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Recursive Bayesian filtering for displacement estimation via output-only vibration measurements

*Saeed Eftekhar Azam[1], Costas Papadimitriou[1], Eleni Chatzi[2]

[1] *Department of Mechanical Engineering, University of Thessaly, Volos, Greece*
[1] *eftekhar@uth.gr*
[2] *Institute of Structural Engineering, ETH Zürich, Zürich, Switzerland*

ABSTRACT

This paper contributes to the prediction of fatigue damage accumulation in metallic structures that undergo vibrations due to unknown input forces during their operational life. In estimating the strain in a point of interest, the displacement at that point and its vicinity would be needed; therefore, a reliable state estimate could lead to reliable fatigue damage identification. The uncertainties that arise due to sensor noise, numerical modeling and the unknown excitation render the unknown state and input estimation a challenging task; rendering the research to achieve a robust solution still in progress. A number of methods and techniques in existing literature have shown promising results in overcoming the issues associated with these uncertainties; however, they are susceptible to instabilities stemming from unobservability issues and accumulation of integration errors in the estimates. In this paper, a dual implementation of the Kalman filter is proposed to estimate the unknown input and states of a linear state-space model. The successive structure of the suggested filter prevents numerical issues attributed to unobservability and rank deficiency, and the regulatory parameter for input estimation furnishes a tool to avoid the so-called drift in the estimated input. Via proper calibration of the proposed tool, a reasonable estimate of the state and thus the fatigue damage could be accomplished.

1. INTRODUCTION

This paper contributes to the problem of state estimation in the entire body of the metallic structures that undergo vibrations due to unknown input forces during their operational life, aiming at prediction of fatigue damage identification. The idea of using the estimated response of the structures for fatigue damage identification was first suggested by Papadimitriou et al. (Papadimitriou, Fritzen, Kraemer, & Ntotsios, 2011); where a technique was introduced that employs the Kalman filter for estimating power spectral densities of the strain in the body of the structure thereby predicting the remaining fatigue life. To estimate the fatigue damage, a time history of the strains in the hotspot points of the structure is required. To estimate the strain in a point of interest, the displacement field at that point is needed; therefore, a reliable state estimate could lead to reliable fatigue damage identification.

The topic of estimation of the states of a partially observed dynamic system in an stochastic frame has been studied by many scientists and there are well developed algorithms to manage both linear (e.g. the Kalman filter (Kalman, 1960)) and nonlinear (e.g. the particle filter (Gordon, Salmond, & Smith, 1993), the unscented Kalman filter (Julier & Uhlmann, 1997)) state-space models. In dealing with structural systems, the states of the system typically comprise displacements and velocities of the response of the system at some points, namely degrees-of-freedom (DOF) on the structure. In practical cases, it is difficult or sometimes impossible to measure displacements and velocities of the system. Hence, when knowledge of the displacements and velocities is required, a state estimation algorithm could be used to infer these quantities from the commonly measured acceleration response, which depends on the states and input of the system through some relation. Among other researchers, Ching and Beck (J. Ching & Beck, 2007), Hernandez (Hernandez, 2011) Smyth and Wu (Smyth & Wu, 2007) and (Reynders & Roeck, 2008) have proposed frameworks for estimating the unknown states of a system using heterogeneous, noisy or incomplete observations, whereas other works also focus on the detection of damage (Gao & Lu, 2006), (Bernal, 2013) though limited structural feedback.

The structural identification task is often one of nonlinear state estimation and parameter identification. In civil engineering the extended Kalman filter (EKF) has been the de facto standard in the past mainly due to its ease of implementation, robustness and suitability for real-time applications. In recent years, however, many alternative techniques have been proposed. In a first extension for alleviating the issues that arise through linearization in the EKF, Julier and Uhlmann (Julier & Uhlmann, 1997) have proposed the unscented Kalman filter (UKF), in which the evolution of the statistics of the state of the system is performed through a sampling scheme. It has been shown by Mariani and Ghisi (Mariani & Ghisi, 2007) that at the price of a higher computational burden, the UKF outperforms the EKF dealing with nonlinear parameter identification problems. Further works demonstrate the potential and versatility of particle-based methods within the context of nonlinear system identification (Eftekhar Azam, Ghisi, & Mariani, 2012) (Jianye Ching, Beck, & Porter, 2006), (E. N. Chatzi & Smyth, 2009), (Eftekhar Azam, Bagherinia, & Mariani, 2012), (E. N. Chatzi & Smyth, 2013), (Eftekhar Azam & Mariani, 2012)..

Although the joint state and parameter identification task is a subject frequently addressed in recent years, the joint identification of state and input information is a topic less treated so far in the literature. Since, the uncertainties that arise due to sensor noise, lacking or imprecise numerical models and the unknown excitations render the unknown input and state estimation a challenging task, the research to achieve a robust solution is still in progress. It this paper, the latter source of uncertainty, i.e. the lack of information regarding the input to the system is the core of the study. In practice, one common approach is to assume the unknown input as a zero mean white Gaussian process and make use of the aforementioned Bayesian techniques for state estimation; however, in many cases this assumption is violated and therefore it may lead to major adverse effects on estimation accuracy. To address this issue, a number of optimal filtering techniques in the presence of unknown input have been proposed. In a pioneering work, Kitanidis developed an unbiased minimum-variance recursive filter for input and state estimation of linear systems without direct transmission; his algorithm did not make any a-priori assumption on the input (Kitanidis, 1987). The latter filter is not globally optimal in the mean square error sense. Hsieh has proposed a new formulation of the Kitanidis filter which is more convenient for practical applications (Hsieh, 2000). Gillijns and De Moor proposed a new filter for joint input and state estimation for linear systems without direct transmission (Gillijns & De Moor, 2007a). Their filter is globally optimal in the minimum-variance unbiased sense. Later Gillijns and De Moor developed a new formulation of the aforementioned filter which included a direct transmission term in its structure (Gillijns & De Moor, 2007b).

In more recent years, Lourens et al. (Lourens, Papadimitriou, et al., 2012) have proposed an extension of the method developed in (Gillijns & De Moor, 2007b) to cope with the numerical instabilities that arise when the number of sensors surpasses the order of the model, i.e. when a large number of sensors is used in combination with a reduced-order model assembled from a relatively small number of modes. This is commonly the case for structural identification problems. The modified algorithm was used to predict and estimate the input force and accelerations of a simulated steel beam, a laboratory test beam and a large-scale steel bridge. It was reported that, although the algorithm provides a reasonable prediction of the accelerations, the input force estimates are affected by spurious low frequency components that must be filtered out in this case. It is worth noting, that in dealing with joint state and parameter estimation, Chatzi and Fuggini (E. F. Chatzi, Clemente;, Accepted for publication) have proposed a technique to cope with the issues related to the spurious low frequency components in the displacement estimates by introducing artificial displacement measurements into the observation vector. Lourens et al. (Lourens, Reynders, De Roeck, Degrande, & Lombaert, 2012) have proposed an augmented Kalman filter (AKF) for unknown force identification in structural systems, and concluded that the augmented Kalman filter is prone to numerical instabilities due to un-observability issues of the augmented system matrix.

In this paper, a dual implementation of the Kalman filter is proposed to estimate the unknown input and states of a linear state-space model. It is assumed that a limited number of noisy acceleration measurements are available. The successive structure of the suggested filter prevents numerical problems attributed to unobservability and rank deficiency of the augmented Kalman filter. Additionally, it is shown that the expert guess on the covariance of the unknown input provides a tool for avoiding the so-called drift effect in the estimated input force and displacements. The drift is linked to the integral nature of these quantities in the presence of acceleration information. The effectiveness and performance of the proposed method is ascertained via a pseudo-experimental analysis carried out on a test shear building. It is concluded that, by fine-tuning the covariance of the fictitious process noise of the unknown input, a successful estimation of the state and thus the fatigue damage can be accomplished.

The paper starts with a section devoted to a brief formulation of the state-space equations for linear dynamical systems. The next section introduces the dual scheme by use of the Kalman filter for estimation of both the unknown input and state of linear state-space models and is followed by a section on the numerical comparison of the dual Kalman filter and the filter proposed by Gillijn and De Moor.

2. Mathematical formulation of the problem

A linear structural dynamics problem is typically formulated using the following continuous time second order differential equation:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{f}(t) = \mathbf{S_p}\mathbf{p}(t) \tag{1}$$

where $\mathbf{u}(t) \in \mathbb{R}^n$ denotes the displacement vector and $\mathbf{K}$, $\mathbf{C}$ and $\mathbf{M} \in \mathbb{R}^{n\times n}$ stand for the stiffness, damping and mass matrix, respectively. $\mathbf{f}(t) \in \mathbb{R}^n$ is the excitation force, which herein is presented as a superposition of time histories $\mathbf{p}(t) \in \mathbb{R}^m$ that are influencing some degrees-of-freedom on the structure as indicated via the $\mathbf{S_p} \in \mathbb{R}^{n\times m}$ matrix, termed the influence matrix.

In practice, when dealing with fine resolution finite element (FE) models, the dimension of the state vector in the Eq. (1) may become relatively large; nonetheless the dynamics of the system could effectively be captured by a significantly smaller number of modes. To suppress the computational costs associated with the large FE models, Eq. (1) is projected in the subspace spanned by a limited number of the undamped eigenmodes of the system. In this regard, consider the eigenvalue problem corresponding to Eq. (1):

$$\mathbf{K\Phi} = \mathbf{M\Phi\Omega}^2 \tag{2}$$

Transforming the coordinate system of Eq. (1) via the following mapping:

$$\mathbf{u}(t) = \mathbf{\Phi}\mathbf{z}(t)$$

where $\mathbf{z}(t) \in \mathbb{R}^m, \mathbf{\Phi} \in \mathbb{R}^{n\times m}$ and pre multiplying by $\mathbf{\Phi}^{\mathrm{T}}$ yields:

$$\mathbf{\Phi}^{\mathrm{T}}\mathbf{M\Phi}\ddot{\mathbf{z}}(t) + \mathbf{\Phi}^{\mathrm{T}}\mathbf{C\Phi}\dot{\mathbf{z}}(t) + \mathbf{\Phi}^{\mathrm{T}}\mathbf{K\Phi}\mathbf{z}(t) = \mathbf{\Phi}^{\mathrm{T}}\mathbf{f}(t) = \mathbf{\Phi}^{\mathrm{T}}\mathbf{S_p}\mathbf{p}(t) \tag{3}$$

By imposing the mass normalization condition $\mathbf{\Phi}^{\mathrm{T}}\mathbf{M\Phi} = \mathbf{I}$, considering $\mathbf{\Phi}^{\mathrm{T}}\mathbf{K\Phi z} = \mathbf{\Omega}^2$ and assuming the damping is proportional, the Eq. (6) can be rewritten:

$$\ddot{\mathbf{z}}(t) + \mathbf{\Gamma}\dot{\mathbf{z}}(t) + \mathbf{\Omega}^2\mathbf{z}(t) = \mathbf{\Phi}^{\mathrm{T}}\mathbf{f}(t) = \mathbf{\Phi}^{\mathrm{T}}\mathbf{S_p}\mathbf{p}(t) \tag{4}$$

where the components of j[th] entry of the diagonal damping matrix $\mathbf{\Gamma}$ are of the form $2\xi_j\omega_j$, in which ξ_j stands for the relevant modal damping ratio. Apparently, a truncated modal space could be substituted in Eq. (4). The aforementioned equation can be discretized in time to constitute a state-space equation, and in so doing the following state vector is introduced:

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{u}(t) \\ \dot{\mathbf{u}}(t) \end{bmatrix}$$

consequently, Eq. (1) can be written in the following form to define the process equation:

$$\dot{\mathbf{x}}(t) = \mathbf{A}_c\mathbf{x}(t) + \mathbf{B}_c\mathbf{p}(t) \tag{5}$$

where the system matrices are:

$$\mathbf{A}_c = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}$$
$$\mathbf{B}_c = \begin{bmatrix} \mathbf{0} \\ -\mathbf{M}^{-1}\mathbf{S_p} \end{bmatrix}$$

Regarding the measurement equation let us consider the most general case by assuming that a combination of the displacements, velocities and accelerations can be measured. Hence, the measurement vector $\mathbf{d}(t)$ assumes the following form:

$$\mathbf{d}(t) = \begin{bmatrix} \mathbf{S_d} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{S_v} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{S_a} \end{bmatrix} \begin{bmatrix} \mathbf{u}(t) \\ \dot{\mathbf{u}}(t) \\ \ddot{\mathbf{u}}(t) \end{bmatrix} \tag{6}$$

where $\mathbf{S_d}$, $\mathbf{S_v}$ and $\mathbf{S_a} \in \mathbb{R}^n$ are the selection matrices for the displacements, velocities and accelerations, respectively. By using equation of motion, Eq. (6) could be transformed into state-space form:

$$\mathbf{d}(\mathrm{t}) = \mathbf{G}_c\mathbf{x}(\mathrm{t}) + \mathbf{J}_c\mathbf{p}(\mathrm{t}) \tag{7}$$

where the output influence matrix and the direct transmission matrix are:

$$\mathbf{G}_c = \begin{bmatrix} \mathbf{S_d} & \mathbf{0} \\ \mathbf{0} & \mathbf{S_v} \\ \mathbf{S_a}\mathbf{M}^{-1}\mathbf{K} & \mathbf{S_a}\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}$$
$$\mathbf{J}_c = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{S_a}\mathbf{M}^{-1}\mathbf{S_p} \end{bmatrix}$$

Recombining Eqs. (5) and (7) through use of the relevant matrices, results into the full order state-space equations that are required to implement the input and state estimation algorithm. In case a reduced order state-space model is needed, a truncated eigenvector space must be substituted in Eq. (3); hence the following variable transformation would be necessary:

$$\mathbf{x}(\mathrm{t}) = \begin{bmatrix} \mathbf{\Phi}_{\mathrm{r}} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_{\mathrm{r}} \end{bmatrix} \boldsymbol{\zeta}(\mathrm{t})$$

where $\boldsymbol{\zeta}(t)$ is the reduced modal state vector:

$$\boldsymbol{\zeta}(\mathrm{t}) = \begin{bmatrix} \mathbf{z}(\mathrm{t}) \\ \dot{\mathbf{z}}(\mathrm{t}) \end{bmatrix}$$

The reduced modal state-space equation in continuous time will have the following form:

$$\dot{\boldsymbol{\zeta}}(\mathrm{t}) = \mathbf{A}_{\mathrm{c}}\boldsymbol{\zeta}(\mathrm{t}) + \mathbf{B}_{\mathbf{c}}\mathbf{p}(\mathrm{t}) \quad (8)$$
$$\mathbf{d}(\mathrm{t}) = \mathbf{G}_{\mathrm{c}}\boldsymbol{\zeta}(\mathrm{t}) + \mathbf{J}_{\mathrm{c}}\mathbf{p}(\mathrm{t}) \quad (9)$$

while the relevant system matrices read:

$$\mathbf{A}_{\mathrm{c}} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{\Omega}^2 & -\mathbf{\Gamma} \end{bmatrix}, \quad \mathbf{B}_{\mathrm{c}} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{\Phi}_{\mathbf{r}}^{\mathrm{T}}\mathbf{S}_{\mathbf{p}} \end{bmatrix}, \quad \mathbf{G}_{\mathrm{c}} = \begin{bmatrix} \mathbf{S}_{\mathbf{d}}\mathbf{\Phi}_{\mathrm{r}} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_{\mathbf{v}}\mathbf{\Phi}_{\mathrm{r}} \\ \mathbf{S}_{\mathbf{a}}\mathbf{\Phi}_{\mathrm{r}}\mathbf{\Omega}^2 & \mathbf{S}_{\mathbf{a}}\mathbf{\Phi}_{\mathrm{r}}\mathbf{\Gamma} \end{bmatrix}, \quad \mathbf{J}_{\mathrm{c}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{S}_{\mathbf{a}}\mathbf{\Phi}_{\mathrm{r}}\mathbf{\Phi}_{\mathbf{r}}^{\mathrm{T}}\mathbf{S}_{\mathbf{p}} \end{bmatrix}$$

To discretize Eqs. (8) and (9), the sampling rate is denoted by $1/_{\Delta\mathrm{t}}$ and the discrete time instants are defined at $\mathrm{t}_k = \mathrm{k}\,\Delta\mathrm{t}$, for $\mathrm{k} = 1, \ldots, \mathrm{N}$. The discrete state-space equation can be expressed by following notation:

$$\boldsymbol{\zeta}_{\mathbf{k+1}} = \mathbf{A}\boldsymbol{\zeta}_{\mathbf{k}} + \mathbf{B}\mathbf{p}_{\mathbf{k}} \quad (10)$$
$$\mathbf{d}_{\mathbf{k}} = \mathbf{G}\boldsymbol{\zeta}_{\mathbf{k}} + \mathbf{J}\mathbf{p}_{\mathbf{k}} \quad (11)$$

where:

$\mathbf{A} = \boldsymbol{e}^{\mathbf{A}_{\mathrm{c}}\Delta\mathrm{t}}$, $\mathbf{B} = [\mathbf{A} - \mathbf{I}]\mathbf{A}_{\mathbf{c}}^{-\mathbf{1}}\mathbf{B}_{\mathrm{c}}$, $\mathbf{G} = \mathbf{G}_{\mathrm{c}}$ and $\mathbf{J} = \mathbf{J}_{\mathrm{c}}$.

3. Recursive Input and state estimation algorithm

Consider the following discrete time state-space equation:

$$\mathbf{x_{k+1}} = \mathbf{A}\mathbf{x_k} + \mathbf{B}\mathbf{p_k} + \mathbf{v_k^x} \tag{12}$$
$$\mathbf{d_k} = \mathbf{G}\mathbf{x_k} + \mathbf{J}\mathbf{p_k} + \mathbf{w_k} \tag{13}$$

where , $\mathbf{v_k^x}$ is the process noise assumed, zero-mean, normally distributed as $\mathbf{v_k^x} \sim \mathbf{N}(\mathbf{0}, \mathbf{Q}^{\mathrm{x}})$, and $\mathrm{w_k}$ is the zero mean, white, measurement noise $\mathrm{w_k} \sim \mathbf{N}(\mathbf{0}, \mathbf{R})$**.** The problem at hand is to estimate the unknown input $\mathbf{p_k}$ and the hidden, or partially observed, state $\mathbf{x_k}$ of the system using the noisy observations $\mathbf{d_k}$ in an online fashion. In doing so, a dual implementation of the Kalman filter is proposed in this section. The proposed scheme could be divided into two stages, with the Kalman filter pertaining to both stages. At each time iteration, a fictitious process equation serving for calibration of the parameters of the system is introduced:

$$\mathbf{p_{k+1}} = \mathbf{p_k} + \mathbf{v_k^p} \tag{14}$$

where $\mathbf{v_k^p}$ is a zero mean white Gaussian process with an associated covariance matrix $\mathbf{Q^p}$. Now, assume that an estimation of the state at time t_k is available; by using Eqs. (13) and (14), a new state-space equation can be obtained, where, the observed quantity is $\mathbf{d_k}$, the new state is $\mathbf{p_k}$ and the actual sought-for state $\mathbf{x_k}$ plays the role of a known input to the system:

$$\mathbf{p_{k+1}} = \mathbf{p_k} + \mathbf{v_k^p}$$
$$\mathbf{d_k} = \mathbf{G}\mathbf{x_k} + \mathbf{J}\mathbf{p_k} + \mathbf{w_k}$$

Through implementation of the Kalman filter, an online estimation of $\mathbf{p_{k+1}}$ could be obtained. Then, once the estimation of $\mathbf{p_{k+1}}$ is performed, it can in a next step be substituted in Eqs. (12) and (13), and a subsequent Kalman filter implementation could be used for estimating $\mathbf{x_{k+1}}$. The general scheme is described in detail in Table 1.

At this point, it is worth noting that, the procedure needs a-priori information on expected value and covariance of the state and input at time $\mathrm{t_0}$. Moreover, similar to the augmented Kalman filter (AKF), the value of the process noise $\mathbf{Q^p}$ for Eq. (14) must be properly chosen so that an accurate estimate of the unobserved state and the unknown input could be achieved. In the jargon of system identification, the covariance noise of the sought-for parameter is sometimes called the tuning knob of the system, and typically heuristic and ad-hoc guidelines are prescribed for a proper adjustment (Bittanti & Savaresi, 2000; Rajamani & Rawlings, 2009). Methods relying on the use of Bayesian techniques, maximizing the likelihood of measurements with respect to the noise parameters have also recently been proposed (Yuen, Hoi, & Mok, 2007). It is additionally, helpful to clarify the nature of the influence of the covariance matrices $\mathbf{Q}^{\mathrm{x}}$, $\mathbf{Q^p}, \mathbf{R}$. The process noise covariance matrices reveal the confidence put on the utilized model of the system. The lower this is, the more accurate the model is considered to be. The observation noise covariance reveals the confidence put in the acquired measurements. The lower this is, the tighter the estimator is forced to fit the recorded data.

In what follows, by use of an illustrative numerical example, the performance of the proposed algorithm, i.e. the Dual Kalman Filter (DKF) formulation, is evaluated against the Gillijns and De Moor filter (GDF), which is deemed as the most stable attempt so far in addressing the joint input and state estimation problem. It will be shown that once the covariance of the fictitious process equation of the input force is tuned properly, the so-called drift in the estimates of the input force and the displacements is disappeared. Moreover, it is shown that the successive structure of the dual Kalman filter does not trigger unobservability issues of the AKF.

Table 1: the general scheme of the two-stage Kalman filter-based input and state estimation algorithm

- Initialization at time t_0: $\hat{\mathbf{p}}_0 = \mathbb{E}[\mathbf{p}_0]$ $\mathbf{P}_0^{\mathbf{p}} = \mathbb{E}[(\mathbf{p}_0 - \hat{\mathbf{p}}_0)(\mathbf{p}_0 - \hat{\mathbf{p}}_0)^{\mathrm{T}}]$ $\hat{\mathbf{x}}_0 = \mathbb{E}[\mathbf{x}_0]$ $\mathbf{P}_0 = \mathbb{E}[(\mathbf{x}_0 - \hat{\mathbf{x}}_0)(\mathbf{x}_0 - \hat{\mathbf{x}}_0)^{\mathrm{T}}]$
- At time t_k, for $k = 1, \ldots, N_t$:   • Prediction stage for the input: 1. Evolution of the input and prediction of covariance input: $\mathbf{p}_k^- = \mathbf{p}_{k-1}$ $\mathbf{P}_k^{p-} = \mathbf{P}_{k-1}^{p} + \mathbf{Q}^{\mathbf{p}}$ • Update stage for the input: 2. Calculation of Kalman gain for input: $\mathbf{G}_k^{p} = \mathbf{P}_k^{p-}\mathbf{J}^{\mathbf{T}}\left(\mathbf{J}\mathbf{P}_k^{p-}\mathbf{J}^{\mathbf{T}} + \mathbf{R}\right)^{-1}$ 3. Improve predictions of input using latest observation: $\hat{\mathbf{p}}_k = \mathbf{p}_k^- + \mathbf{G}_k^{p}(\mathbf{d}_k - \mathbf{G}\,\hat{\mathbf{x}}_{k-1} - \mathbf{J}\mathbf{p}_k^-)$ $\mathbf{P}_k^{p} = \mathbf{P}_k^{p-} - \mathbf{G}_k^{p}\mathbf{J}\mathbf{P}_k^{p-}$ • Prediction stage for the state: 4. Evolution of state and prediction of covariance of state: $\mathbf{x}_k^- = \mathbf{A}\,\hat{\mathbf{x}}_{k-1} + \mathbf{B}\hat{\mathbf{p}}_k$ $\mathbf{P}_k^- = \mathbf{A}\mathbf{P}_{k-1}\mathbf{A}^{\mathbf{T}} + \mathbf{Q}^{\mathbf{x}}$ • Update stage for the state: 5. Calculation of Kalman gain for state: $\mathbf{G}_k^{\mathbf{x}} = \mathbf{P}_k^-\mathbf{G}^{\mathbf{T}}(\mathbf{G}\mathbf{P}_k^-\mathbf{G}^{\mathbf{T}} + \mathbf{R})^{-1}$ 6. Improve predictions of state using latest observation: $\hat{\mathbf{x}}_k = \mathbf{x}_k^- + \mathbf{G}_k^{\mathbf{x}}(\mathbf{d}_k - \mathbf{G}\mathbf{x}_k^- - \mathbf{J}\hat{\mathbf{p}}_k)$ $\mathbf{P}_k = \mathbf{P}_k^- - \mathbf{G}_k^{\mathbf{x}}\mathbf{G}\mathbf{P}_k^-$

4. Simulated example

To assess the performance of the proposed algorithm, a 4 DOF shear building (see Figure 1) with the following system properties is adopted: the value of the mass of each floor is assumed to be $625\ tones$, and the inter-storey stiffness of each floor is equal to 10^9 Kgf/m. Additionally, the modal damping ratio of each mode is assumed to be 2%.

Throughout the numerical analysis section, it is assumed that only accelerations of the response of the structure at some of the storey levels are available. This is the common case in structural dynamics; in practice the displacements and velocities are difficult, or even sometimes impossible to measure. Therefore, the problem lies in estimating the displacements and velocities of all stories of the structure by using noisy observations acquired from acceleration sensors. In this section, it is assumed that the acceleration time history of the last floor is measured. The optimization of the spatial distribution of the sensors (Papadimitriou & Lombaert, 2012) is not within the scope of this research but would be an interesting issue to explore in a follow-up investigation.

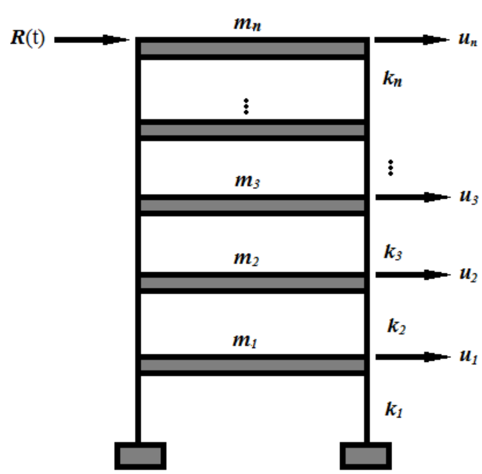

Figure 1: schematic view of a shear-type building

To account for measurement errors, a zero mean white noise is added to the results of the direct analysis of structure. By varying the level of the covariance of the added noise different signal-to-noise ratios could be obtained. The first four undamped natural frequencies of the system are reported in the Table 2.

The common trend in the state-of-the-art algorithms for unknown input and state estimation available in the literature (e.g. (Gillijns & De Moor, 2007a, 2007b; Hsieh, 2000;

Kitanidis, 1987)), is to avoid using any a-priori knowledge on the statistics of the input force, in an attempt to render the online estimation more practical.

In order to compare the performance of different schemes, the time histories of the sought-for states are cross-compared. In the following examples, it is assumed that a single noisy acceleration observation from the 4^{th} floor of the shear building is available, and as for the enforced excitation, and a harmonic load is applied to the same floor.

Table 2: The first four undamped natural frequencies of the structure

vibration mode index	1	2	3	4
Undamped natural frequency (Hz)	2.21	6.37	9.75	11.96

Regarding harmonic excitation, assume that a constant amplitude sinusoidal excitation is applied to the last floor of the building:

$$a_m \sin 2\pi\omega t$$

where a_m denotes the amplitude of the force and ω stands for the associated frequency. In current study, the $\omega = 0.5$ and $a_m = 5 \times 10^7\ Kgf$. Figure 2 shows the observed process, which is the actual acceleration time history of the last floor contaminated with a zero mean white Gaussian process featuring a standard deviation equal to $0.01\ {}^{m}/_{s^2}$.

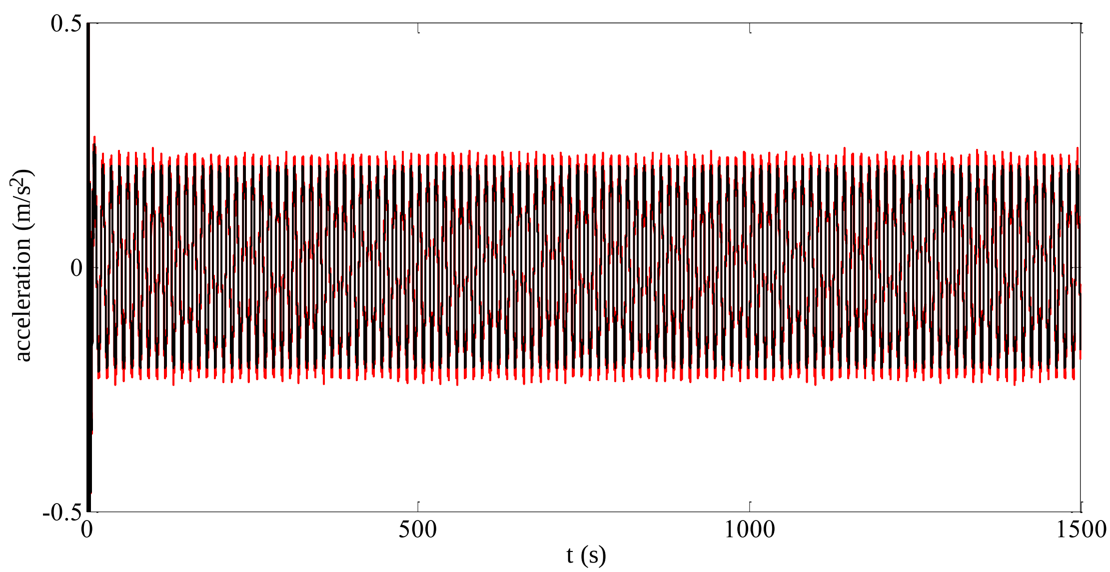

Figure 2: Noisy acceleration time history of the last floor

Figure 3, shows the estimated displacement time histories of the fourth floor furnished by AKF, GDF and DKF. The $\mathbf{P}_0$, $\mathbf{Q}^{\mathbf{x}}$ and $\mathbf{R}$ are set to 10^{-10}, 10^{-10} and 10^{-4}, respectively, whereas the $\mathbf{Q}^{\mathbf{p}}$ used in DKF is set to $10^{12} Kgf^2$. Concerning the AKF, the variations of the latter parameter does not result in an improved estimation, hence the results are shown are relevant to same value used for DKF. It is apparently seen that, the estimates provided by AKF are severely affected by unobservability of the displacement as the filter fails to provide any estimate of it. The estimates furnished by GDF are also affected by a low frequency trend, which is attributed to the accumulation of the observation errors in the double integration. However, the DKF seems to appropriately cope with issues observed in AKF and GDF.

The input force estimations by AKF, GDF and DKF are shown in Figure 4.The same trend observed in displacement time histories is seen in the force estimation as well.

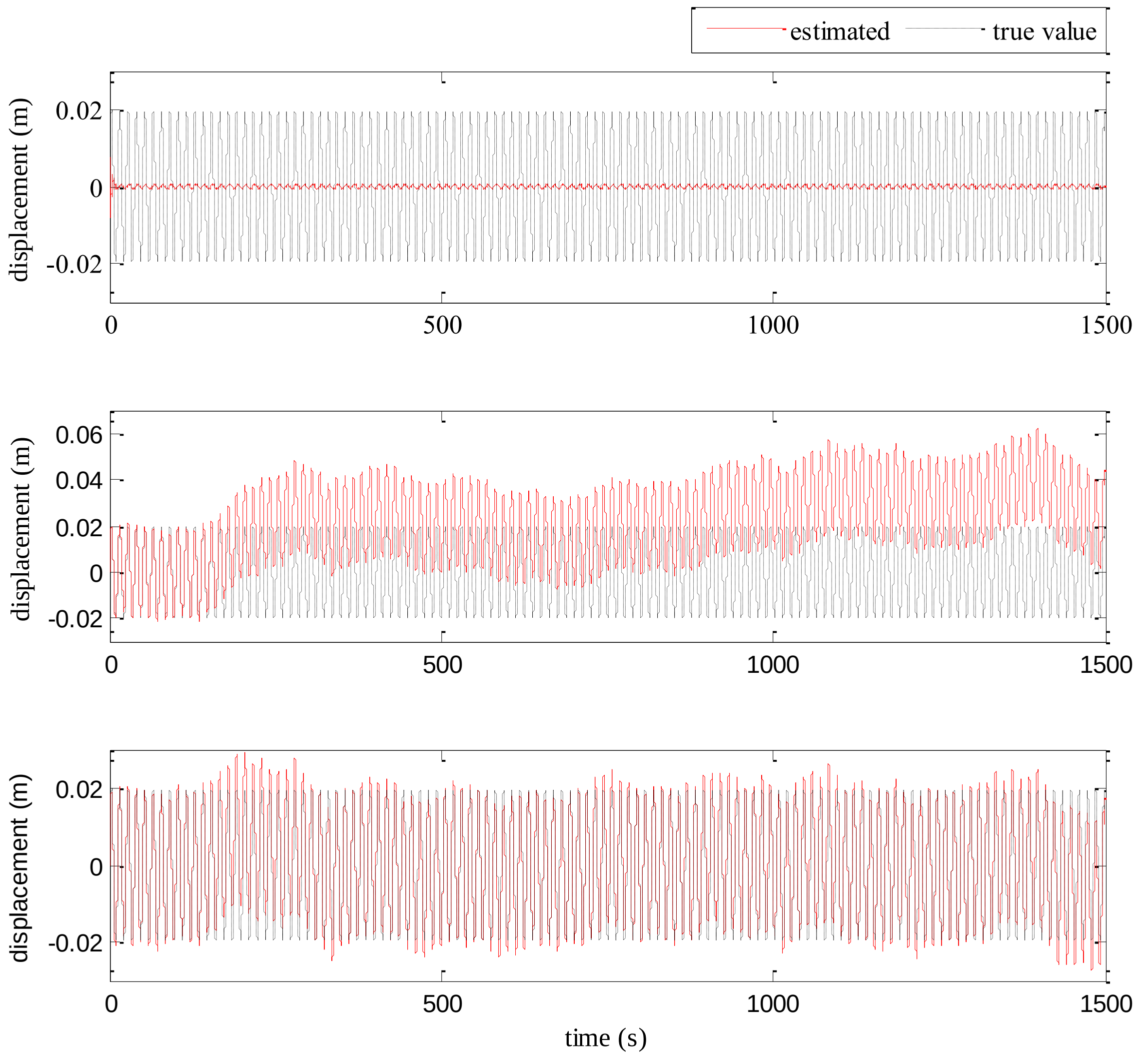

Figure 3: Displacement time history of the fourth floor estimated by the AKF (top), GDF (middle) and DKF (bottom) in the case of the unknown input

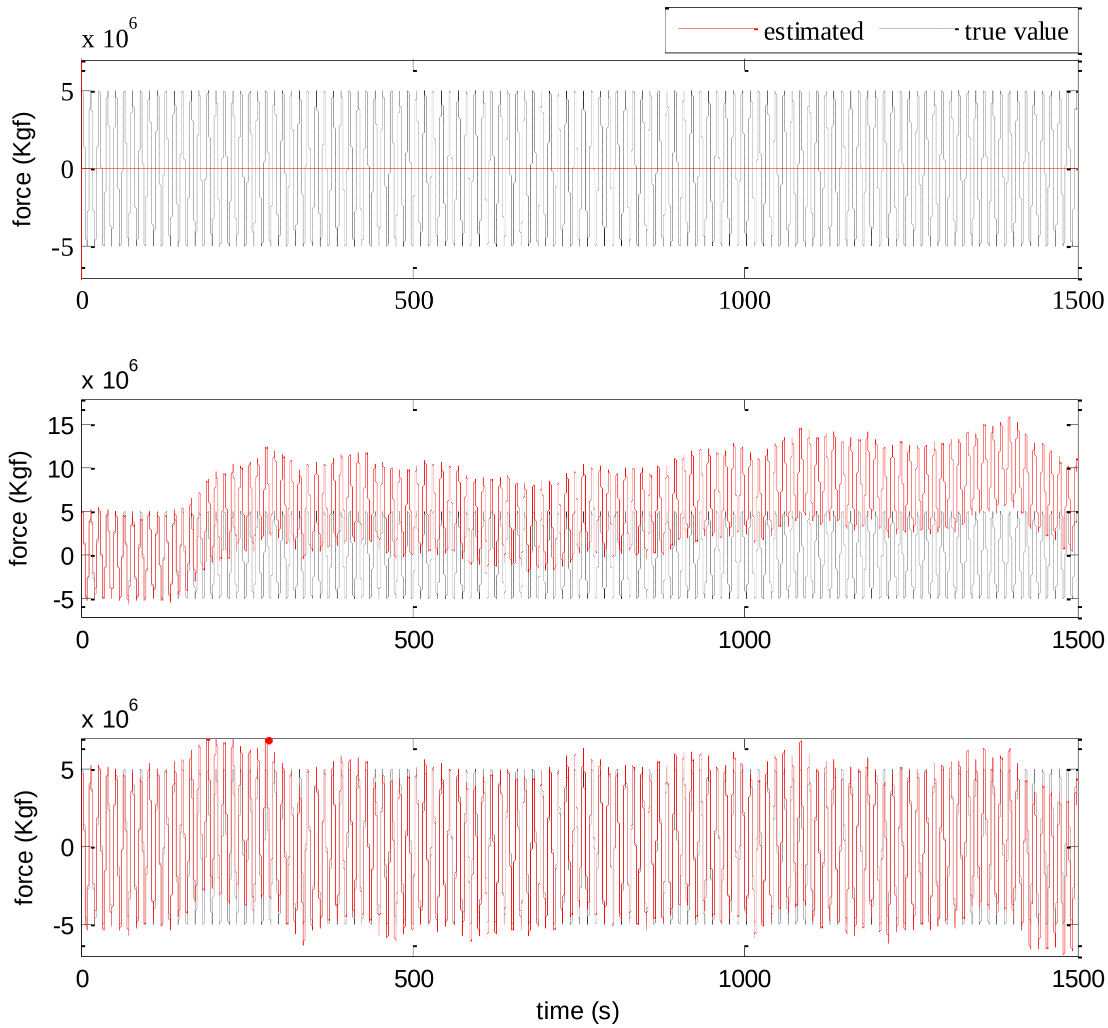

Figure 4: Input force time history of the fourth floor estimated by the AKF (top), GDF (middle) and the DKF (bottom) in the case of the unknown input (AKF)

5. Conclusions and remarks

In this paper, a dual implementation of the Kalman filter is proposed to estimate the unknown input and states of a linear state-space model. The effectiveness of the proposed filter is investigated through a pseudo-experimental tests. The proposed filter is confronted with the most recent methods applied to the problem. It is shown that, the successive structure of the suggested filter prevents numerical issues attributed to un-observability and rank deficiency, and the regulatory parameter for input estimation furnishes a tool to avoid the so-called drift in the estimated input. By fine-tuning the regulatory parameters of the proposed technique, a reasonable estimate of the state and thus the fatigue damage could be accomplished.

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7. References

Bernal, D. (2013). Kalman filter damage detection in the presence of changing process and measurement noise. *Mechanical Systems and Signal Processing, 39*(1-2), 361-371.

Bittanti, S., & Savaresi, S. M. (2000). On the parameterization and design of an extended Kalman filter frequency tracker. *IEEE Transactions on Automatic Control, 45*(9), 1718-1724.

Chatzi, E. F., Clemente;. (Accepted for publication). Online correction of drift in Structural Identification using Artificial White Noise observations and an Unscented Kalman Filter. *Smart Structures and Systems, "Errors / uncertainties in sensors for Structural Health Monitoring"*.

Chatzi, E. N., & Smyth, A. W. (2009). The unscented Kalman filter and particle filter methods for nonlinear structural system identification with non-collocated heterogeneous sensing. *Structural Control and Health Monitoring, 16*(1), 99-123.

Chatzi, E. N., & Smyth, A. W. (2013). Particle filter scheme with mutation for the estimation of time-invariant parameters in structural health monitoring applications. *Structural Control and Health Monitoring, 20*(7), 1081-1095.

Ching, J., & Beck, J. L. (2007). Real-time reliability estimation for serviceability limit states in structures with uncertain dynamic excitation and incomplete output data. *Probabilistic Engineering Mechanics, 22*(1), 50-62.

Ching, J., Beck, J. L., & Porter, K. A. (2006). Bayesian state and parameter estimation of uncertain dynamical systems. *Probabilistic Engineering Mechanics, 21*(1), 81-96. doi: http://dx.doi.org/10.1016/j.probengmech.2005.08.003

Eftekhar Azam, S., Bagherinia, M., & Mariani, S. (2012). Stochastic system identification via particle and sigma-point Kalman filtering. *Scientia Iranica, 19*(4), 982-991.

Eftekhar Azam, S., Ghisi, A., & Mariani, S. (2012). Parallelized sigma-point Kalman filtering for structural dynamics. *Computers and Structures, 92-93*, 193-205.

Eftekhar Azam, S., & Mariani, S. (2012). Dual estimation of partially observed nonlinear structural systems: A particle filter approach. *Mechanics Research Communications, 46*, 54-61.

Gao, F., & Lu, Y. (2006). A Kalman-filter based time-domain analysis for structural damage diagnosis with noisy signals. *Journal of Sound and Vibration, 297*(3–5), 916-930. doi: http://dx.doi.org/10.1016/j.jsv.2006.05.007

Gillijns, S., & De Moor, B. (2007a). Unbiased minimum-variance input and state estimation for linear discrete-time systems. *Automatica, 43*(1), 111-116.

Gillijns, S., & De Moor, B. (2007b). Unbiased minimum-variance input and state estimation for linear discrete-time systems with direct feedthrough. *Automatica, 43*(5), 934-937.

Gordon, N. J., Salmond, D. J., & Smith, A. F. M. (1993). Novel approach to nonlinear/non-Gaussian Bayesian state estimation. *Radar and Signal Processing, IEE Proceedings F, 140*(2), 107-113.

Hernandez, E. M. (2011). A natural observer for optimal state estimation in second order linear structural systems. *Mechanical Systems and Signal Processing, 25*(8), 2938-2947.

Hsieh, C. S. (2000). Robust two-stage Kalman filters for systems with unknown inputs. *IEEE Transactions on Automatic Control, 45*(12), 2374-2378.

Julier, S. J., & Uhlmann, J. K. (1997). *A new extension of the Kalman filter to nonlinear systems.* Paper presented at the Int. symp. aerospace/defense sensing, simul. and controls.

Kalman, R. E. (1960). A new approach to linear filtering and prediction problems. *Journal of basic Engineering, 82*(1), 35-45.

Kitanidis, P. K. (1987). Unbiased minimum-variance linear state estimation. *Automatica, 23*(6), 775-778.

Lourens, E., Papadimitriou, C., Gillijns, S., Reynders, E., De Roeck, G., & Lombaert, G. (2012). Joint input-response estimation for structural systems based on reduced-order models and vibration data from a limited number of sensors. *Mechanical Systems and Signal Processing, 29*, 310-327.

Lourens, E., Reynders, E., De Roeck, G., Degrande, G., & Lombaert, G. (2012). An augmented Kalman filter for force identification in structural dynamics. *Mechanical Systems and Signal Processing, 27*(1), 446-460.

Mariani, S., & Ghisi, A. (2007). Unscented Kalman filtering for nonlinear structural dynamics. *Nonlinear Dynamics, 49*(1-2), 131-150.

Papadimitriou, C., Fritzen, C. P., Kraemer, P., & Ntotsios, E. (2011). Fatigue predictions in entire body of metallic structures from a limited number of vibration sensors using Kalman filtering. *Structural Control and Health Monitoring, 18*(5), 554-573.

Papadimitriou, C., & Lombaert, G. (2012). The effect of prediction error correlation on optimal sensor placement in structural dynamics. *Mechanical Systems and Signal Processing, 28*, 105-127.

Rajamani, M. R., & Rawlings, J. B. (2009). Estimation of the disturbance structure from data using semidefinite programming and optimal weighting. *Automatica, 45*(1), 142-148. doi: http://dx.doi.org/10.1016/j.automatica.2008.05.032

Reynders, E., & Roeck, G. D. (2008). Reference-based combined deterministic–stochastic subspace identification for experimental and operational modal analysis. *Mechanical Systems and Signal Processing, 22*(3), 617-637. doi: http://dx.doi.org/10.1016/j.ymssp.2007.09.004

Smyth, A., & Wu, M. (2007). Multi-rate Kalman filtering for the data fusion of displacement and acceleration response measurements in dynamic system monitoring. *Mechanical Systems and Signal Processing, 21*(2), 706-723. doi: http://dx.doi.org/10.1016/j.ymssp.2006.03.005

Yuen, K.-V., Hoi, K.-I., & Mok, K.-M. (2007). Selection of noise parameters for Kalman filter. *Earthquake Engineering and Engineering Vibration, 6*(1), 49-56. doi: 10.1007/s11803-007-0659-9