

Simultaneous geometry and size optimization of diagrids against equivalent wind loading

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ABSTRACT

Diagrid is known as an efficient resistant system for tall buildings which has received much attention in recent years. The present work reveals a problem formulation to optimize performance characteristics of such a system against equivalent lateral wind loading. The design vector is defined to include not only structural member sizing but also variation in geometry and topology of diagrids among the building height. In order to solve this problem, a *pseudo-random directional search* is utilized. An especial coding technique is also used to efficiently handle practical variation of diagrid modules. Performance of this method is compared with some well-known optimization methods in a number of three dimensional structural models. The results show superior efficiency of the proposed algorithm over the standard particle swarm optimization and also better performance of the achieved optimal designs with respect to sizing-only designs to resist the imposed loadings with minimal consumption of structural material.

1. INTRODUCTION

Diagrid system is shown to be more efficient than many other lateral resistant systems in tall buildings (Ali and Moon 2007; Boake 2014; Mele et al. 2014), however, its performance is highly dependent to its layout design among the structure's height and width. Moon et al. (2007) developed a simplified methodology for stiffness-based sizing design of diagrid members. Several parametric studies were performed on diagrid configurations in tall buildings with different aspect ratios (Moon 2008, 2012). Shahrouzi et al. (2015) treated similar problems utilizing *Mine Blast Algorithm*. It was concluded that the corresponding structural efficiency can be maximized by optimization of grid geometries.

The present work concerns this matter by formulating it as an optimization problem to be solved. The proposed formulation allows even non-uniform diagrid angles for a better search toward the optimal design. Such a systematic optimization of the structure includes both member sizing and geometry of the employed modules, simultaneously. Since cardinality of the alternatives' space is extensively more than can be solved by trail and error, some optimization methods are utilized here including

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Pseudo-random Directional Search. Comparison of results shows superior performance of PDS both in structural responses and algorithmic scope of view.

2. PROBLEM FORMULATION

In optimal design of a tall building with diagrids, it is desired to minimize the material consumption provided that the code-based stress and deflection requirements are satisfied. Such a problem is formulated here so that member sizing and digrid layout can be simultaneously altered to obtain feasible lightest structure.

$$\text{Minimize } f = \rho \sum_i A_i L_i \times (1 + K_p \sum_j \max(0, g_j(\underline{X}))^2), \quad (1)$$

The first term in Eq. (1) denotes total weight of a structure with the i^{th} -member length of L_i and cross sectional area of A_i where ρ stands for the material density. The second applies penalty for every j^{th} violated constraint in the typical form of $g_j(\underline{X}) \leq 0$. K_p is the prescribed penalty coefficient. The behavioral constraints include design-code requirements for the resulted stress and displacement of the model. They are usually normalized to the allowable limits of member stress and story sway, respectively. The design vector is structured as in Eq. (2).

$$\underline{X} = \begin{Bmatrix} \underline{S} \\ \underline{Z} \end{Bmatrix}, \quad (2)$$

Section indices to be associated for structural members are denoted by $\underline{S} = \{s_1, \dots, s_m\}$ while $\underline{Z} = \{z_1, \dots, z_n\}$ corresponds the geometrical part of such a design vector. In the present work, the integer value of any z_d means the number of storey levels covered by the d^{th} module of diagrid system. The d^{th} index varies from with 1 for the lowermost module to n for the one connected to the roof level.

2. OVERVIEW OF DIRECTIONAL SEARCH

Several meta-heuristic algorithms have already been introduced to solve engineering optimization problems. A majority of these methods apply graduate improvement of a current search candidate via the following equation.

$$\underline{X}^{new} = \underline{X}^{old} + \underline{V}^{new}, \quad (3)$$

\underline{X}^{new} and \underline{X}^{old} are the design vectors at the current and previous iterations of the main algorithm, respectively. The present work concerns two algorithms in this class of search methods with different procedures in calculating \underline{V}^{new} ; i.e. the velocity vector. Both the following algorithms are initiated with a random population of particles provided that they fall within lower and upper bounds of the design variables.

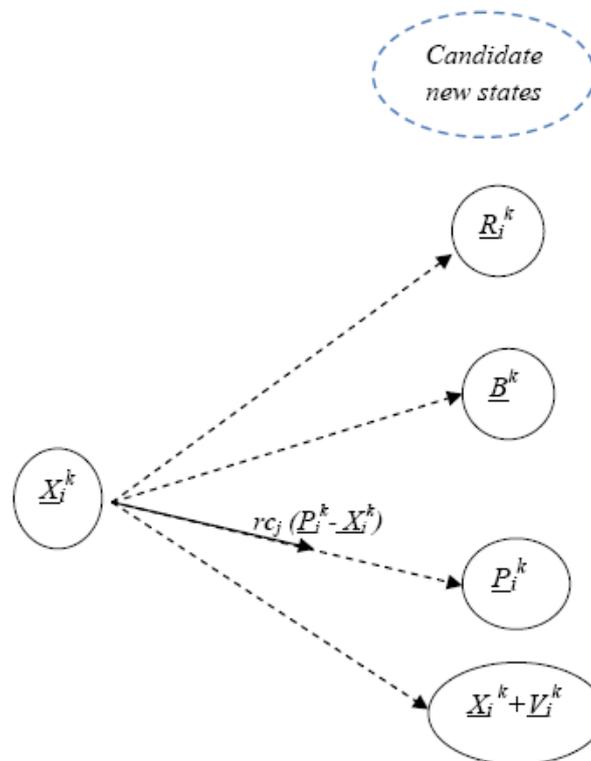


Fig. 1 Schematic selection of states by a search agent

2.1 Particle Swarm Optimization

PSO is a well-known population-based meta-heuristic inspired by swarm intelligence of bird flocks (Kennedy and Eberhart, 2001). Every artificial bird or particle denotes a candidate solution which takes the corresponding value of the design variables as its position vector. In order to update the position of the i^{th} bird in the k^{th} iteration, PSO applies the following formula.

$$\underline{V}_i^{k+1} = c_i \underline{V}_i^k + rand \times c_c \times (\underline{P}_i^k - \underline{X}_i^k) + rand \times c_s \times (\underline{B}^k - \underline{X}_i^k), \quad (4)$$

Three terms exist in Eq. (4) as:

- **Inertial term:** that is moving in the same direction of the previous movement, $\underline{V}_i^{old} = \underline{V}_i^k$.
- **Cognitive term:** in which a particle notices its best experienced position up to the current iteration. Such a position is denoted by \underline{P}_i^k or the i^{th} particle in the population.
- **Social term:** which simulates moving toward \underline{B}^k ; i.e. the global best position among all the particles up to the current iteration.

In Eq. (4) c_i , c_c and c_s stand for the prescribed inertial, cognitive and social coefficients, respectively and $rand$ function generates uniform random numbers between 0 and 1.

At every iteration, the new position of an i^{th} particle is updated by Eq. (3) and Eq. (4). It is accomplished for all the particles and the procedure is repeated for a prescribed number of iterations. The global best vector at the final iteration is then announced as the optimal solution.

2.2 Pseudo-random Directional Search

PSO is in fact a directional search which applies new velocity for each particle by vector-sum of Eq. (4) in any walk in the design space. Another way of generating new solutions is to select only one direction at a walk step, instead of applying vector-sum of such terms. It is utilized in *Pseudo-random Directional Search*, PDS, to allow the particle making any its walk at a distinct direction among the prescribed movement terms in the algorithm (Shahrouzi 2011). A modified variant of PDS is introduced as follows with fewer parameters to be tuned for engineering design.

According to the utilized PDS, at the $k+1^{\text{th}}$ iteration, any i^{th} particle moves by the following velocity formula.

$$\underline{V}_i^{k+1} = 2 \times \text{rand} \times (\underline{S}_j^k - \underline{X}_i^k), \quad (5)$$

Each term in PDS terminology is called a state. The j^{th} state is selected among $\underline{S}_i^k \in \{\underline{X}_i^k + \underline{V}_i^k, \underline{P}_i^k, \underline{B}^k, \underline{R}_i^k\}$ by Eq. (6) where \underline{R}_i^k is a random particle in the current population.

$$j = \begin{cases} \arg \max_i (P_{i,l}) & \text{if } r \leq q_0 \\ j^P & \text{if } q_0 < r \leq q_1 \\ j^R & \text{otherwise} \end{cases}, \quad (6)$$

in which, q_0, q_1 are positive control parameters less than unity and $r = \text{rand}$. $P_{i,l}$ is a probability measure for choosing the l^{th} state by the i^{th} particle. It is calculated as:

$$P_{i,l} = \frac{\tau_{i,l}}{\sum_{h=1}^{\text{NumStates}} \tau_{i,h}}, \quad (7)$$

j^P is a state number resulted by a *Roulette Wheel* procedure using such probability values. $\tau_{i,h}$ denotes the existing amount of pheromone on the edge connecting the vertex i on the particles side to the vertex h on the states side of a bi-partite graph.

The algorithm uses indirect information share by pheromone deposit and evaporation via the following relation:

$$\tau_{i,h}^{(k+1)} = (1-\alpha) \times (\tau_{i,h}^{(k)} + \alpha), \quad (8)$$

Both the evaporation and deposit ratios are implemented here by α . The pheromone matrix $[\tau_{i,h}]$ is initiated with 0 at diagonals and 1 at the other components. During the optimization, it is reinitiated when the minimum pheromone falls below a prescribed value. The algorithm is started with random velocities and is repeated for N_{\max} number of iterations.

3. NUMERICAL SIMULATION

Performance of the aforementioned algorithms in the optimal design of diagrids is evaluated treating a number of structural models under gravitational and wind loading. Material properties are taken as in Table 1 and general notation of the treated model is given in Table 2.

For all the examples, typical story height of 3.00 m and bay length of 4.00 m are applied. A dead load of $DL = 600\text{ kgf/m}^2$ and a live load of $LL = 200\text{ kgf/m}^2$ is distributed at any floor and further implemented in the following load combinations:

- 1) DL
- 2) $DL + LL$
- 3) $DL \pm 0.84WL_x, DL \pm 0.84WL_y$ (9)
- 4) $DL + 0.75(LL \pm 0.84WL_x), DL + 0.75(LL \pm 0.84WL_y)$

in which the wind loads WL are calculated due to *Iranian National Building Code* for a base wind speed of 130 km/h . It is distributed height-wise among all three dimensional faces of the treated models (Fig. 2).

Table 1: Properties of the employed structural material

Material grade	Density $\rho(\text{kg} / \text{m}^3)$	Poisson ratio ν	Stiffness modulus $E(\text{kgf} / \text{m}^2)$	Yield stress $F_y(\text{kgf} / \text{m}^2)$
St-37 Steel	7850	0.3	2.1×10^{10}	2.4×10^7

Table 2: Notation of the treated models

Model ID	# of Stories	# of X-bays	# of Y-bays
20st	20	6	6

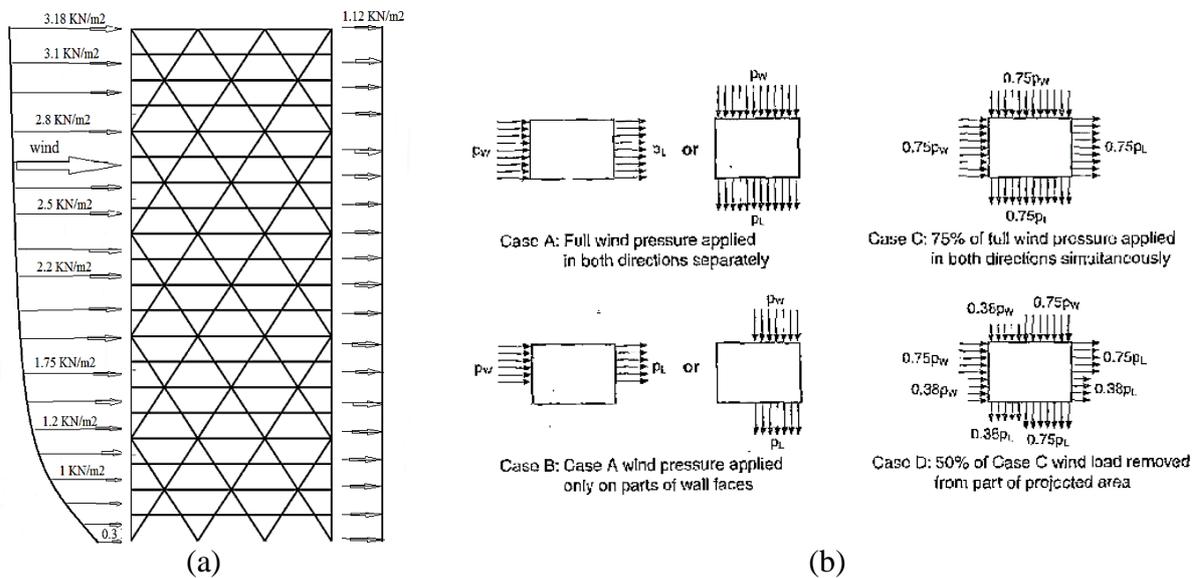


Fig. 2. Wind pressure and suction among the (a) height and (b) plan of a typical 20st model

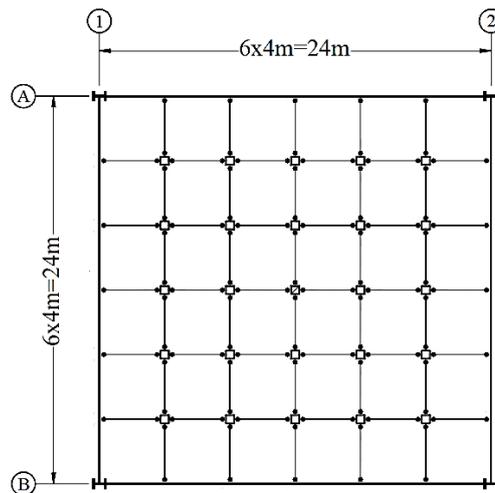


Fig. 3. Plan of the building

Optimization parameters for each example are tuned and the results are derived using several trial runs. Table 3 reports such coefficients for the employed PSO. According to Table 4, the proposed PDS algorithm is run with one more control parameter than PSO. Note that tuned values for cognitive and social coefficients in PSO are almost 2 while the inertial coefficient is linearly decreased from 0.9 to 0.1 during iterations of the algorithm.

Table 3: Applied control parameters of PSO

Population	N_{\max}	c_i	c_c	c_s
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Size				
11	250	0.9~0.1	2.1	1.9

Table 4: Applied control parameters of PDS

Population Size	N_{\max}	q_0	q_1	α
11	250	0.3	0.9	0.1

Structural members are symmetrically divided into 3 groups: beams, columns and diagrid bracings. For each group 16 sections is available to be selected during the optimization. For example there are so 16^{10} options for designing the 20st model; that means a quite large search space.

A number of issues are then investigated including the type of optimization algorithms and formulations. For the latter case, two types of problem formulation are implemented: at the 1st type, diagrid angles are kept fixed while in the 2nd, they are released so that non-uniform modules can arise in the optimal design.

3.1 Performance of the algorithms in simultaneous size and geometry optimization

In this example, 20st model is treated using the problem formulation of Eq. (1). Convergence history of the best result in Fig. 4, shows superior efficiency and effectiveness of PDS over *Particle Swarm Optimization*, *Harmony Search* (Lee and Geem, 2005) and *Mine Blast Optimization* (Sadollah et al. 2012).

Furthermore, a statistical study is performed by several independent runs leading to the results of Table 5. It confirms that PDS has obtained the least weight among the best results; however, it has been almost similar to PSO in the mean and worst results in this example. According to Fig.4, it is also evident that the least computation belongs to HS in the charge of the worst result. Setting aside the HS, the 1st rank belongs to PDS both in lower time consumption and better final result.

Table 5: Statistical results of structural weight (tons) for 20st model by different optimization algorithms

Method	Best weight	Mean weight	Worst weight
PDS	148	162	172
PSO	154	162	170
MBO	172	182	209
HS	182	194	215

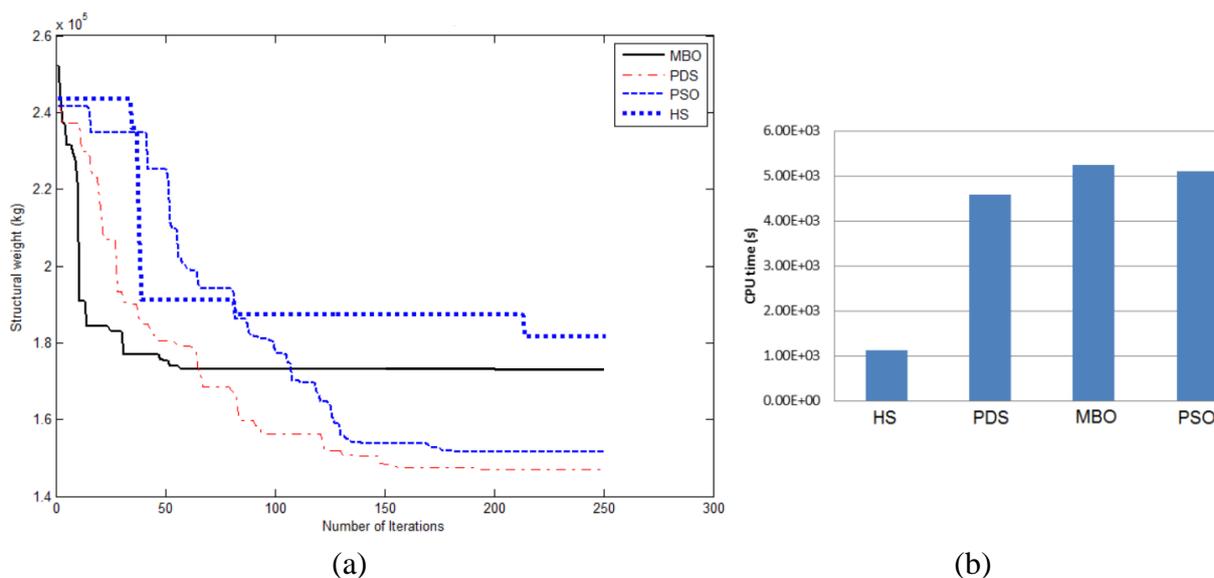


Fig. 4 Comparison of (a) convergence and (b) elapsed time for the 20th optimal design

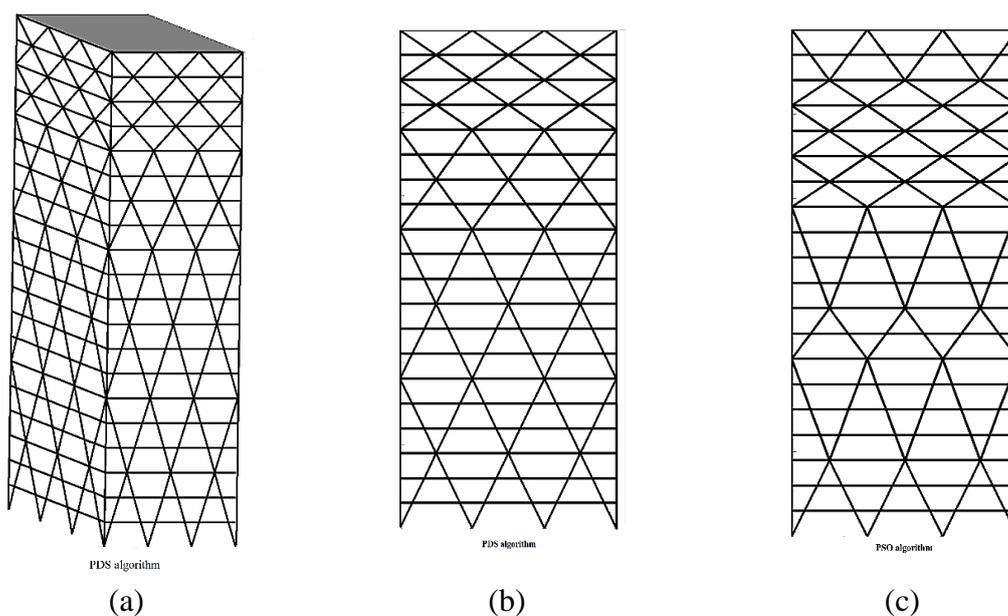


Fig. 5 Non-uniform optimal diagrid layouts of 20st by (a),(b) PDS and (c) PSO

Fig. 5 shows compares PSO and PDS methods which have both resulted in non-uniform diagrid angles among the structures' height. It may also be noticed that such a variation in diagrid angles is smoother in the PDS result with respect to PSO. Note that the arrange of diagrids among height is taken constant in transverse faces of the structure to be more practical.

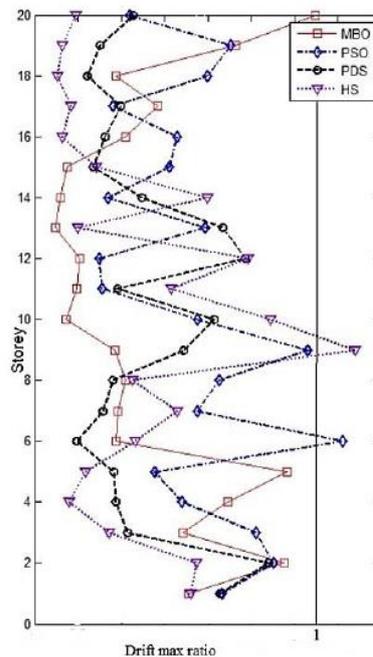


Fig. 6 Comparison of drift ratios for the optimal design of 20st example with non-uniform modules

Structural responses for the optimal designs of 20st between the employed algorithms are compared in Fig. 6. It is evident that PDS has led to lower story sways than PSO. It is worth mentioning that such a result is even obtained with less consumption of structural material. It shows superior performance of PDS with respect to PSO; not only in weight minimization but also in reduction of structural responses.

3.2 Effect of releasing modules uniformity in optimization

The 20st model is retreated here with another type of optimization; that is structural sizing with constant diagrid-angles which results in uniform modules only. Fig.7 shows results of via two types of optimal diagrid design by PDS; first with uniform modules and second when such uniformity is released. For the sake of true comparison, the number of diagrid modules is kept the same in such a test.

In the first type, only sizing optimization is performed for the model with fixed diagrid angle. While both layout and sizing is optimized in the second type. Although, the second case has resulted in non-uniform modules, a uniform trend of decreasing diagrid angle is observed from the base level to the roof.

Fig.8 shows that the optimized non-uniform module design has been superior in minimizing structural weight when all the code-based stress and deflection constrained are met. According to Fig.8b, non-uniform design of diagrid has resulted in lower story drifts; regarding the maximal response even with less structural weight than

uniform-module design. It can also be noted that the trend of drift variation with the building height is smoother for the optimized non-uniform diagrid. The matter confirms necessity of releasing diagrid angles during optimization and its better structural performance and to allow satisfying design constraints with lower material consumption.

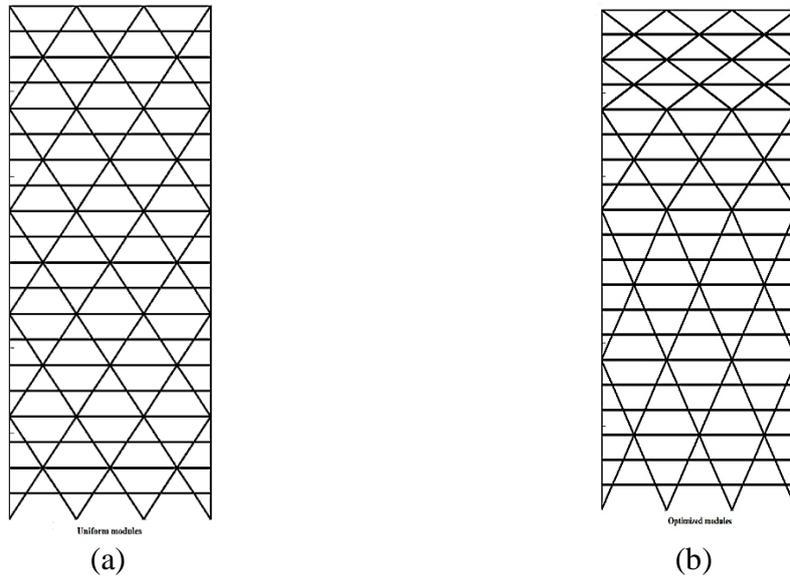


Fig. 7 Results of PDS for 20st in (a) uniform and (b) non-uniform diagrid design

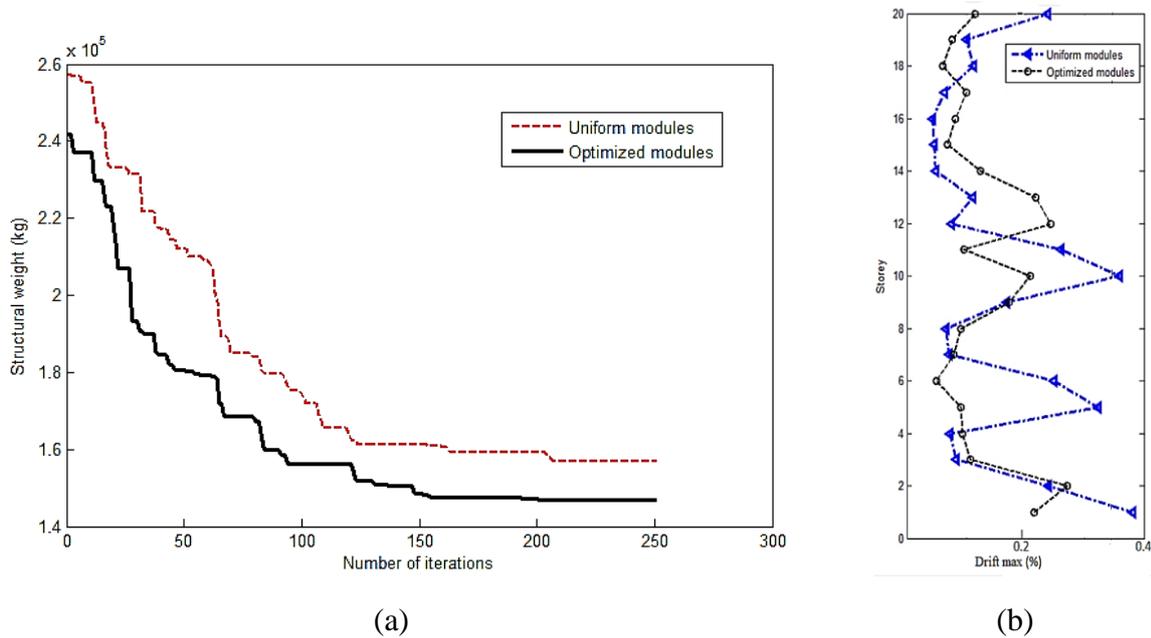


Fig. 8 Comparison between uniform and non-uniform diagrid optimization regarding (a) trend of weight minimization and (b) storey drift profile at the final design

4. CONCLUSIONS

Two types of formulations are used in this research for optimal design of diagrids. In the first type, only size of structural members are taken as design variable while in the second, both sizing and layout variables are taken into account using suitable encoding to practical position of diagrid nodes.

The second problem allows both uniform and non-uniform diagrid angles, while it is fixed in the first type. PDS as a meta-heuristic algorithm was developed and utilized to solve the problem in addition to the standard PSO. It is declared that the proposed method can be best tuned for the problem in hand. Comparison of convergence curves for a set of rather similar control parameters showed that PDS can successfully escape from local optima trap toward higher quality global solutions than PSO with the same number of iterations.

It is found that the best designs in all cases the 2nd problem include non-uniform modules of diagrid system. In order to concisely study this matter, the optimal design was repeated using the 1st formulation; i.e. sizing-only with fixed configuration of diagrid. Although the number of diagrid modules was kept the same between the uniform and non-uniform diagrid designs, the latter could achieve lower minimal weight.

As another interesting result, such non-uniform design could better withstand the story sways against wind loading satisfying codified stress and deflection requirements; even with its lower weight than uniform design. Hence, simultaneous optimization of geometry and sizing is essential to reveal superior results with proper spatial distribution of diagrid modules. Diagrid modules in such optimal designs are usually non-uniform and become denser in the upper stories in order to provide them sufficient stiffness to withstand consequent drifts. Therefore, the angle of diagrid modules with the horizon is recommended to be reduced with the increase of story level so that every upper module covers less number of stories.

In the light of the current study, it is concluded that the proposed PDS can reveal optimal designs of diagrid system regarding both sizing and configuration design with considerable performance improvement and weight minimization. Superiority of PDS over other treated algorithms was declared confirming that the proposed method provides proper efficiency and effectiveness in such structural optimization problems by its adaptive selection of walking states.

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