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A Span-wise Coherence Model for Gusty Loads on Elongated Bluff Bodies

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ABSTRACT

A double-exponent model, which is derivate from the classical aerofoil aerodynamics theory, is proposed to describe the spanwise coherence for gusty loads on elongated bluff bodies. It can reveal an important feature that gusty loads are more correlated than oncoming gusty components, and is validated in wind tunnel experiments.

1. INTRODUCTION

It is known that gusty loading on elongated bodies, such as the wing of large aeroplanes, long-span bridges and other line-like civil structures, is more correlated than the turbulent components. This phenomenon was found by Nettleton (1971) when he conducted an experimental investigation for aerodynamic forces on a motionless NACA0012 aerofoil immersed in grid-generated turbulent flow. A similar result was also observed by Li et al (2015) for the lift on NACA0015 aerofoil and Bearman (1971) for the flat-plate. For bluff bodies, many researchers (Jakobsen 1997; Kimura et al. 1997; Larose 1997) found the similar phenomenon for gusty loads on bridge decks. To describe the spatial distribution of gusty loads and determine aerodynamic admittance of bridge decks, several coherence models, for example Jakobsen's and Kimura's models, were proposed. It is noted that those models are based on the theoretical or empirical coherence models of gusty wind and introduce floating parameters to modify the length scale of turbulence and the frequency. However, the coherence of gusty loads may be different from the one of gusty components due to the contribution of aerodynamic admittance.

In this paper, inspired by Graham's two wave number aerodynamic admittance of thin aerofoil (Graham, 1971) and Ribner's general expression on gusty loads (Ribner, 1956), a theoretical double-exponent coherence model of the lift on an aerofoil is derived. It can demonstrate that gusty loads must be more correlated than oncoming turbulent component, which is an inherent characteristics of aerodynamic loads on the aerofoil. The proposed coherence model is then extended to describe the correlation of lifts on bluff bodies. Finally wind tunnel tests of an aerofoil and rectangular cylinders are conducted to validate the accuracy of the proposed coherence model wind tunnel tests.

2. SPANWISE COHERENCE OF THE LIFT ON THIN AEROFOIL

Ribner (1956) suggested a more general unsteady aerodynamic force model to consider the three-dimensional effects of turbulence. The three-wavenumber spectrum of lift can be written as

$$S_L\left(\vec{k}\right)=\left|\chi_L\left(\vec{k}\right)\right|^2 S_w\left(\vec{k}\right) \tag{1}$$

where $\vec{k}$ is the wavenumber vector having components k_1 , k_2 , k_3 in x, y and z directions respectively, $k_1 = n/U$, n is frequency (unit: Hz), U is mean wind velocity, $\left|\chi\left(\vec{k}\right)\right|^2$ is three-wavenumber aerodynamic admittance, $S_w\left(\vec{k}\right)$ is three-wavenumber spectrum of vertical turbulent component.

If geometrical dimensions in z direction is far less than the other two directions, the lift force, aerodynamic admittance and turbulence can be simplified into functions of just two-wavenumber in a horizontal plane. The cross-spectrum of the lift can be expressed as the form of convolution using the inverse Fourier transform,

$$S_L\left(k_1,\Delta y\right)=\left(\rho U b C_L^{'}\right)^2\left[\left|\chi_L\left(k_1,\Delta y\right)\right|^2 * \mathrm{Coh}_w(k_1,\Delta y)\right]S_w(k_1) \tag{2}$$

Mugridge's AAF (Mugridge, 1971) is expressed in terms of a correlation factor to Sears' function, which is of high accuracy as $k_1 < 1/\pi$ compared with Graham's exact result (Graham, 1970) that is

$$\left|\chi_L\left(k_1,k_2\right)\right|^2=\frac{1}{1+2\pi\tilde{k}_1}\left(\frac{\lambda_1^2}{\lambda_1^2+k_2^2}\right)\ ;\ \lambda_1=\sqrt{k_1^2+2/\pi^2}\Big/\left(2\pi b\right) \tag{3}$$

Therefore, $\left|\chi_L\left(k_1,\Delta y\right)\right|^2$ could be expressed in terms of the inverse Fourier transform of Eq. (3) as the following:

$$\left|\chi_L\left(k_1,\Delta y\right)\right|^2=\frac{\pi\lambda_1}{1+2\pi\tilde{k}_1}\mathrm{Coh}_{\mathrm{AAF}}\left(k_1,\Delta y\right) \tag{4}$$

Where the coherence function of aerodynamic admittance is

$$\mathrm{Coh}_{\mathrm{AAF}}\left(k_1,\Delta y\right)=\exp\left(-2\pi\lambda_1\Delta y\right) \tag{5}$$

Jakobsen (1997) proposed the following empirical coherence function for gusty wind,

$$\mathrm{Coh}_w\left(k_1,\Delta y\right)=\exp\left(-2\pi\mathrm{A}_w\Delta y\right) \tag{6}$$

where, $\mathrm{A}_w = \left[\sqrt{c_2^2 + \left(c_3 k_1 L_w^x\right)^2}\right]^{c_1} \Big/ \left(2\pi L_w^x\right)$ and c_1, c_2, c_3 are floating parameters that need to be fitted, L_w is the integral length scale of turbulence.

Inserting Eqs.(4)-(6) into Eq.(2), the closed-form coherence of lift force can be obtained:

$$\mathrm{Coh}_L\left(k_1, \Delta y\right) = \frac{\lambda_1 \exp\left(-2\pi \mathrm{A}_w \Delta y\right) - \mathrm{A}_w \exp\left(-2\pi \lambda_1 \Delta y\right)}{\lambda_1 - \mathrm{A}_w} \tag{7}$$

Now we can evaluate the difference between the lift force and vertical turbulence,

$$\Delta_{\mathrm{Coh}} = \mathrm{Coh}_L - \mathrm{Coh}_w = \frac{\mathrm{A}_w\left[\exp\left(-2\pi \mathrm{A}_w \Delta y\right) - \exp\left(-2\pi \lambda_1 \Delta y\right)\right]}{\lambda_1 - \mathrm{A}_w} \tag{8}$$

In which, Δ_{Coh} is apparently positive, i.e. $\Delta_{\mathrm{Coh}} > 0$, for the aerofoil with arbitrarily positive A_w and λ_1 ($\mathrm{A}_w \neq \lambda_1$). It indicates that the lift on an aerofoil with finite span is more correlated than that of the turbulence, which is the inherent property of the unsteady aerodynamic force acting on the aerofoil passing throng arbitrary turbulent field.

3. EMPIRICAL SPANWISE COHERENCE OF BLUFF BODY

For bluff bodies Graham's 3D AAF is obviously invalid. However, the mathematical expression with respect to the lift distribution and coherence function of thin aerofoil still makes sense and can be taken as an empirical model.

Eq. (7) can be extended to define the spanwise coherence of gusty loads on bluff bodies by introducing three floating parameters into λ_1,

$$\lambda_1 = \sqrt{a_1 \cdot \left(2\pi b k_1\right)^{a_2} + a_3} \Big/ \left(2\pi b\right) \tag{9}$$

Where a_1, a_2, a_3 are floating parameters to be fitted using tested data. Meanwhile, Jakobsen's model is also used to describe the coherence of turbulence.

4. EXPERIMENTAL VALIDATION

Wind tunnel tests were carried out in grid-generated turbulence. The fluctuating pressures on strips with different separations for NACA0015 and two rectangular sections were measured. The measured results of lift coherence are fitted by the proposed double-exponent model (as shown in Fig.1-2).

5. CONCLUSION

Based on Ribner's three-dimensional theory, a double-exponent coherence model of lift on an aerofoil is proposed by means of inverse Fourier transform, which is of strictly physical significance and is extended to describe the spatial distribution of the lift on elongated bluff bodies. The wind tunnel tests confirm that the improved double-exponent is of high accuracy, which validates Graham's theoretical 3D AAF in the perspective of correlation.

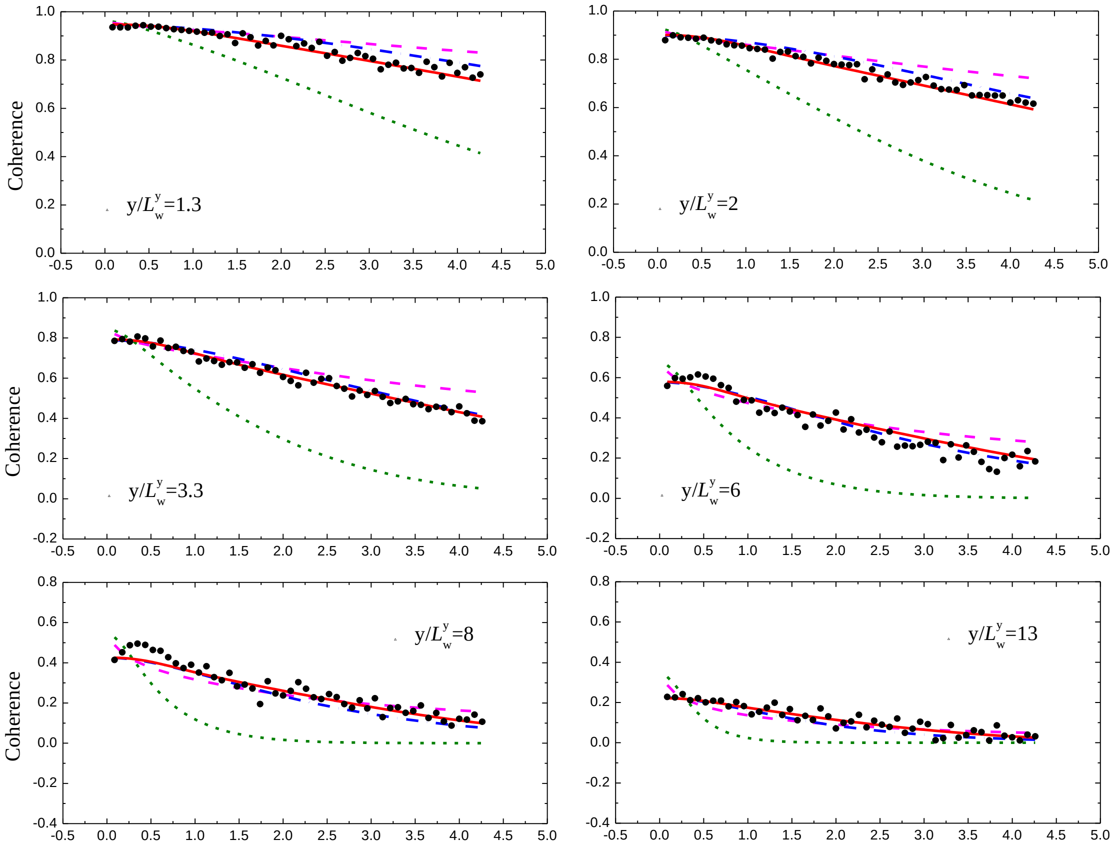

Fig. 1 Comparisons between direct measurements of the coherence of lift force on aerofoil with NACA0015 cross-section (▪) and theoretical calculations by improved double-exponent coherence model: (short dashed), double-exponent coherence with Mugridge's parameter; (dashed), Blake's parameter; (dash-dotted), three floating parameters; (solid), six floating parameters

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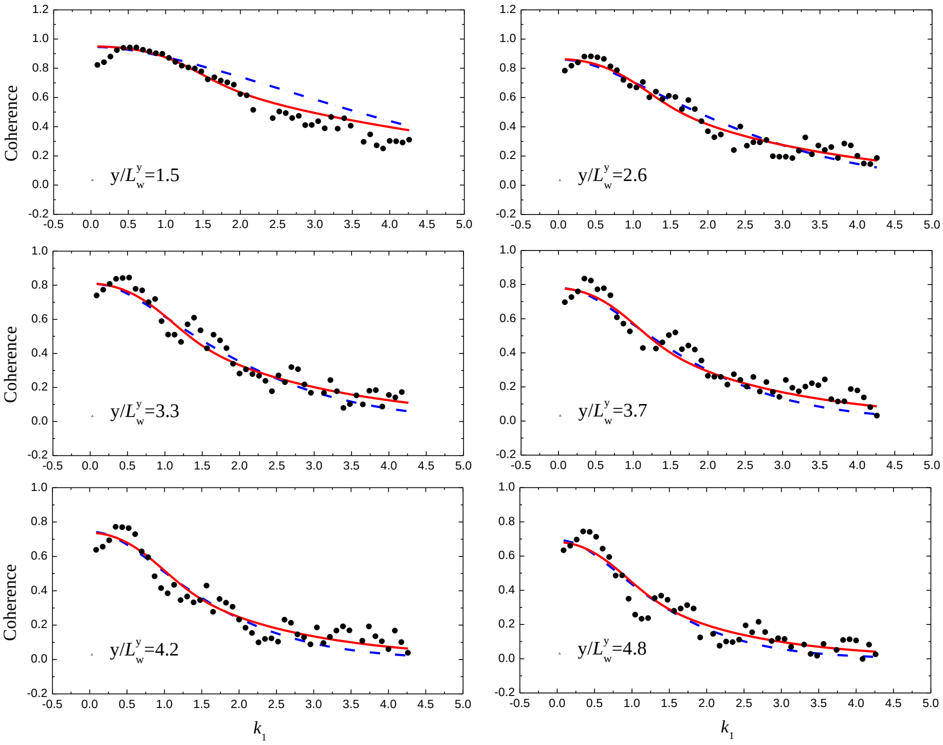

Fig. 2 Comparisons between direct measurements of the coherence of lift force on rectangular cylinder (▪) and the double-exponent coherence models of three floating parameters (dash-dotted) and six floating parameters (solid).

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