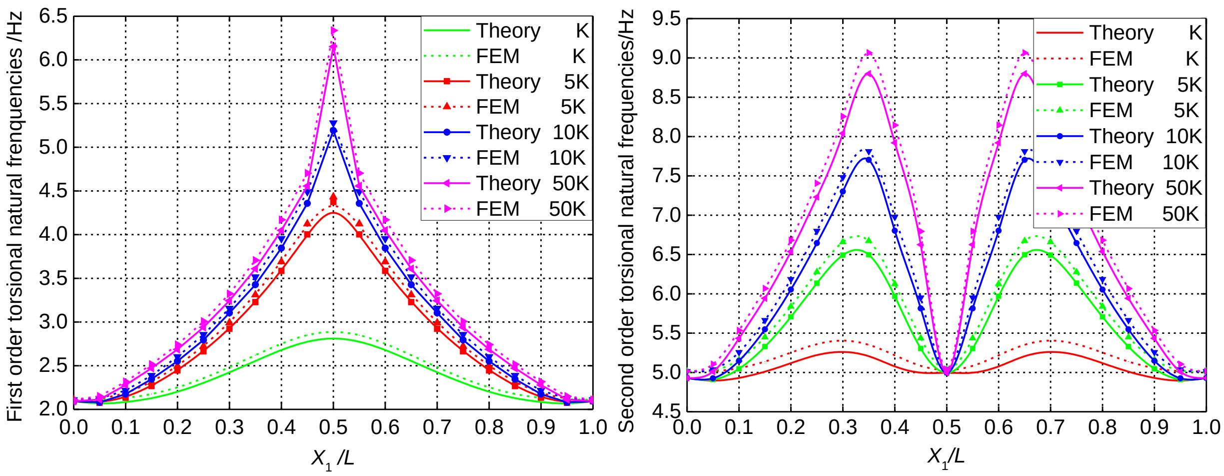



（a）First order （b）Second order

Fig.6 The lowest two torsional natural frequencies of the coupled system with H-section hanger and wind-resistant cables in different locations versus $\varepsilon$

Table 3 The maximum torsional natural frequencies of the first two orders for the hanger(Hz)

| $\varepsilon$ | First order | Second order |
|---|---|---|
| 0 | 2.09 | 4.93 |
| K(1.41e3) | 2.97 | 5.39 |
| 5K(7.05e3) | 4.49 | 6.87 |
| 10K(1.41e4) | 5.28 | 7.54 |
| 50K(7.05e4) | 6.34 | 8.86 |
| Increased percentage ( $\varepsilon = 50K$ ) | 203.35% | 79.66% |

As shown in Fig.6 and Table 3, some rules can be deduced:

(1) The theoretical solutions are highly anastomosed with finite element method computations of lowest two torsional natural frequencies, which proves the accuracy and stability of theoretical solutions to solve the torsional natural frequencies of coupled system with H-section hanger and wind-resistant cables. Due to considering the 0.5m length of high strength bolts field at the end of the hanger in finite element model, resulting in the effective length of the hanger decreased partly, the finite element method computations have a little bigger than theoretical solutions.

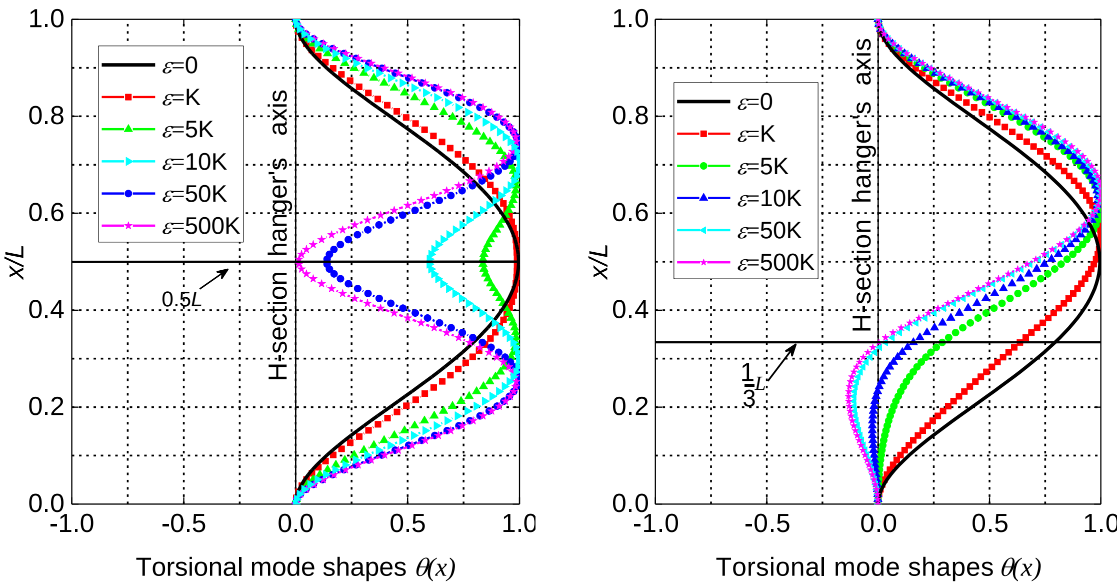
(2) The lowest two torsional natural frequencies of coupled system are increased gradually along with the values of $\varepsilon$ added. Observing the axial coordinates where each curve reach the extreme values, the higher torsional natural frequencies are obtained when the wind-resistant cables are in the potion of the biggest node coordinate values of the mode shapes, but the torsional natural frequencies are not affected by the wind-resistant cables when it is located in model node of H-section

hanger (when $x/L$=0.5, the second order torsional frequencies are common with the $\varepsilon$ values changed in Fig.6(b)). Hence, considering the first order torsional vibration control in real engineering, the wind-resistant cables should be installed in the middle length of the hanger, and it should be set in the third or two thirds length of the hanger when the second order torsional vibration need to be restrained.

(3) As is shown in table 3, the first order torsional natural frequencies can be improved to the highest 3.03 times of the original value which don't have wind-resistant cables attached in hanger, and the second torsional natural frequencies have the highest 1.8 times of its original value, which have highly improvements of lowest two torsional frequencies.

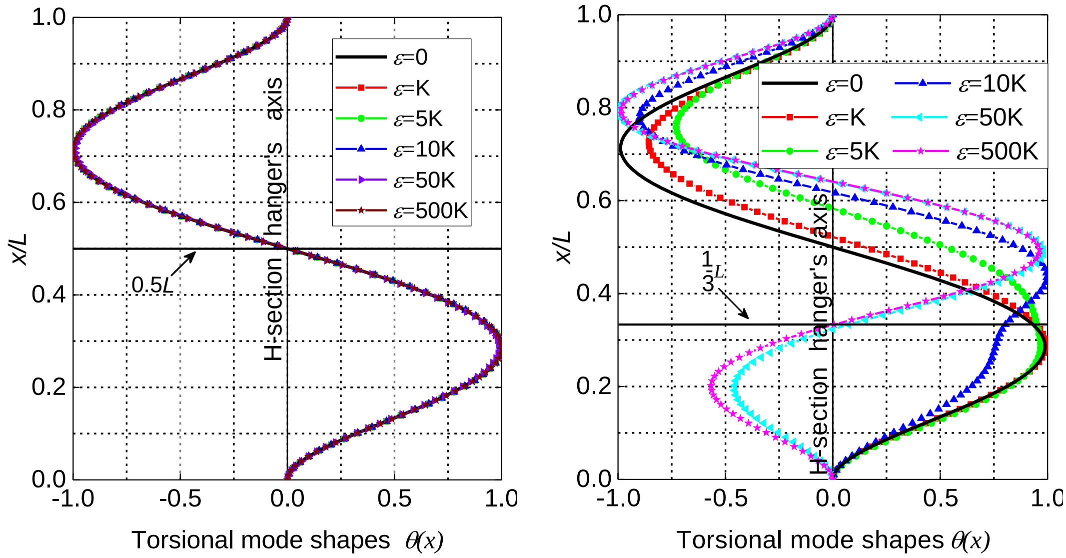
## 3.2 The effects of wind-resistant cables' locations and rigidities for torsional mode shapes

Due to the section 3.1, the torsional natural frequencies are improved to the biggest values when the wind-resistant cables are installed in the positions of maximum mode shapes' node, and the influence rules of lowest two mode shapes with $\varepsilon$ increased are analyzed when the wind-resistant cables are in the 1/2 and 1/3 lengths of hanger, which are shown in Fig. 7 and Fig. 8.




（a） $X_1/L=1/2$ （b） $X_1/L=1/3$

Fig.7 First order torsional mode shapes of the H-section hanger




(a) $X_1/L=1/2$ (b) $X_1/L=1/3$

Fig.8 Second order torsional mode shapes of the H-section hanger

From Fig. 7 and Fig. 8:

(1) When $X_1/L=1/2$ and $\varepsilon=0,K$, the maximum mode function values of first order torsional mode shapes is in the position of 0.5L, and its node coordinates of mode shapes are decreased to 0 gradually with $\varepsilon$ increased, which have two amplitude positions of mode shapes including $x/L=0.25$ and $x/L=0.75$ respectively. When $X_1/L=1/3$, the node coordinates of amplitudes for first order torsional mode shapes have emerged negative values with $\varepsilon$ increased, and the torsional displacement turn to be 0 at the position of wind-resistant cables ($\theta(1/3)=0$). The more bigger $\varepsilon$ values, the more stronger constraint force of the wind-resistant cables to H-section hanger, and the position can be fixed finally.

(2) When $X_1/L=1/2$, the second order torsional mode shapes have no changing with $\varepsilon$ increased, this can coincide with the results which the second torsional natural frequencies are invariant when the wind-resistant cables are in mid-point of the hanger obtained in section 3.1 and Fig. 7. When $X_1/L=1/3$ and $\varepsilon=50K,500K$, the second order torsional mode curves have two points of intersections with H-section hanger's axis.

## 4. DAMPING SOLUTIONS OF THE FIRST ORDER TORSIONAL VIBRATION

Due to Eq. (7), the $\varepsilon$, non-dimensional equivalent spring rigidity of wind-resistant cables, is determined modulus, length and area of wind-resistant cables, which can easily change the length and area of wind-resistant cables by steel strand parameters. In real bridge engineering, first order torsional natural frequencies can be improved preferentially, and the best installed positions of wind-resistant cables is in hangers' mid-point owing to section 3.1. Changing the length and area of wind-resistant cables to make $\varepsilon$ increase, Fig. 9 shows the curves of lowest two torsional natural frequencies with $\varepsilon$ increased, which also describe the finite element method computations to compare.

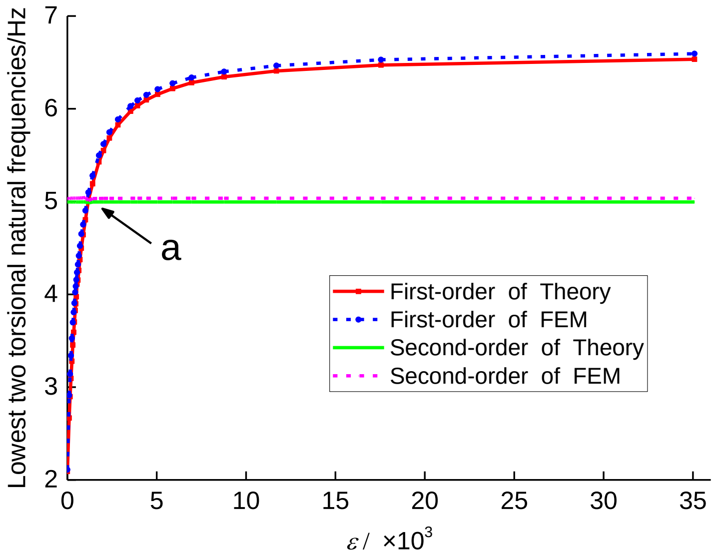



Fig.9 Natural frequencies of the coupled system with a hanger and mid-point wind-resistant cables versus $\varepsilon$

As is shown in Fig. 9,

(1) The curves of the first order torsional natural frequencies are growing rapidly at the beginning with the high amplification, but latter the tendency become slow and turn into stable finally with $\varepsilon$ increased, which in this time the fundamental torsional frequencies reach the maximum values. Because the wind-resistant cables are installed in middle hanger, the second order torsional natural frequencies are kept the same values and invariant.

(2) The first torsional natural frequencies will exceed the second after the point of "a" which is a point intersection of two curves with $\varepsilon$ increased. And the first order torsional natural frequencies keep increasing and the second order emerge first after the point of "a", which change the maiden orders.

In the above, for damping the first order torsional natural frequencies, the natural frequency value corresponding the point of "a" is treated as the objective torsional frequency, which the corresponding rigidity is treated as objective stiffness. Then backing the objective rigidity into Eq. (7), the requisite length and area of wind-resistant cables can be calculated.

## 5. CONCLUSIONS

Based on the open thin-walled sections theory of Vlasov and the “compatibility conditions” in the position of wind-resistant cables, the theoretical model of coupled system with a hanger under the fixed-fixed boundaries and horizontal wind-resistant cables is established. The effects and rules of lowest two torsional natural frequencies and its mode shapes are analyzed with the locations and rigidities of wind-resistant cables changed. The following conclusions can be drawn:

(1) The established theoretical approach to solve the torsional natural frequencies of coupled system with a hanger and horizontal wind-resistant cables is verified the greatly accuracy.

(2) For improving first order torsional natural frequencies of H-section hanger, the wind-resistant cables should be installed in the middle length of hanger, which in this position the fundamental torsional frequencies increase rapidly at the beginning, and the incremental trend is slow gradually, which the hanger is fixed in the position of wind-resistant cables and turned to be two independent segments completely with the torsional rigidity of wind-resistant cables increased finally. And the maximum value of fundamental torsional natural frequency for H-section hanger in this numerical results is 3.03 times of its original value. Due to the special position which in mid-point of H-section hanger of the wind-resistant cables, the first order torsional frequencies surpass the second order when the equivalent torsional rigidities of wind-resistant cables are greater than a certain threshold, which can inverse the wind-resistant cables’ parameters according to objective torsional frequency which is the intersection value of the first and second order frequencies’ curves.

(3) With the invariant wind-resistant cables rigidity, the best second order torsional frequencies can be obtained when the cables are installed in 1/3 or 2/3 length of hanger. And there are two points with hanger’s axis to the second order torsional mode shapes with the wind-resistant cables rigidities increased. The maximum value of second torsional natural frequency for H-section hanger in this numerical results is 1.80 times of its original value.

## REFERENCES

Keller P, Higgins C, Lovejoy S.C. (2015), “Evaluation of torsional vibrations in steel truss bridge members.” Journal of Bridge Engineering-ASCE, Vol, 20(9): 04014102.

Chen Z.Q., Liu M.G., Hua X.G., et al. (2012), “Flutter, galloping, and vortex-induced vibrations of H-Section hangers.” Journal of Bridge Engineering-ASCE, Vol 17(3): 500-508.

Prasad S., O'neil T. (1984), “Vlasov theory of electrostatic modes in a finite length electron column.” Physics of Fluids, Vol, 27(1): 206-213.

Matsumoto M., Shirato H., Mizuno K., et al. (2008), “Flutter characteristics of h-shaped cylinders with various side-ratios and comparisons with characteristics of rectangular cylinders.” Journal of Wind Engineering and Industrial Aerodynamics, Vol, 96(6): 963-970.

Matsumoto M., Shirato H., Hirai S. (1992), “Torsional flutter mechanism of 2-d h-shaped cylinders and effect of flow turbulence.” Journal of Wind Engineering and Industrial Aerodynamics, Vol, 41(1): 687-698.

Maher F.J., Wittig L.E. (1980), “Aerodynamic response of long H-sections.” J. Struct. Div., Vol, 106(1): 183-198.

Ulstrup C.C. (1980), “Aerodynamic lessons learned from individual bridge members.” Annals of the New York Academy of Sciences, Vol, 352(1): 265-281.

Huang Z.H. Nicholas P.J. (2011), “Damping_of_taut-cable_systems_effects_of linear elastic spring support.” Journal of Engineering Mechanics, Vol, 137(7): 512-518.

Zhou H.J., Yang X., Sun L.M., et al. (2015), “Free vibrations of a two-cable network with near-support dampers and a cross-link.” Structural Control and Health Monitoring, Vol, 22(9): 1173-1192.

Javaid A, Cheng S. H. (2013), “Effect of cross-link stiffness on the in-plane free vibration behavior of a two-cable network.” Engineering Structures, Vol, 52: 570-580.

Ruscheweyh H., Hortmanns M., Schnakenberg C. (1996), “Vortex-excited vibrations and galloping of slender elements.” Journal of Wind Engineering and Industrial Aerodynamics, Vol, 65(1): 347-352.

Lin H.P., Chang S.C. (2005), “Free vibration analysis of multi-span beams with intermediate flexible constraints.” Journal of Sound and Vibration, Vol, 281(1): 155-169.

Ambrosini R.D., Riera J.D., Danesi R.F. (2000), “Modified Vlasov theory for dynamic analysis of thin-walled and variable open section beams.” Engineering Structures, Vol, 22(8): 890-900.

Aleksandar P., Dragan L. (2012), “Flexural-torsional vibration analysis of axially loaded thin-walled beam.” Journal of the Brazilian Society of Mechanical Sciences and Engineering, Vol, 34(3): 262-268.

Ulstrup C.C. (1978), “Natural frequencies of axially loaded bridge members.” Journal of the Structural Division, Vol, 104(2): 849-857.

Lee J.K., Jeong S., Lee J. (2014), “Natural frequencies for flexural and torsional vibrations of beams on pasternak foundation.” Soils and Foundations, Vol, 54(6): 1202-1211.

Mehrdad M. (2015), “New analytical approach for determination of flexural, axial and torsional natural frequencies of beams.” Structural Engineering and Mechanics, Vol, 55(3): 655-674.

Gokdag H., Kopmaz O. (2005), “Coupled bending and torsional vibration of a beam with in-span and tip attachments.” Journal of Sound and Vibration, Vol, 287(3): 591-610.

M.Tahmaseb T.K., Supun J., Seyed M.H. (2014), “On the flexural-torsional vibration and stability of beams subjected to axial load and end moment.” Shock and Vibration, Vol, 2014:153532 (10).