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Parametric Identification of a Cable-stayed Bridge using Substructure Approach

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ABSTRACT

Parametric identification of structures is one of the important aspects of structural health monitoring. Most of the techniques available in the literature have been proved to be effective for structures with small degree of freedoms. However, the problem becomes challenging when the structure system is large, such as bridge structures. Therefore, it is highly desirable to develop parametric identification methods that are applicable to complex structures. In this paper, the LSE based techniques will be combined with the substructure approach for identifying the parameters of a cable-stay bridge with large degree of freedoms. Numerical analysis has been carried out and the results demonstrate that the proposed approach is capable of identifying the structural parameters with reasonable accuracy.

1. INTRODUCTION

Parametric identification based on vibration characteristics provides useful information for both real-time online monitoring and overall offline evaluation of structures. The modal parameters (such as damping and frequency) of structure vibrations are dependent variables that relate to physical parameters (such as mass and stiffness). Therefore, the accurate identification of structure parameters is the premise of a reasonable structure health monitoring system. However, the problem becomes challenging when the structure system is large and complex, for example, bridge structures where the number of degree of freedom (DOF) is huge, because most of the identification methods available in the literature have better accuracy and adaptability for relatively small DOF structural systems (e.g. Caravani et al, 1977, Yang and Lin, 2005). Therefore, in order to ensure accurate identification of structural parameters, it is desired to decompose the complex structure having multiple DOFs into smaller parts such

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that the number of unknowns is limited within a certain range, which is known as the substructure approach.

Since 1960s, many scholars have proposed various substructure theories constantly for the dynamic analyses of structural responses. In particular, Koh et al (1991, 2003) conducted systematic studies on the parametric identification of structures with different scales and conditions using genetic algorithm and found that the accuracy of substructure identification approach is higher than full structure identification method and requires much less computation time.

In this paper, the substructure approach proposed in Koh et al (2003) will be combined with the least square estimation (LSE) method given in Yang and Lin (2005) for identifying the parameters of a cable-stay bridge with large DOFs. Numerical simulation studies are carried out for three substructures extracted from the 2-dimentional (2D) finite element model of the bridge. The result shows that the proposed identification method has high accuracy without measurement noises.

2. FUNDAMENTAL THEORY

2.1 Substructure approach

The equation of motion (EOM) for a complete structural system can be written as

$$\boldsymbol{M}\ddot{\boldsymbol{u}}(t)+\boldsymbol{C}\dot{\boldsymbol{u}}(t)+\boldsymbol{K}\boldsymbol{u}(t)=\boldsymbol{F}(t) \tag{1}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{K}$ are the mass, damping and stiffness matrices, respectively, $\boldsymbol{u}(t)$ is the displacement vector and ***F***(t) is the excitation force vector.

Consider a substructure extracted from the system, the corresponding EOM may be written using partition matrices as follows

$$\begin{aligned}\begin{pmatrix}\boldsymbol{M}_{ff} & \boldsymbol{M}_{fr} & \boldsymbol{0}\\ \boldsymbol{M}_{rf} & \boldsymbol{M}_{rr} & \boldsymbol{M}_{rg}\\ \boldsymbol{0} & \boldsymbol{M}_{gr} & \boldsymbol{M}_{gg}\end{pmatrix}\begin{bmatrix}\ddot{\boldsymbol{u}}_f(t)\\ \ddot{\boldsymbol{u}}_r(t)\\ \ddot{\boldsymbol{u}}_g(t)\end{bmatrix}+\begin{pmatrix}\boldsymbol{C}_{ff} & \boldsymbol{C}_{fr} & \boldsymbol{0}\\ \boldsymbol{C}_{rf} & \boldsymbol{C}_{rr} & \boldsymbol{C}_{rg}\\ \boldsymbol{0} & \boldsymbol{C}_{gr} & \boldsymbol{C}_{gg}\end{pmatrix}\begin{bmatrix}\dot{\boldsymbol{u}}_f(t)\\ \dot{\boldsymbol{u}}_r(t)\\ \dot{\boldsymbol{u}}_g(t)\end{bmatrix}\\ +\begin{pmatrix}\boldsymbol{K}_{ff} & \boldsymbol{K}_{fr} & \boldsymbol{0}\\ \boldsymbol{K}_{rf} & \boldsymbol{K}_{rr} & \boldsymbol{K}_{rg}\\ \boldsymbol{0} & \boldsymbol{K}_{gr} & \boldsymbol{K}_{gg}\end{pmatrix}\begin{bmatrix}\boldsymbol{u}_f(t)\\ \boldsymbol{u}_r(t)\\ \boldsymbol{u}_g(t)\end{bmatrix}=\begin{bmatrix}\boldsymbol{F}_f(t)\\ \boldsymbol{F}_r(t)\\ \boldsymbol{F}_g(t)\end{bmatrix}\end{aligned} \tag{2}$$

where the subscripts 'f' and 'g' denote the DOFs at the two interface ends of the substructure, referred to as interface DOFs. The remaining DOFs are denoted by subscript 'r' and referred to as internal DOFs. Since we are interested in identifying the parameters within the substructure, only the second submatrix equation of Eq. (2) will be used, i.e.,

$$\begin{aligned}\boldsymbol{M}_{rf}\ddot{\boldsymbol{u}}_f(t)+\boldsymbol{M}_{rr}\ddot{\boldsymbol{u}}_r(t)+\boldsymbol{M}_{rg}\ddot{\boldsymbol{u}}_g(t)+\boldsymbol{C}_{rf}\dot{\boldsymbol{u}}_f(t)+\boldsymbol{C}_{rr}\dot{\boldsymbol{u}}_r(t)+\boldsymbol{C}_{rg}\dot{\boldsymbol{u}}_g(t)\\ +\boldsymbol{K}_{rf}\boldsymbol{u}_f(t)+\boldsymbol{K}_{rr}\boldsymbol{u}_r(t)+\boldsymbol{K}_{rg}\boldsymbol{u}_g(t)=\boldsymbol{F}_r(t)\end{aligned} \tag{3}$$

Or

$$\begin{aligned}&\boldsymbol{M}_{rr}\ddot{\boldsymbol{u}}_r(t)+\boldsymbol{C}_{rr}\dot{\boldsymbol{u}}_r(t)+\boldsymbol{K}_{rr}\boldsymbol{u}_r(t)\\&=\boldsymbol{F}_r(t)-\boldsymbol{M}_{rf}\ddot{\boldsymbol{u}}_f(t)-\boldsymbol{M}_{rg}\ddot{\boldsymbol{u}}_g(t)-\boldsymbol{C}_{rf}\dot{\boldsymbol{u}}_f(t)-\boldsymbol{C}_{rg}\dot{\boldsymbol{u}}_g(t)-\boldsymbol{K}_{rg}\boldsymbol{u}_g(t)-\boldsymbol{K}_{rf}\boldsymbol{u}_f(t)\end{aligned} \tag{4}$$

For parametric identification, the right hand side of Eq. (4) can be treated as input (excitation) to the substructure, while the output responses are $\ddot{\boldsymbol{u}}_r(t), \dot{\boldsymbol{u}}_r(t), \boldsymbol{u}_r(t)$ correspondingly (Koh 1991).

2.2 Least square estimation (LSE)

Supposed $\boldsymbol{\theta}(t)$ is an n-parametric vector involving n unknown parameters, including damping, stiffness, and nonlinear parameters, i.e.,

$$\boldsymbol{\theta}(t)=\left[\theta_1(t) \quad \theta_2(t) \quad \cdots \quad \theta_n(t)\right]^T \tag{5}$$

The observation equation associated with the equations of motion in Eq. (3) or (4) can be written as

$$\boldsymbol{\varphi}\left[\ddot{\boldsymbol{x}}(t), \dot{\boldsymbol{x}}(t), \boldsymbol{x}(t); t\right]\boldsymbol{\theta}(t)+\boldsymbol{\varepsilon}(t)=\boldsymbol{y}(t) \tag{6}$$

where $\ddot{\boldsymbol{x}}(t), \dot{\boldsymbol{x}}(t), \boldsymbol{x}(t)$ are m-measured acceleration, velocity, displacement response vectors; $\boldsymbol{y}(t)$ is m-measured output vector; $\boldsymbol{\varepsilon}(t)$ is m-model noise vector contributed by the measurement noise and possible model errors; and $\boldsymbol{\varphi}[\ ]$ is $(m\times n)$ observation matrix.

At each time instant $t=t_{k+1}=(k+1)\Delta t$, Eq. (6) can be discretized as

$$\boldsymbol{\varphi}_{k+1}\boldsymbol{\theta}_{k+1}+\boldsymbol{\varepsilon}_{k+1}=\boldsymbol{y}_{k+1} \tag{7}$$

Combining all equations in Eq.(7) for $k+1$ time instants, and assuming that $\boldsymbol{\theta}_{k+1}$ is a constant vector, one obtains

$$\boldsymbol{\phi}_{k+1}\boldsymbol{\theta}_{k+1}+\boldsymbol{E}_{k+1}=\boldsymbol{Y}_{k+1} \tag{8}$$

where

$$\boldsymbol{\phi}_{k+1}=\begin{pmatrix}\boldsymbol{\varphi}_1\\\boldsymbol{\varphi}_2\\\vdots\\\boldsymbol{\varphi}_{k+1}\end{pmatrix}, \quad \boldsymbol{E}_{k+1}=\begin{pmatrix}\boldsymbol{\varepsilon}_1\\\boldsymbol{\varepsilon}_2\\\vdots\\\boldsymbol{\varepsilon}_{k+1}\end{pmatrix}, \quad \boldsymbol{Y}_{k+1}=\begin{pmatrix}\boldsymbol{y}_1\\\boldsymbol{y}_2\\\vdots\\\boldsymbol{y}_{k+1}\end{pmatrix}; \tag{9}$$

Let $\hat{\boldsymbol{\theta}}_{k+1}$ be the estimate of $\boldsymbol{\theta}_{k+1}$ at $t_{k+1} = (k+1)\Delta t$, the recursive solution for $\hat{\boldsymbol{\theta}}_{k+1}$ can be obtained as

$$\hat{\boldsymbol{\theta}}_{k+1}=\hat{\boldsymbol{\theta}}_k+\boldsymbol{K}_{k+1}\left(\boldsymbol{y}_{k+1}-\boldsymbol{\varphi}_{k+1}\hat{\boldsymbol{\theta}}_k\right) \tag{10}$$

in which

$$\boldsymbol{K}_{k+1} = \boldsymbol{P}_k\boldsymbol{\varphi}^T{}_{k+1}\left(\boldsymbol{I}+\boldsymbol{\varphi}_{k+1}\boldsymbol{P}_k\boldsymbol{\varphi}^T{}_{k+1}\right)^{-1} \tag{11}$$

$$\boldsymbol{P}_{k+1} = \left(\boldsymbol{I}-\boldsymbol{K}_{k+1}\boldsymbol{\varphi}_{k+1}\right)\boldsymbol{P}_k \tag{12}$$

where $\boldsymbol{K}_{k+1}$ is the LSE gain matrix, Eqs. (10)-(12) are the recursive solution of classic LSE method (Yang 2005).

3. NUMERICAL MODEL OF A CABLE-STAYED BRIDGE

In this paper, the Kezhushan Bridge which is one of the main navigation channels of Donghai bridge located in Shanghai, China will be studied by numerical simulation. The bridge is 710 meters long with a main span of 332 meters and two side spans of 139 meters each and the full wide is 35 meters. It is a steel-concrete composite beam structure with two pylons and double cable planes. Each of the pylons is a reinforced concrete structure with a height of 105 meters. Cables are shaped into sectors and disposed symmetrically. Each cable plane has 64 (2×32) cables. The general layout is shown in Fig. 1.

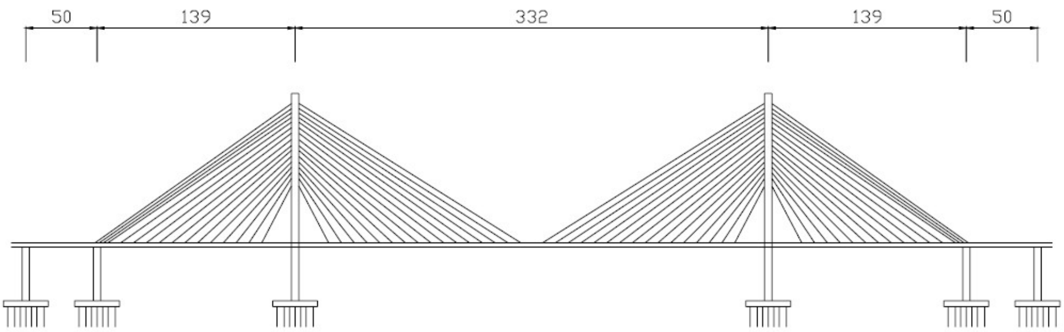

Fig 1 The general layout of Kezhushan Bridge

3.1 Simplified model

For the simplification of analysis, a 2D model is established in the numerical study, consisting of a beam, two towers and 64 cables. The cross section of a tower or a cable is two times of original, since a pair of parallel cables or towers is combined into one. Axial deformation of beam and tower elements is ignored.

The finite element model of the bridge is set up with numbering of nodes and elements shown in Fig 2. The beam element is chosen between adjacent cable nodes and numbered from left to right as 1-67 where the nodes are numbered as 1-68. On the upper tower, the elements are selected between adjacent cable nodes, while on the middle and lower tower, it is divided into 14 elements equally. Therefore, the entire tower has 30 elements and 31 nodes numbered as 68-97 and 69-99 respectively for the left tower, and as 98-127 and 100-130 respectively for the right tower. Each cable is taken as one element numbered from left to right as 128-191.

Since only vertical excitation is considered in this paper, the beam node will have just vertical DOF while the tower node has horizontal DOF, and all the nodes at the boundaries are constrained.

In summary, the entire model has 191 elements, 130 nodes and 252 DOFs.

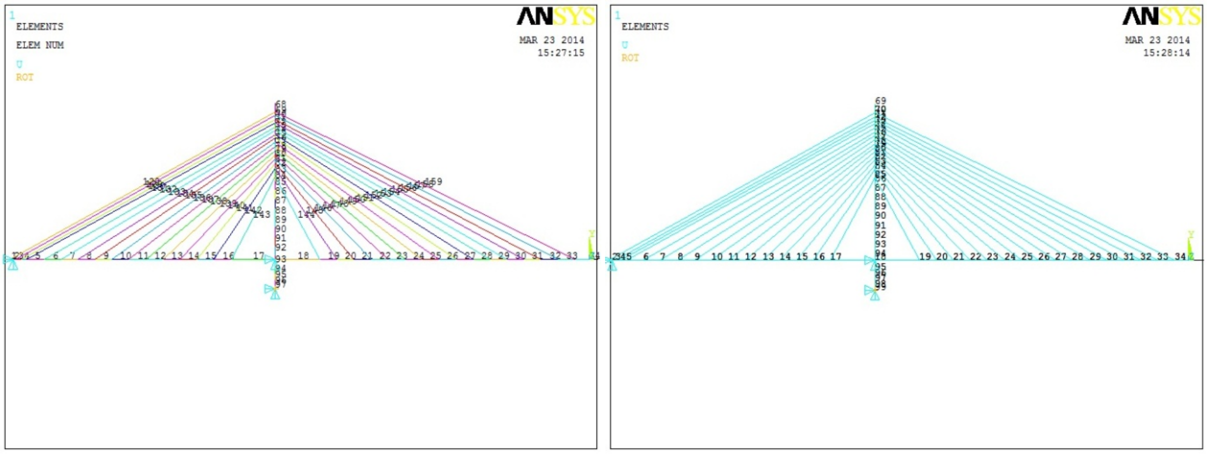

Fig 2 Numbering of elements and nodes of half model

3.2 Element matrices

<u>Beam and tower element stiffness matrix</u> The geometric stiffness should be considered for element stiffness matrix because of the existence of axial forces. Hence

$$\overline{\boldsymbol{K}}^{e} = \boldsymbol{K}^{e} - \boldsymbol{K}_{G}^{e} \tag{13}$$

in which

$$\boldsymbol{K}^{e} = \frac{2EI}{l^3}\begin{pmatrix} 6 & 3l & -6 & 3l \\ 3l & 2l^2 & -3l & l^2 \\ -6 & -3l & 6 & -3l \\ 3l & l^2 & -3l & 2l^2 \end{pmatrix} \tag{14}$$

$$\boldsymbol{K}_G^e = \frac{N}{30l}\begin{pmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & -l^2 \\ -36 & -3l & 36 & -3l \\ 3l & -l^2 & -3l & 4l^2 \end{pmatrix} \tag{15}$$

where N is element axial force, E is modulus of elasticity, I is moment of inertia, l is element length.

Beam and tower element mass matrix Suppose element mass is distributed evenly along length, a consistent mass matrix can be used as follows

$$\boldsymbol{M}^e = \frac{\overline{m}l}{420}\begin{pmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{pmatrix} \tag{16}$$

where $\overline{m}$ is the linear density of element.

Cable element stiffness matrix A cable element only has two DOFs, a vertical DOF at the beam side and a horizontal DOF at the tower side, and the stiffness matrix can be written as follows

$$\boldsymbol{K}^e = \frac{E_{eg}A}{l}\begin{pmatrix} \sin^2\alpha & -\sin\alpha\cos\alpha \\ -\sin\alpha\cos\alpha & \cos^2\alpha \end{pmatrix} \tag{17}$$

where E_{eg} is the modulus of elasticity modified by Ernst equation given in Eq. (18), A is the area of section, and α is the horizontal inclination of cable.

$$E_{eg} = \frac{E}{1+\dfrac{\gamma^2 l_h^{\,2} E}{12\sigma^3}} \tag{18}$$

where σ is the initial tension stress of cable, γ is the bulk density, and l_h is the horizontal length of cable.

Cable element mass matrix The mass matrix of cable element can be obtained based on linear interpolation as follows

$$\boldsymbol{M}^e = \begin{pmatrix} \dfrac{1}{3}\overline{m}l\sin^2\alpha & \dfrac{1}{6}\overline{m}l\sin\alpha\cos\alpha \\ \dfrac{1}{6}\overline{m}l\sin\alpha\cos\alpha & \dfrac{1}{3}\overline{m}l\cos^2\alpha \end{pmatrix} \tag{19}$$

The frequencies of the first six modes of the bridge computed using the simplified 2D model are compared with those obtained from a 3-dimensional (3D) model in Dong (2010), as listed in Table 1. It shows that the 2D model can be used to represent the dynamic characteristics of the bridge with reasonable accuracy.

Table 1 Frequencies of the first six modes of the bridge

mode	Simplified model	ANSYS 3D model	error
1	0.4018Hz	0.3979Hz	0.98%
2	0.5403Hz	0.5079Hz	6.38%
3	0.7985Hz	0.8124Hz	-1.71%
4	0.9832Hz	0.9215Hz	6.70%
5	1.1104Hz	1.0320Hz	7.60%
6	1.3027Hz	1.2253Hz	6.32%

4. IDENTIFICATION OF STRUCTURAL PARAMETERS

Because of the symmetry of the cable-stayed bridge, only the parameters of half of the model will be indentified through three substructures. A vertical white noise excitation is applied at each beam node as shown in Fig 3, with loading period of 10 seconds, and the corresponding responses are measured with sampling frequency of 1000Hz.

The parameters to be identified are the stiffness of all elements, namely EI for beam and tower elements and EA for cable elements.

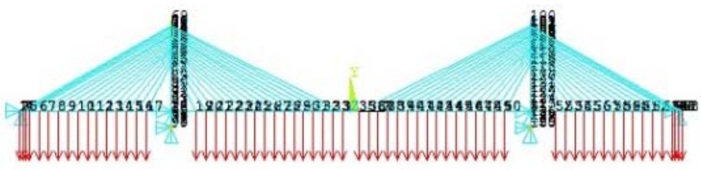

Fig 3 White noise excitation

<u>Substructure 1</u> The beam of left span and the cables attached to it are considered as substructure 1 as shown in Fig 4.

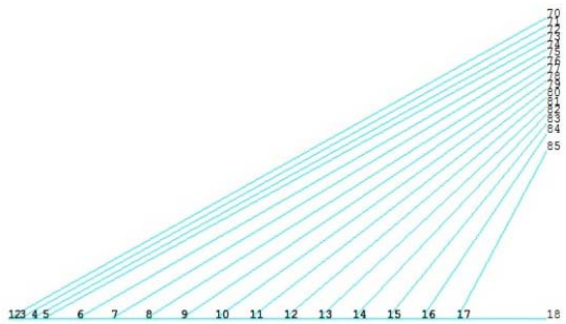

Fig 4 Substructure1

The results of identified structural parameters are presented in Table 2, where NO.1-17 list the stiffness of beam elements EI ($10^{11} N \cdot m^2$) and NO.18-33 gives that of cable elements EA ($10^9 N$).

Table 2 Identification parameters of substructure 1

Beam	Theoretical	Estimated	Error(%)	Cable	Theoretical	Estimated	Error(%)
1	1.1508	1.1508	0.000	18	2.2100	2.2100	0.000
2	1.1508	1.1508	0.000	19	1.9900	1.9900	0.000
3	1.1508	1.1508	0.000	20	2.0000	2.0000	0.000
4	1.1508	1.1508	0.000	21	1.7600	1.7600	0.000
5	1.1508	1.1508	0.000	22	2.4200	2.4200	0.000
6	1.1508	1.1508	0.000	23	2.4500	2.4500	0.000
7	1.1508	1.1508	0.000	24	2.5000	2.5000	0.000
8	1.1508	1.1508	0.000	25	2.5400	2.5400	0.000
9	1.1508	1.1508	0.000	26	2.6500	2.6500	0.000
10	1.1508	1.1508	0.000	27	2.6300	2.6300	0.000
11	1.1508	1.1508	0.000	28	2.6900	2.6900	0.000
12	1.1508	1.1508	0.000	29	2.6000	2.6000	0.000
13	1.1508	1.1508	0.000	30	2.6100	2.6100	0.000
14	1.1508	1.1508	0.000	31	2.3700	2.3700	0.000
15	1.1508	1.1508	0.000	32	2.4000	2.4000	0.000
16	1.1508	1.1508	0.000	33	4.0200	4.0200	0.000
17	1.1508	1.1508	0.000				

The time tracking of the identification processes of NO.1 beam element and NO.128 cable element are shown in Fig 5 (a) and (b) respectively for illustration.

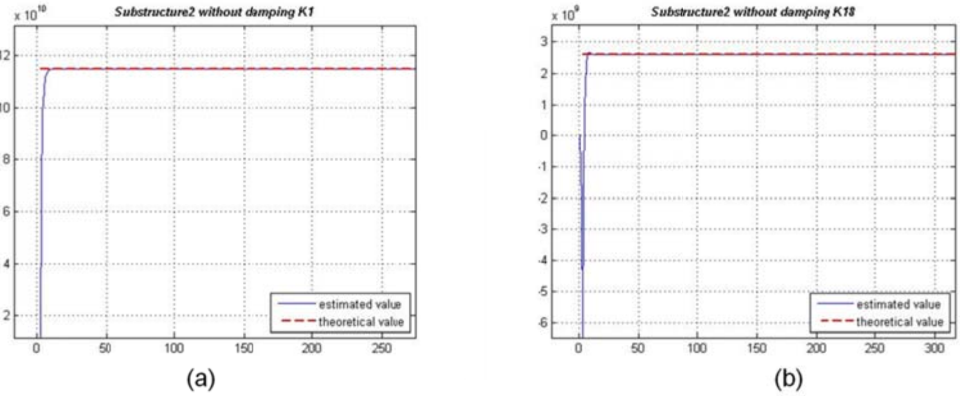

Fig 5 The time tracking of identification processes of substructure 1

Substructure 2 The beam of right span and the cable attached to it are extracted as substructure 2, as shown in Fig 6. The identification results are listed in Table 3.

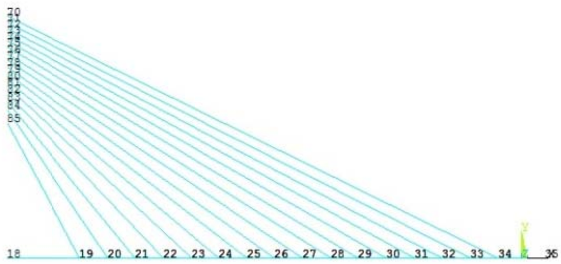

Fig 6 Substructure2

Table 3 Identification parameters of substructure 2

Beam	Theoretical	Estimated	Error(%)	Cable	Theoretical	Estimated	Error(%)
1	1.1508	1.1508	0.000	18	2.6000	2.2100	0.000
2	1.1508	1.1508	0.000	19	1.9700	1.9900	0.000
3	1.1508	1.1508	0.000	20	1.7200	2.0000	0.000
4	1.1508	1.1508	0.000	21	1.5600	1.7600	0.000
5	1.1508	1.1508	0.000	22	1.4000	2.4200	0.000
6	1.1508	1.1508	0.000	23	1.6100	2.4500	0.000
7	1.1508	1.1508	0.000	24	1.8000	2.5000	0.000
8	1.1508	1.1508	0.000	25	1.7000	2.5400	0.000
9	1.1508	1.1508	0.000	26	1.6700	2.6500	0.000
10	1.1508	1.1508	0.000	27	1.3900	2.6300	0.000
11	1.1508	1.1508	0.000	28	1.1900	2.6900	0.000
12	1.1508	1.1508	0.000	29	1.1300	2.6000	0.000
13	1.1508	1.1508	0.000	30	1.6300	2.6100	0.000
14	1.1508	1.1508	0.000	31	1.5000	2.3700	0.000
15	1.1508	1.1508	0.000	32	1.1900	2.4000	0.000
16	1.1508	1.1508	0.000	33	1.0200	4.0200	0.000
17	1.1508	1.1508	0.000				

<u>Substructure 3</u> The tower and all the cables attached to it are extracted as substructure 3, as shown in Fig 7, and the corresponding identification results are summarized in Table 4.

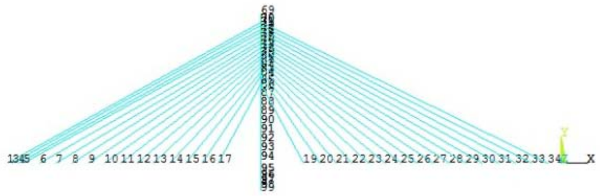

Fig 7 Substructure3

Table 4 Identification parameters of substructure 3

Tower	Theoretical	Estimated	Error(%)	Cable	Theoretical	Estimated	Error(%)
1	3.2362	3.2362	0.000	16	3.2362	3.2362	0.000
2	3.2362	3.2362	0.000	17	7.7504	7.7504	0.000
3	3.2362	3.2362	0.000	18	7.7504	7.7504	0.000
4	3.2362	3.2362	0.000	19	7.7504	7.7504	0.000
5	3.2362	3.2362	0.000	20	7.7504	7.7504	0.000
6	3.2362	3.2362	0.000	21	7.7504	7.7504	0.000
7	3.2362	3.2362	0.000	22	7.7504	7.7504	0.000
8	3.2362	3.2362	0.000	23	7.7504	7.7504	0.000
9	3.2362	3.2362	0.000	24	7.7504	7.7504	0.000
10	3.2362	3.2362	0.000	25	7.7504	7.7504	0.000
11	3.2362	3.2362	0.000	26	7.7504	7.7504	0.000
12	3.2362	3.2362	0.000	27	10.981	10.981	0.000
13	3.2362	3.2362	0.000	28	10.981	10.981	0.000
14	3.2362	3.2362	0.000	29	10.981	10.981	0.000
15	3.2362	3.2362	0.000	30	10.981	10.981	0.000

It can be seen from Tables 2-4 and Fig 5 that, without measurement noises, the LSE method can estimate the structural parameters with very high accuracy.

5. CONCLUSIONS

In this paper, the LSE method has been used combined with a substructure approach for the identification of structural parameters of a cable-stay bridge with large DOFs. Numerical analysis has been carried out based on the simplified 2D model of the bridge under vertical excitations. Three substructures are extracted from the full finite element model of the bridge and the parameters of each substructure are estimated. The results show that the proposed identification method has a high accuracy without measurement noises, while further studies have to be carried out to verify the proposed approach by considering different external excitations and introducing measurement noises.

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