







$$u_{11} = \frac{\partial^2 u}{\partial \theta_1^2}, \quad u_{22} = \frac{\partial^2 u}{\partial \theta_2^2} \quad (10)$$

$$p = \pi(\theta_1, \theta_2) \quad (11)$$

$$p_1 = \frac{\partial p}{\partial \theta_1}, \quad p_2 = \frac{\partial p}{\partial \theta_2} \quad (12)$$

$$L_{20} = \frac{\partial^2 L}{\partial \theta_1^2}, \quad L_{02} = \frac{\partial^2 L}{\partial \theta_2^2} \quad (13)$$

$$L_{30} = \frac{\partial^3 L}{\partial \theta_1^3}, \quad L_{03} = \frac{\partial^3 L}{\partial \theta_2^3} \quad (14)$$

$$\sigma_{11} = (-L_{20})^{-1}, \quad \sigma_{22} = (-L_{02})^{-1} \quad (15)$$

And

$$\begin{aligned} E(u(\theta) | \vec{t}) &= u(\bar{\theta}_1, \bar{\theta}_2) + \frac{1}{2}(u_{11}\sigma_{11} + u_{22}\sigma_{22}) + p_1 u_1 \sigma_{11} + p_2 u_2 \sigma_{22} \\ &+ \frac{1}{2}(L_{30} u_1 \sigma_{11}^2 + L_{03} u_2 \sigma_{22}^2 + L_{21} u_2 \sigma_{11} \sigma_{22} + L_{12} u_1 \sigma_{22} \sigma_{11}) \end{aligned} \quad (16)$$

where,  $\bar{\theta}_1$  and  $\bar{\theta}_2$  are the classical MLEs for  $\theta_1$  and  $\theta_2$ , respectively (Miladinovic 2009).

## 5. Forecast model of extreme value wind speed

Thus, a Bayesian approximate estimate for  $\bar{t}_B$  is given by

$$\bar{t}_B = \bar{\mu} + 4.6\bar{\sigma} + p_2 u_2 \sigma_{22} + \frac{1}{2}(L_{30} \sigma_{11}^2 + L_{03} u_2 \sigma_{22}^2 + L_{21} u_2 \sigma_{11} \sigma_{22} + L_{12} u_1 \sigma_{22} \sigma_{11}) \quad (17)$$

where,  $\bar{\mu}$  and  $\bar{\sigma}$  are the classical MLEs for  $\mu$  and  $\sigma$ , respectively.

## 6. Numerical Analysis

In this section, a numerical study is presented to compare the maximum likelihood and Bayes estimates for determining the wind speed model subject to specified reliability. The numerical simulation was carried out in the following manner (Miladinovic 2009):

(1) Under the assumption that the scale parameter  $\sigma$  and the location parameter  $\mu$  behave independently and randomly, 100 location parameters  $\mu$  were simulated from the normal distribution. Location parameters from the normal distribution with mean 25 and variance equal to 4 were simulated.

(2) The scale parameter was assumed following the uniform distribution.  $\sigma$  was let equal to 2.

(3) Using the acquired 100 pairs of  $\mu$  and  $\sigma$ , 200 observations from the Gumbel PDF were generated and both the maximum likelihood and Bayes estimates of the wind speed were calculated.

(4) For comparison intention, the absolute value of the difference between the true wind speed and the corresponding ML and Bayes estimates for 99% reliability was calculated.

Because of the size of the simulation, some of all the numerical results are listed in table 1 under 99% reliability. In table 1, the size of the prior sample used to calculate the Bayes estimate  $\mu_B$  was presented, while  $\bar{\sigma}$  and  $\bar{\mu}$  are the ML estimates of the scale parameters and location.  $t_{\frac{1}{T}}$  is the true wind speed,  $\bar{t}_{\frac{1}{T}}$  is the ML estimates of wind speed,  $\bar{t}_B$  is the Bayes estimation value of wind speed,  $\left| \frac{t_{\frac{1}{T}} - \bar{t}_{\frac{1}{T}}}{t_{\frac{1}{T}}} \right|$  and  $\left| \frac{t_{\frac{1}{T}} - \bar{t}_B}{t_{\frac{1}{T}}} \right|$  represent the absolute value of the difference between the true wind speed, and maximum likelihood and Bayes wind speed estimates, respectively.

**Table 1** Numerical Study of the Gumbel wind speed

$\mu_B$	$\bar{\mu}$	$\sigma_B$	$\bar{\sigma}$	$t_{\alpha}$	$\bar{t}_{\alpha}$	$\bar{t}_B$	$\left  \frac{t_{\frac{1}{T}} - \bar{t}_{\frac{1}{T}}}{t_{\frac{1}{T}}} \right $	$\left  \frac{t_{\frac{1}{T}} - \bar{t}_B}{t_{\frac{1}{T}}} \right $
14.9081	15.1877	2	1.9201	24.2	24.0202	24.3331	0.74%	0.55%

In Table 1, the Bayes estimate is closer to the true wind speed than its maximum likelihood counterpart.

## 7. Engineering application

Based on the reliability analysis model of aerostatic instability from Ge (1997), wind speed is obtained by the Bayesian method, and the reliability indices of four long-span cable-stayed bridges are calculated by the JC method.

### 7.1 Limit state function

A limit state function can be represented by the critical aerostatic instability speed  $U_{cr}$ , minus the extreme value of wind speed  $U_e$ , or

$$G = U_{cr} - U_e = C_w U_f - G_s U_b \quad (18)$$

in which  $C_w$  is the wind conversion factor from a scaled model to the prototype structure, and  $U_f$  is the basic aerostatic instability speed which is experimentally

determined with some uncertainties of the structural properties, and  $G_s$  is the gust speed factor for taking account of the influence of wind gust speed and its horizontal correlation, and  $U_b$  is the basic wind speed at the bridge deck location, which can be determined as, for example, the hourly average or 10-min average from the past statistics at the site.  $C_w$ ,  $U_f$ ,  $G_s$  and  $U_b$  are random variables and must be described by probability distribution functions (Ge 2000).

**Wind conversion factor** The uncertainty related to the modeling of wind properties is treated by the wind conversion factor, which involves both mean and turbulent wind properties, such as mean wind profiles, intensities and scales of turbulence, functional distribution of velocity spectra, and spatial correlation between velocity components and so on. All these properties have their own random characteristics and may or may not be statistically dependent upon each other.

According to the comparison of model test results and full-scale observations, no clear evidence of deviation has been found through the process of conversion from model to prototype, though a significant variability certainly exists. Therefore, a normal distribution function with the following parameters could be chosen to approximately describe the wind conversion factor

$$E[C_w] = \mu_C = 1.0 \quad (19)$$

$$\sigma[C_w] = \sigma_C = 0.05\mu_C = 0.05 \quad (20)$$

**Basic aerostatic instability speed** A proper distribution function for the basic aerostatic instability speed may be estimated from wind tunnel tests of a scaled model, modified by the influence of possible structural uncertainties, such as mass, stiffness, material characteristics, manufacturing processes and mathematical idealization such as boundary conditions. All of these components influenced on dynamic properties of the structure and its critical conditions, may be simply described by the following empirical formula:

$$U_f = K_{td} f_t B \quad (21)$$

in which,  $B$ = width of the bridge deck;  $f_t$ = frequency of the lowest symmetric torsional model of the bridge deck; and  $K_{td}$  is expressed as

$$K_{td} = \sqrt{\frac{\pi^3}{2} \mu \left(\frac{r}{b}\right)^2 \frac{1}{C_M(0)}} \quad (22)$$

where  $\mu$  is the mass parameter and  $\mu = m / \pi \rho b^2$ , where  $m$  is the mass per unit length of the stiffened girder,  $\rho$  is the air density,  $b$  is the half width of the bridge deck;  $r$  is the radius of gyration of the deck cross-section, and is defined as

$r = \sqrt{I_m / m}$ , where  $I_m$  is mass moment of inertia per unit length of bridge deck; and  $C'_M(0)$  = gradient of torque moment coefficient  $C_M$  of main girder associated with zero wind attack angle.  $C_M$  is determined by the wind tunnel test.

Based on general experience, the basic aerostatic instability speed  $U_f$  may be statistically described as a log-normally distributed variable with the following properties:

$$E[U_f] = \mu_{U_f} = [U_{cr}] \quad (23)$$

$$\sigma[U_f] = \sigma_{U_f} = 0.127 \mu_{U_f} \quad (24)$$

where  $[U_{cr}]$  is the flutter speed directly determined by the empirical formula, the finite element method or the wind tunnel tests with a scaled aero-elastic model.

**Gust speed factor** Based on general experience, it is assumed that the gust speed factor  $G_s$  may be described by a normally distributed variable with the following distribution parameters:

$$E[G_s] = \mu_G = 1.0 \quad (25)$$

$$\sigma[G_s] = \sigma_G = 0.07 \mu_G \quad (26)$$

where the value of  $\mu_G$  is given Wind-resistant Design Specification for Highway Bridges (JTG/T D60-01-2004).

**Basic wind speed** For a very wide class of parent distributions including the daily wind records, the cumulative distribution functions of the extreme values taken from large random samples tends to converge to certain limiting forms of asymptotic extreme-value distributions: Type I or the Gumbel distribution, Type II or the Frechet distribution or Type III or the Weibull distribution. The best-fitted distribution of the wind speeds in most meteorological stations in China has been confirmed to follow a Gumbel distribution expressed as

$$F(U_b) = \exp[-\exp(-\frac{U_b - b}{a})] \quad (27)$$

where the parameters  $1/a$  and  $b$  are the dispersion and model, respectively. The mean value  $\mu$  and  $\sigma$  standard deviation of this distribution are related to these parameters by

$$\mu = \gamma a + b = 0.5772a + b \quad (28)$$

$$\sigma = \frac{\pi}{\sqrt{6}} a \quad (29)$$

in which  $\gamma = 0.5772\dots$  is Euler's constant.

### 7.2 Reliability analysis of four cable-stayed bridges

The parameters of the random variables of  $C_w$ ,  $U_f$ ,  $G_s$  and  $U_b$  of each bridge are shown in Table 2 through 5.

**Table 2** Random variables and their statistical properties of Sutong Yangtze Bridge - L=1088m

Random variables	Mean	Coefficients of variation	Distribution type	References
$C_w$	1	0.05	Normal	(Su 2013)
$U_f$	313 (empirical formula) 200 (finite element)	0.127	Lognormal	(Su 2013 and SLDRCE 2002)
$G_s$	1.18	0.1	Normal	(Su 2013 and SLDRCE 2002)
$U_b$	33.6322 (Bayes estimates) 32.4717 (ML estimates)	0.15	Extreme type I	(Su 2013 and SLDRCE 2002)

**Table 3** Random variables and their statistical properties of Second Jiujiang Bridge - L=818m

Random variable	Mean	Coefficients of variation	Distribution type	References
$C_w$	1	0.05	Normal	(Su 2013)
$U_f$	297 (empirical formula) 160 (finite element)	0.127	Lognormal	(Su 2013 and SLDRCE 2012)
$G_s$	1.19	0.1	Normal	(Su 2013 and SLDRCE 2012)
$U_b$	25.0194 (Bayes estimates) 25.6458	0.15	Extreme type I	(Su 2013 and SLDRCE 2012)



(ML estimates)

**Table 4** Random variables and their statistical properties of Edong Yangtze Bridge - L=926m

Random variables	Mean	Coefficients of variation	Distribution type	References
$C_w$	1	0.05	Normal	(Su 2013)
$U_f$	189.8 (empirical formula) 109 (finite element)	0.127	Lognormal	(Su 2013 and SLDRCE 2006)
$G_s$	1.21	0.1	Normal	(Su 2013 and SLDRCE 2006)
$U_b$	22.3157 (Bayes estimates) 23.0178 (ML estimates)	0.15	Extreme type I	(Su 2013 and SLDRCE 2006)

**Table 5** Random variables and their statistical properties of Second Humen Bridge - L=1200m

Random variables	Mean	Coefficients of variation	Distribution type	References
$C_w$	1	0.05	Normal	(Su 2013)
$U_f$	251 (empirical formula) 194 (finite element)	0.127	Lognormal	(Su 2013 and WTLCU 2011)
$G_s$	1.17	0.1	Normal	(Su 2013 and WTLCU 2011)
$U_b$	25.8739 (Bayes estimates) 24.9112 (ML estimates)	0.15	Extreme type I	(Su 2013 and WTLCU 2011)

On the basis of the JC approach, a computational program was developed to compute the reliability indices  $\beta$ , which are numerically listed in Table 6 and compared to the traditional safety factors defined as  $K = [U_{cr}] / \mu_G U_d$  (JTG 2004), which listed in Table 7.

**Table 6** Reliability indices of four cable-stayed bridges

Bridge name	empirical formula		finite element analysis	
	Bayes estimates	ML estimates	Bayes estimates	ML estimates
Sutong Yangtze bridge	7.9137	8.0468	6.2571	6.3845
Second Jiujiang bridge	8.8140	8.7182	6.4905	6.4005
Edong Yangtze bridge	7.4807	7.3651	5.4599	5.3495
Second Humen Bridge	8.1034	8.2480	7.1368	7.2775

**Table 7** Safety factors of four cable-stayed bridges

Bridge name	empirical formula		finite element analysis	
	Bayes estimates	ML estimates	Bayes estimates	ML estimates
Sutong Yangtze Bridge	7.89	8.17	5.04	5.22
Second Jiujiang Bridge	9.98	9.73	5.37	5.24
Edong Yangtze Bridge	7.03	6.81	4.04	3.91
Second Humen Bridge	8.29	8.61	6.41	6.66

It can be concluded from Table 6 and 7 that the reliability indices of Bayes estimates are smaller than those of ML estimates for Sutong Yangtze Bridge and Second Humen Bridge, while the reliability indices of Bayes estimates are larger than those of ML estimates for Second Jiujiang Bridge and Edong Yangtze Bridge, which

are all same as the safety factors. In addition, the reliability indices of empirical formula are larger than those of finite element analysis for all the cable-stayed bridges. Finally, the reliability indices of Bayes estimates are either smaller or larger than those of ML estimates for the long-span cable-stayed bridges.

## 8. conclusion

In this paper, a forecast method based on Bayesian theory of wind speed was introduced, upon which the reliability analysis model was established. The reliability indices were calculated using the JC approach based on the empirical formula and finite element analysis, in which the results of Bayes estimates and ML estimates were compared and indicate that probability assessment and reliability analysis of aerostatic instability of cable-stayed bridges under stochastic wind loading based on Bayesian theory are more precise and scientific, which is significantly valuable in the application of practical engineering projects.

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