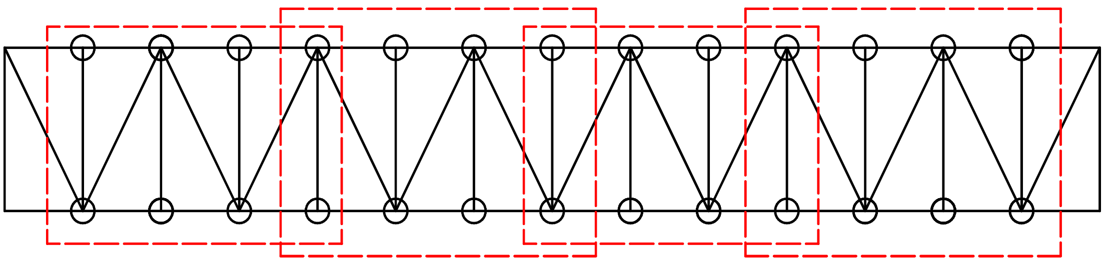

Fig. 4 Group 3 sensor topology (top view)

For the selected sensor topology, it can be found that MAC between the exact and estimated mode shapes obtained in RDT/ITD with local extremum triggering condition is consistent with those obtained in NExT/ITD, as listed in Table 2. Note that the 5^{th} and 6^{th} modes are not identified in both RDT- and NExT-based decentralized data aggregation approaches due to the low energy by the given responses. These two modes are dominated by a transverse motion; it is neither well excited nor well observed, because the input excitation is vertically applied and only vertical accelerations are measured. All identified MAC values are close to unit. As such, the decentralized implementation of RDT with local extremum triggering condition accurately estimates modal properties.

Table 2 MAC between the exact and estimated mode shapes

Mode	decentralized data aggregation	
	NExT	RDT (Local extremum Tri.)
1	0.9945	0.9994
2	0.9983	0.9977
3	0.9991	0.9886
4	0.9550	0.9663
7	0.9661	0.9776

5. CONCLUSIONS

The RDT-based decentralized data aggregation approach was improved for efficient modal identification from response data of structural system under nonstationary ambient vibration, and verified numerically on a three-dimensional truss model. In addition, the choice of the triggering condition is significant to reduce the data communication. The triggering condition is chosen whose trigger crossing has least data transmitting between the leaf nodes and the cluster head in a local sensor community. The performance of improved RDT was assessed in terms of (1) accuracy of the estimated modal properties and (2) efficiency in the wireless data communication. The NExT-based and standard RDT-based decentralized data aggregation approaches were selected as reference for comparison. Simulation results demonstrate the efficacy of the improved RDT-based decentralized data aggregation approach.

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