

Analytical Investigation of the Variation of Natural Frequencies with Temperature

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ABSTRACT

It is well known through the observations of field measurements that natural frequencies of a bridge structure vary depending on the temperature change. However, few analytical investigations have been carried out to verify this observed phenomenon. In addition, it is also known that temperature varies through the thickness of a bridge deck system as reflected in the design codes, but the effects of temperature change on the natural frequencies of a bridge structure have not been analytically investigated properly. This paper introduces an analytical approach to predict the variation of natural frequencies due to temperature change through the thickness of a bridge. The proposed analytical method is applied to evaluate the properness of the design codes by comparing the determined analytical natural frequencies with the measured data.

1. INTRODUCTION

It is a kind of common sense that natural frequencies of a bridge structure vary depending on the temperature change even though the determined level of variation is much different from one case to the other. Laboratory test results usually showed a high percentage variation in the natural frequencies due to temperature change (Kim *et al.*, 2003) while the natural frequencies of a long-span bridge usually do not vary much by 1~2% annual variation (Kim *et al.*, 2013). However, a few systematic approaches have been introduced in the literature to investigate this well-known phenomenon analytically. In addition, the temperature variation through the thickness as reflected in the design codes has not been much considered in the analytical investigations so far.

The paper reviews some available design codes considering the temperature variation through the thickness and introduces an analytical procedure with the some parameters influencing on the results. The proposed analytical method is examined by comparing the analytical results with the measured data with temperature variation through the thickness.

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2. REVIEW OF DESIGN CODES

Some available design codes reflecting the temperature variation through the thickness of a bridge are reviewed and compared.

2.1 Korean Bridge Design Codes (KBDC)

Currently both two different design codes are allowed in Korea; KDBC-2008 (KSCE, 2008) and the other new KBDC-2012 (KRTA, 2012) based on the limit state design concept. Thus, the temperature variations given in KBDC-2008 and KBDC-2012 are compared in Fig. 1 and Fig. 2, respectively. Basically KBDC-2008 does not much consider the variation of temperature through the thickness while KBDC-2012 allows parts of linear variations through the thickness.

2.2 AASHTO(2007)

AASHTO-2007 is basically the same as KBDC-2012 because KBDC-2012 adopts the same concept as described in Fig. 2.

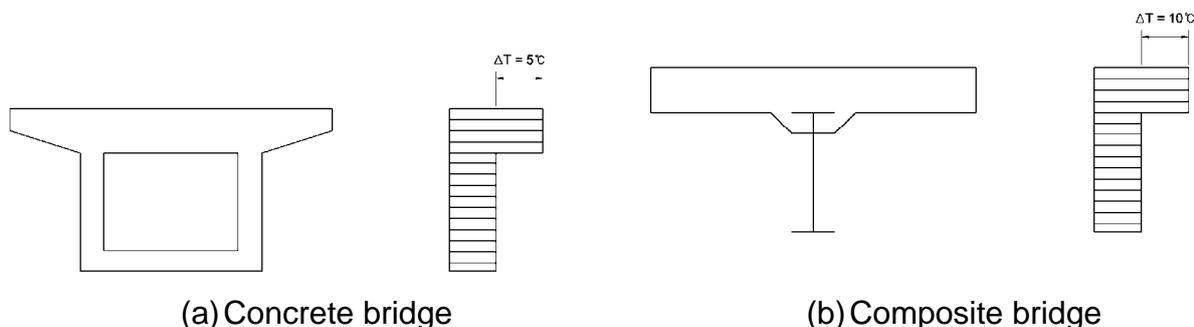


Fig. 1 Temperature variation in the design code of KBDC-2008

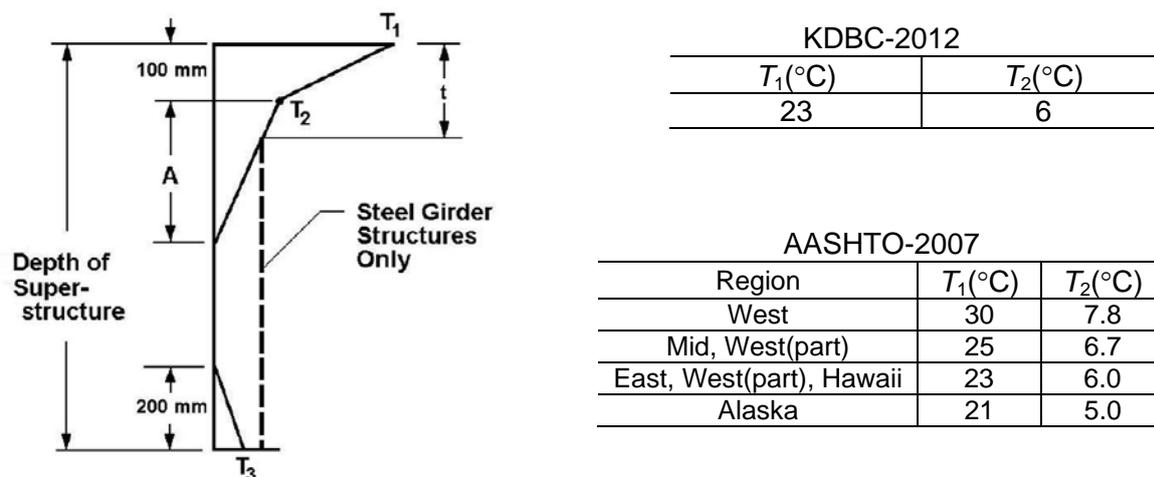


Fig. 2 Temperature variation in the design codes of KBDC-2012 and AASHTO-2007

3. ANALYTICAL APPROACH

3.1 Parameters considered

Three different parameters are considered in the analysis; including (a) initial thermal stresses, (b) structural deformation due to temperature change, and (c) variation in the elastic modulus of the materials. The previous investigation by Kim *et al.* (2013) demonstrated that the use of all these three parameters together could minimize the error between the analytical results and the measured data when a uniform temperature change through the section was assumed. The variation of elastic modulus of concrete was predicted by using an equation given by CEB-FIP design guideline as Eq. (1).

$$E_{ci}(T) = E_{ci}(1.06 - 0.003T / T_0), \quad (1)$$

where $E_{ci}(T)$ = elastic modulus at T , E_{ci} = elastic modulus at 20°C, and $T_0 = 1^\circ\text{C}$.

3.2 Procedure for the analysis

The procedure for the eigenvalue and stress analysis is described in the flowchart of Fig. 3. The first step for the analysis is the model-updating based on the measured response. After a model is updated, then each analytical procedure can be applied as described in the flowchart. For the static stress analysis, only the parameter of elastic modulus is considered in the current paper.

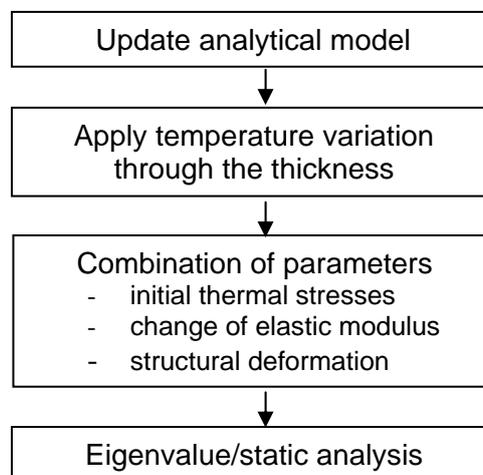


Fig. 3 Flowchart for the analysis

4. EXAMINATION OF THE PROPOSED ANALYTICAL APPROACH

4.1 Comparison of analytical results with the measured data

Xia *et al.* (2011) carried out a laboratory test on a simple RC slab shown in Fig. 4. The temperature variation through the thickness of the RC slab was measured as shown in the same figure. The temperature changed between 18.8°C and 36.7°C for almost one day.

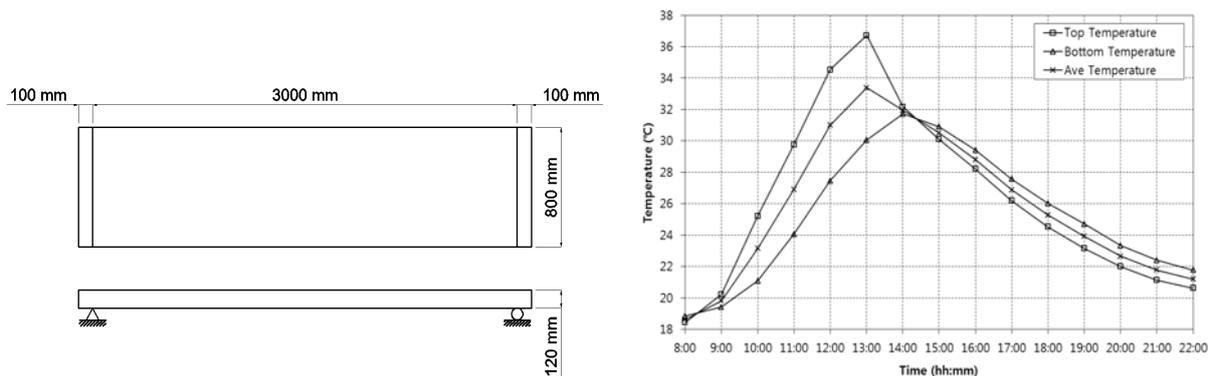


Fig. 4 RC slab for the laboratory test (Xia *et al.*, 2011)

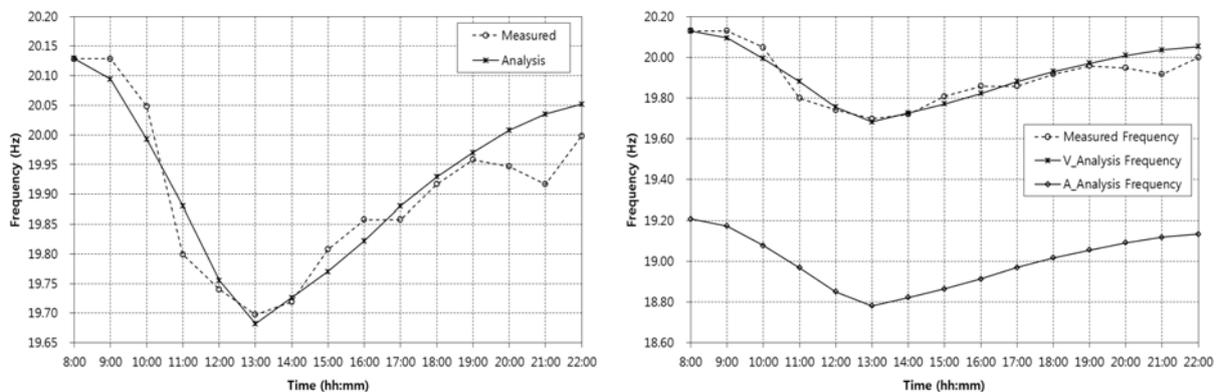


Fig. 5 Comparison of the first natural frequencies varying in time

The analytical natural frequencies and mode shapes were determined by assuming a linear temperature variation through the thickness because only the top and bottom temperatures were measured as shown in Fig. 4. The measured and analytical first natural frequencies varying in time are compared in Fig. 5. Even if some gaps can be observed around the end of the measurements, both data fit well in overall.

The right-hand figure of Fig. 5 compares the previous results with another analytical ones by assuming a uniform temperature through the thickness with the average value of the top and bottom temperatures at every time. A gap between the results can clearly demonstrate that the temperature variation through the thickness should be considered in the analysis.

4.2 Comparison of results through a simulation study on a PSC beam bridge

To compare the analytical results by two different design codes of KBDC-2008 and AASHTO-2007, a simulation study was carried out with a simply supported PSC beam bridge as shown in Fig. 6. KBDC-2012 is considered as the same as AASHTO-2007 so that it is skipped from the comparison. For the comparison, another temperature variation obtained by the thermal transfer analysis using ABAQUS was also applied to determine stress resultants and natural frequencies as shown in the left-hand figure of Fig. 7.

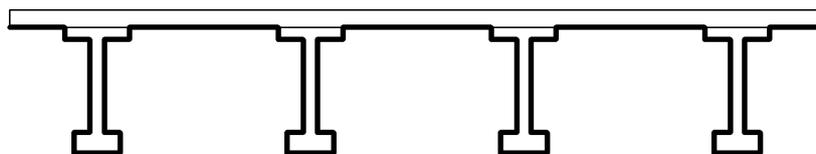


Fig. 6 A simply supported PSC beam bridge for the simulation study

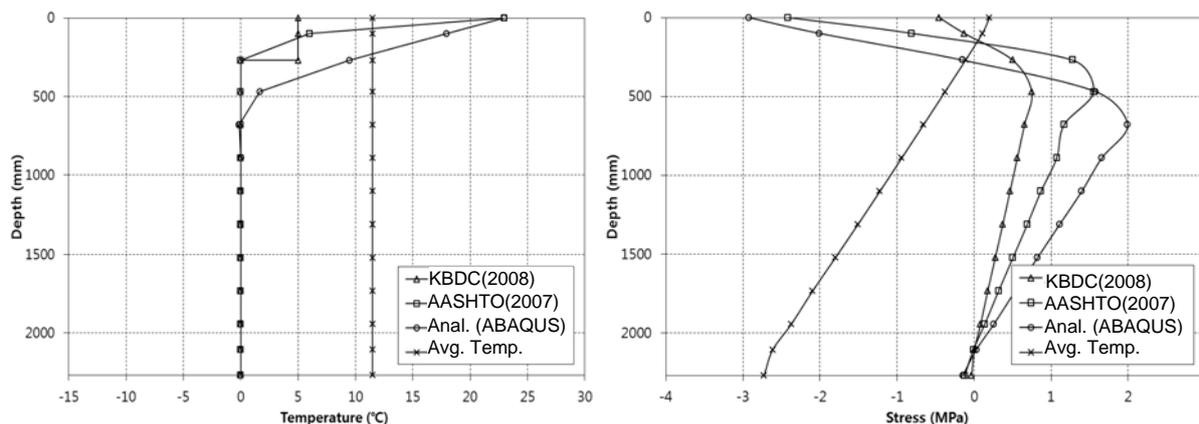


Fig. 7 Temperature distribution and the stress resultants through the thickness

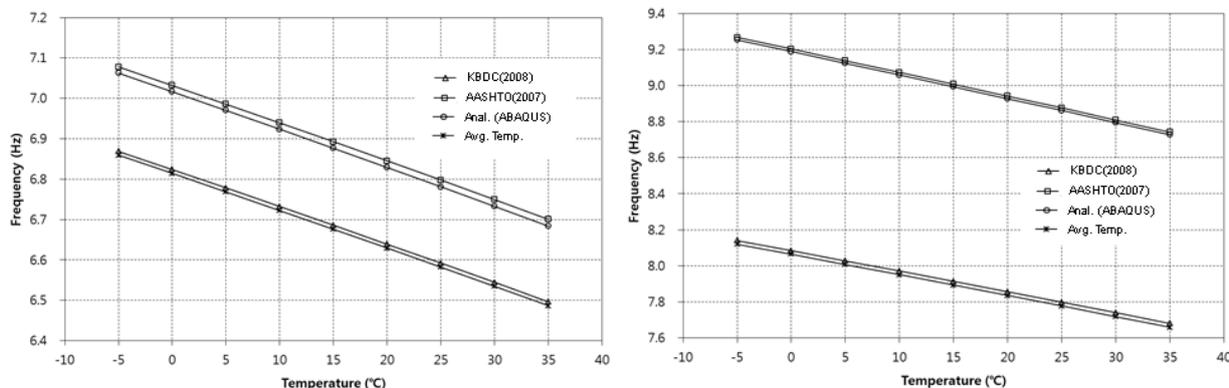


Fig. 8 Comparison of the first and the second natural frequencies.

The left-hand figure of Fig. 7 shows different temperature variations depending on the design codes. And the right-hand figure of Fig. 7 shows the computed stress resultants for each temperature variation through the thickness. At first, it can be observed that the stresses by the uniform temperature are linearly varying through the thickness but a little far from the other resultants. The stresses of the other cases sharply change around the top surface. Especially, two stress resultants by AASHTO-2007 and ABAQUS thermal analysis change very sharply around the top. Even though it is not cleared to conclude which case is closer to the actual situation, the resultants with a uniform temperature may be the worst results comparatively.

The computed natural frequencies with different temperature variations described in the left-hand figure of Fig. 7 are compared in Fig. 8 for the first and the second modes.

From both figures, it can be observed that the computed natural frequencies using AASHTO-2007 and ABAQUS thermal analysis are close each other, and the other two cases are close, respectively. From the experience of Fig. 5 of the comparison with measured data, however, it may be reasonable to conclude that the natural frequencies with temperature variation through the thickness by AASHTO-2007 or ABAQUS thermal analysis may represent the possible actual situation better.

5. CONCLUSIONS

A procedure of considering temperature change in time and variation through the thickness was proposed and examined through some case studies in the paper. The study on the comparison of analytically computed natural frequencies and the measured data clearly demonstrated that it is required to consider the temperature variation through the thickness. When a uniform temperature through the thickness of the slab was assumed, the natural frequencies were computed with the values equally distant from the measured data marching with time.

A simulation study to investigate the distribution of stress resultants through thickness was also carried out in the study by applying different design codes. Since it is reasonable to assume that AASHTO-2007 may provide better results, it could be concluded that the stresses vary sharply around the top surface with rapid temperature variation through the thickness.

ACKNOWLEDGEMENTS

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