$$B = \begin{bmatrix} 0 & 0 & -\frac{1}{m_1} & \frac{1}{m_1} + \frac{1}{m_d} \end{bmatrix}^T \quad (5)$$

is a location vector of *u(t)*

$$H = \begin{bmatrix} 0 & 0 & \frac{1}{m_1} & -\frac{1}{m_1} \end{bmatrix}^T \quad (6)$$

is a location vector of *f(t)*.

## 3. OPTIMUM PARAMETERS OF PTMD

Optimum parameters of PTMD for minimizing rms responses of the primary structure are mass ratio, tuning natural frequency ratio and damping ratio of PTMD to primary structure. While the basic concepts of how PTMD suppressing primary vibrating structures has been established, the optimum parameters of PTMD could be different for different primary structures and external loading conditions (Ayoringde 1980). Warburton investigated the optimum parameters of PTMD for minimizing the rms responses of the undamped primary structure under random loads(Warburton 1981,1982). Krenk derived the optimum parameters of PTMD for minimizing the rms response of the damped primary structure under random loads under the condition that the mass ratio is small and the primary structure's damping ratio is less than that value of PTMD (Krenk 2008) as follows

$$f_{opt} = \frac{1}{1+\mu} \quad (7)$$

$$\xi_{opt} = \frac{\sqrt{\mu}}{2} \quad (8)$$

where, $f_{opt}$= optimum tuning frequency ratio; $\xi_{opt}$= optimum damping ratio; $\mu$=mass ratio. It was pointed out that tuning the natural frequency of PTMD to the fundamental natural frequency in the primary structure is more effective than tuning to different natural frequencies(Kareem 1995).
Optimum parameters of ATMD for minimizing the rms responses of the main structure are similar to that for PTMD.

## 4. LINEAR QUADRATIC REGULATOR CONTROLLER

The LQR control method is a widely used modern optimal control technique in structural vibration control problems (Dorato 1995). In LQR control law, all continuous time state-space variables are available and linear dynamic equations of motion of the system can be written in terms of the state-space formulation as shown in Eq. (3). The external force term in Eq.(3) can be treated as a noise input hence Eq. (3) can be written as (Dorato 1995).

$$\dot{X}(t) = AX(t) + Bu(t) \tag{9}$$

The object of LQR control law is seek to find out a state-feedback optimal control force *u(t)* that minimize the deterministic cost functional J maintaining the state close to the zero state. The cost functional J is given by

$$J = \int_0^\infty \left( X(t)^T QX(t) + u(t)^T Ru(t) \right) dt \tag{10}$$

where Q is a positive semi-definite state weighting matrix and R is a positive definite control weighting matrix. where Q and R are positive semi-definite and positive definite weighting matrices. The term $X(t)^T QX(t)$ in Eq.(10) is a measure of control accuracy and the term $U(t)^T Ru(t)$ is a measure of control effort. Minimizing J with keeping the system response and the control effort close to zero needs appropriate choice of the weighting matrices Q and R(Suhardjo 1992). If it is desirable that the system response be small , then large values for the elements of Q should be chosen with selecting the matrix Q to be diagonal and to make the diagonal element large value for any respective state variable to be small. If it wants the control energy be small, then large values of the elements of R should be chosen(Suhardjo 1992).

The state-feedback optimal control force *u(t)* is derived as(Dorato 1995).

$$u(t) = -KX(t) \tag{11}$$

where $K = R^{-1}B^T P$

In Eq. (11), *K* is called an optimal controller gain and *P* is the unique, symmetric, positive semi-definite solution to the algebraic Riccati equation(ARE) given by

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{12}$$

Then the closed-loop system using the optimal control force u(t) becomes

$$\begin{aligned} X &= (A-BK)X(t) \\ &= AcX(t) \end{aligned} \tag{13}$$

where Ac is the closed-loop system matrix
In LQR control law, the cost functional J keep minimize means that larger value of state weighting matrix Q makes the state X(t) must be smaller, that is, the poles of the closed-loop system matrix Ac further left in the s-plane so that the state X(t) decays faster to zero. On the other hand larger value of control weighting matrix R makes the control force *u(t)* be smaller(Dorato1995).

## 5. NUMERICAL SIMULATION OF FLUCTUATING ALONG-WIND LOADS

Fluctuating along-wind load treated as a random process of stationary Gaussian white noise can be simulated numerically in time domain using along-wind load power spectral density data .That is particularly useful fir some response estimation which are more or less narrow banded random process such as an along-wind response of a tall building. The numerical simulation procedure presented in this work is taken from Shinozuka (Shinozuka 1987).

$$f(t) = \sum_{k=1}^{N} \sqrt{2S_F(\omega_1)\Delta\omega}\cos(\omega_k t + \phi_t) \tag{14}$$

where $S_F(\omega_1)$ = the value of the spectral density of along-wind load corresponding to the first modal resonant frequency.

$\Delta\omega = (\omega_u - \omega_\ell)/N$

$\omega_k = \omega_\ell + (k - 1/2)\Delta\omega$

$\omega_u$ = upper frequency of S(ω)

$\omega_\ell$ = lower frequency of S(ω)

$\Phi_t$ = uniformly distributed random numbers between 0~2π

N = number of random numbers

The along-wind load power spectral density used in in equation (14) is that by G.Solari as follows[18]

$$S_F(n) = \left[\rho BHC_D\bar{V}(h)\sigma_v(h)K_b\right]^2 S_{veq}^{\ *}(n) \tag{15}$$

Where

$$S_{veq}*(n) = \frac{Sv(h;n)}{\sigma_v^{\ 2}(h)} L\left[0.4\frac{nC_xB}{\bar{V}(h)}\right]$$

$$\frac{1}{C_D^{\ 2}}\left[C_W^{\ 2} + 2C_WC_\ell L\left[\frac{nC_yD}{\bar{V}(h)}\right] + C_\ell^{\ 2}\right] \ L\left[0.4\frac{nC_zH}{\bar{V}(h)}\right]$$

Where

$$L(\eta) = \frac{1}{\eta} - \frac{1}{2\eta^2}(1 - e^{-2\eta})$$

$$K_b = \frac{1}{H\bar{V}(h)\sigma_V(h)}\int_0^H \bar{V}(z)\sigma_V(z)\psi_1(z)dz$$

$$\frac{nS_v(z;n)}{\sigma_v^{\,2}(z)} = \frac{6.868\dfrac{fL_v}{z}}{(1+10.302\dfrac{fL_v}{z})^{5/3}}$$

where

$$f = \frac{nz}{\overline{V}(z)}$$

$C_x$, $C_z$ = lateral and vertical exponential decay coefficients
$C_y$ = cross-correlation coefficient of pressure acting on the windward and leeward face
$L_v(h)$ = integral length scale of turbulence at height h
ρ = air density
B = width of building
H = height of building
h = reference height of building
$C_D$ = drag coefficient
$C_m$, $C_e$ = absolute values of mean pressure coefficients on windward and leeward face
$\overline{V}$ = mean wind velocity
$\sigma_v$ = standard deviation of longitudinal turbulence
n = frequency
$S_F(n)$ = power spectrum of first fluctuating modal force

## 6. NUMERICAL EXAMPLE

This numerical example is from "Numerical Examples" in reference (Solari 1993). The tall building's height H=180 m, width B=60m, depth D=30m, first modal natural frequency $n_1$=0.27Hz, critical damping ratio=0.015etc. h=120m, V(h)=40.96m/s, $\sigma_v(h)$=5.39m/s, $L_v(h)$=582.48m, $C_x$=16, $C_z$=10, $C_w$=0.8, $C_\ell$=0.5, $K_b$=0.5, etc Another data for along-wind load and properties of building were in (Solari 1993). The optimum parameters of ATMD were considered as the same value of PTMD. The optimum parameters of ATMD, with a different mass ratio $\mu_{AP}$ of ATMD to PTMD $\mu_{AP}$=0.01, 0.03, 0.05, 0.1, 0.3, 0.5, tuning frequency $f_{opt=}$and damping ratio $\xi_{opt}$=0.05.
The numerically simulated along-wind load and response without ATMD are shown in Fig.2 and Fig.3. The ms response without ATMD shown in Fig.3 is 0.0274, which is good approximation to that of Solari's closed form response of 0.027m (Solari 1993). For estimating LQR controller , the weighting matrix Q and R are selected as

$$Q = 1.0 * 10^{8} * \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad R = [1.0 * 10^{-12}]$$

The controlled responses with ATMD with a different mass ratio $\mu_{AP}$ of ATMD to PTMD $\mu_{AP}$=0.01,0.03,0.05,0.1,0.3,0.5 are presented in Fig.4~Fig.9.The controlled rms responses with ATMD are reduced to 20%~28% of the rms response without ATMD, which shows that ATMD is effective in mitigating wind-induced vibrations of a tall building.

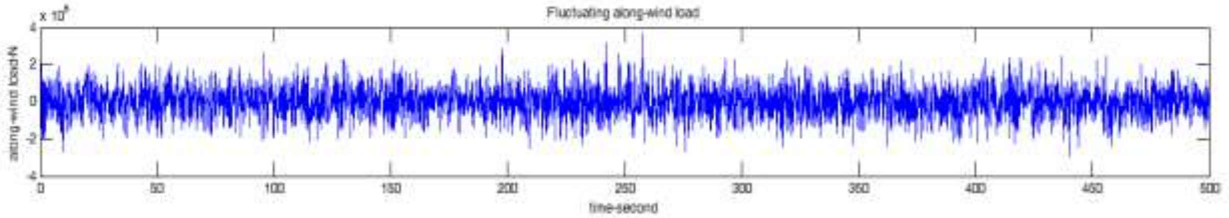


Fig.2 Fluctuating along-wind loads

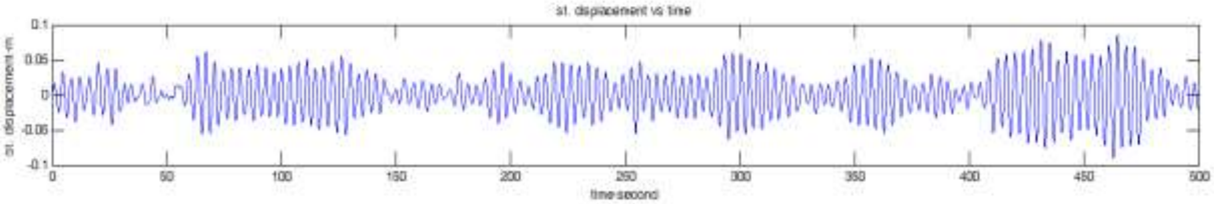


Fig.3 Along-wind responses without ATMD ( rms=0.0274)

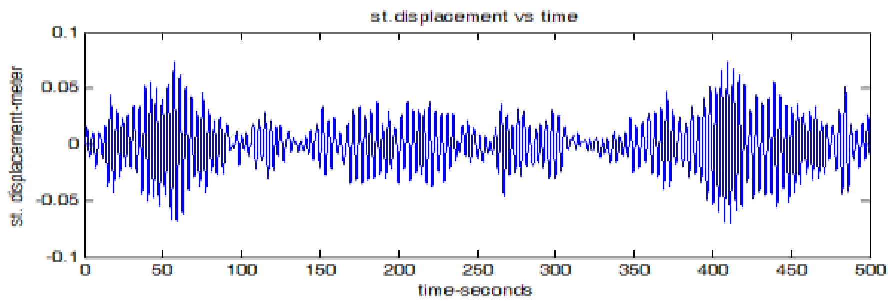


Fig.4 Along-wind responses with ATMD ($\mu_{AP}$=0.01, rms=0.0222)

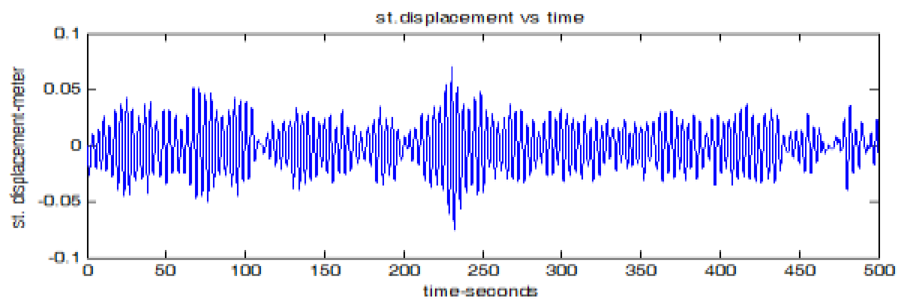


Fig.5 Along-wind responses with ATMD ($\mu_{AP}$=0.03, rms=0.0200)

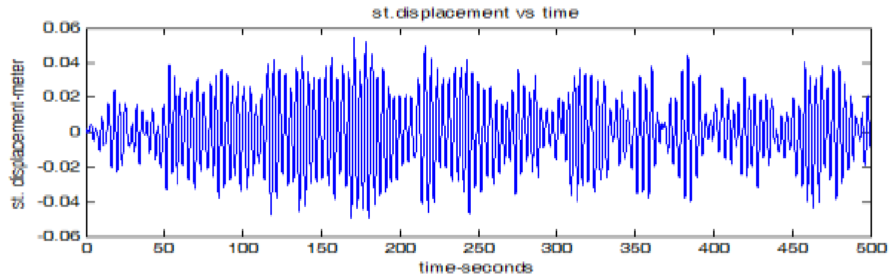


Fig.6 Along-wind responses with ATMD ($\mu_{AP}$=0.05, rms=0.0196)

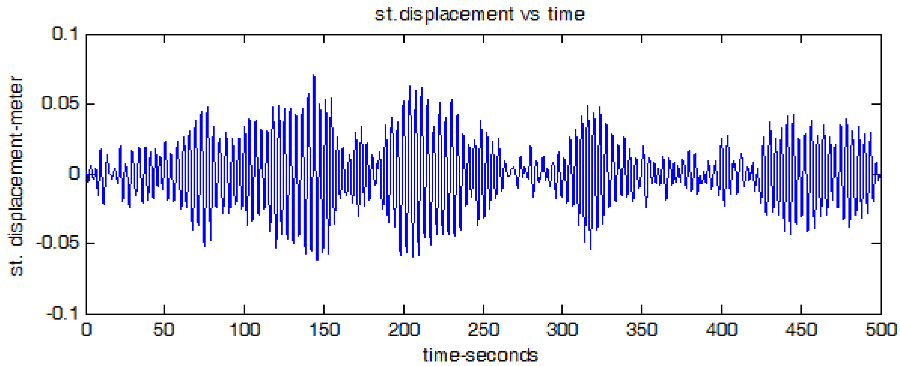


Fig.7 Along-wind responses with ATMD ($\mu_{AP}$=0.1, rms=0.0219)

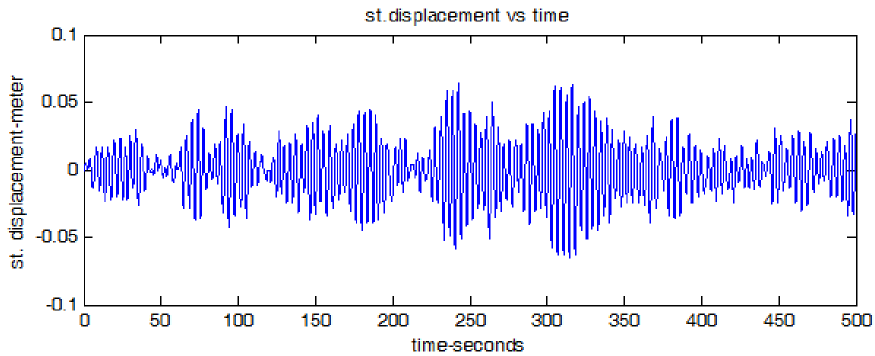


Fig.8 Along-wind responses with ATMD ($\mu_{AP}$=0.3, rms=0.0210)

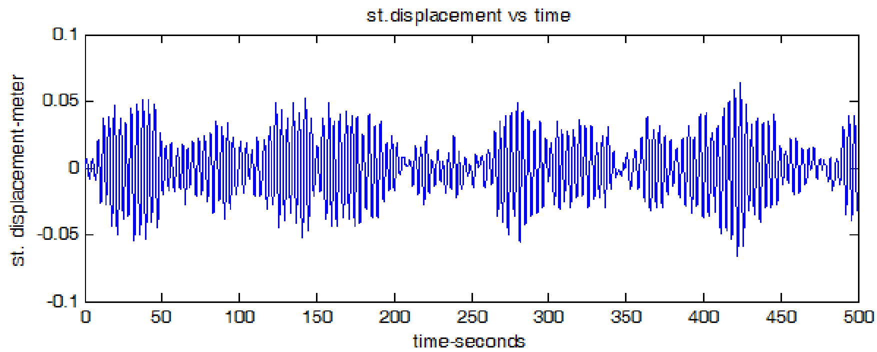


Fig.9 Along-wind responses with ATMD ($\mu_{AP}$=0.5, rms=0.0216)

## 7. CONCLUSIONS

The performance of ATMD for mitigating along-wind responses of a tall building is investigate. Optimal control force generated by actuator of ATMD is estimated by LQR controller Fluctuating along-wind load is simulated numerically using along-wind load spectra by Solari. The rms responses with ATMD are reduced to 20%~28% of the rms response without ATMD. Therefore, ATMD system is effective in mitigating wind-induced vibration of a tall building.

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## REFERENCES

Frahm, H. (1909), “ Device for damped vibration of bodies”, U.S. Patent No. 989958. October 30.

Den Hartog, J.P. (1956) ,Mechanical Vibration, 4th edn. McGraw-Hill, New York,

Ayoringde, E.O. and Warburton, G.B. (1980), “Minimizing structural vibrations with absorbers”, Earthquake Engineering and Structural Dynamics, 8, 219-236.

Warburton,G.B.(1981),”Optimum absorber parameters for minimizing vibration response”, Earthquake Engineering and Structural Dynamics, 9, 251-262.

Warburton,G.B.(1982),”Optimum absorber parameters for various combinations of respose and excitation parameters”, Earthquake Engineering and Structural Dynamics, 10,381-401.

Housner, G.W., Bergman, L.A., Caughey, T.K., Chassiakos, A.G., Claus, R.O., Masri,S.F., Skelton, R.E., Soong, T.T. ,Spencer, B.F. and Yao, J.T.P. (1997), “Structural control: past, present, and future”, Journal of Engineering Mechanics ,ASCE, 123(9), 897-971.

Krenk, S. and Hogsberg, J. (2008), “Tuned mass damped structures under random load”, Probabilistic Engineering Mechanics, 23: 408-415.

Kareem,A. and Kline,S.(1995),”Performance of multiple mass damper under random loading”,Jour.of Structural Engineering,ASCE,121(2),348-361.

McNamara, R.J. (1977), “Tuned mass dampers for buildings”, Journal of the Structural Division. 103: 1785-1798.

Wang,C.M.,Yan,N. and Balendra,T.(1999),”Control on dynamic structural response using active-passive composite- tuned mass damper”, Journal of Vibration and Control,5,475-489.

Chang.,J.C.H and Soong,T.T (1980), “Structural Control Using Active Tuned Mass Dampers”, Jour. of Engineering. Mech. Div. ASCE,106 , 1091-1098.

Dorato,G., Abdallah, C.and Cerone,V.(1995),” Linear Quadratic Control”, Prentice Hall, New Jersey.

Suhardjo, J.Spencer, B.F. and .Kareem, A (1992),.”Active Control of Wind-Excited Buildings: A Frequency Domain Based Design Approach, Jour.Wind Eng. Ind. Aerodyn. 41-44, 1985-1996.

Ankireddi, S. and.Yang, H.T.Y.(1996), “Simple ATMD Control Methodology for Tall Buildings Subject to Wind Loads”, J. Struct. Engng. ASCE 12283-91.

Ankireddi, S.and Yang, H.T.Y.(1997),” Multiple Objective LQG Control of Wind-Excited Buildings”, Jour of Struct. Engng. ASCE.123, 943-951.

Ricciardelli, F.,.Pizzimenti, A.D. and Mattei, M. (2003),” Passive and Active Mass Damper Control of the Response of Tall Buildings to Wind Gustiness”, Jour. of Engng. Struct, 25, 1199-1209.

Yang, J.N., Lin, S., Kim,J.H. and. Agrawal, A.K. , (2002), “Optimal Design of Passive Energy Dissipation Systems Based on H and H2 Performances,” J.Earthquake Engng.Struct.Dyn. 31921-936.

Yang, J.N., Lin, S.and Jabbari, F. (2003), “H2-based Control Strategies for Civil Engineering Structures”, J. Struct.Control 10, 205-230.

Stavroulakis,G.E., Marinova,., D.G., Hadjigeorgiou, E., Foutsitzi, G. and Bantiotopoulos, C.C. (2006), ”Robust Active Control against Wind-Induced Structural Vibrations”, J.Wind Eng. Ind. Aerodyn, 94,895-907.

Solari,G.(1993), “Gust Buffeting.2: Dynamic Alongwind Respose”. Jour.of Structural Engineering, ASCE,119(2), 383-398.

Shinozuka, M. (1987), “Stochastic fields and their digital simulation”, edited by Schueller, G.I. & Shinozuka, M. Martinus Nijhoff Publishers, Stochastic Methods in Structural Dynamics: 93-133.