Tab.1 Specification parameters

T_c/kN	T_m/kN	l/m	l_d/m
20	15	50	0.6
a/m	h_c/m	m	
0.3	1.5	0.9	

A wind tunnel test have been taken to test displacement in wind direction of contact wire. A model of 3 posts and 2 spans have been made to test wind deviation, one of the most important measurement index of catenary system (Xie and Zhi et al. 2015). Since the limitation of experimental instruments, only the midpoint have been tested on the contact wire. The model is shown in Fig.3, including the midpoint D1z. A uniform flow has been taken into account, with the wind velocities of 4m/s, 6m/s, 8m/s, 10m/s, 12m/s and 14m/s. Z direction means the direction of wind.

Fig.3 Arrangement of the Test Points

Fig.4 shows the wind deviation in the middle of contact wire ($D1_z$) in different wind velocities in uniform flow. It can be seen that the displacement increases nonlinearly with the wind velocities, which is due to the fact that the pressure on the line posed by wind is square function of the wind velocity. When the wind velocities are 10m/s and 14m/s, the tested mean wind deviations are 18.03mm and 41.10mm respectively which were converted to be 81.14mm and 184.95mm in the wind velocities of 21.2m/s and 29.68m/s respectively.

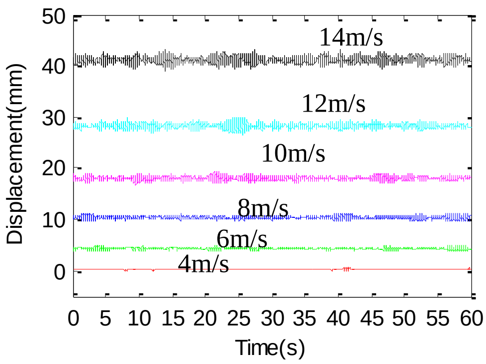

Fig.4 Time evolution of wind deviation in uniform flow

Although wind tunnel can show a relatively convinced result, it costs a lot and the results are limited by many facts including experimental site, measuring equipment and many uncontrollable factors. Take this test for example, for reason of test condition, the maximum wind speed can only reach 14m/s, which is relatively small to the deviation research. Moreover, the tested point cannot present the largest deviation, which only shows the displacement in mid span. So the theoretical research of catenary system is a vital task to figure out the largest deviation.

3. CALCULATION OF WIND DEVIATION IN DIFFERENT COUNTRIES

Wind deviation of catenary system is influenced by many factors, including the types of catenary, materials of wires, length of lateral position and wind situation. In the catenary system, droppers are used to link contact wire and messenger wire, so they transfer the forces between the wires. It is a complicate work to determine the forces, as well as the wind deviation of contact wire.

Catenary system has the shape of “Z” along the road, since the ends of wires have a distance from the central road, which is called the length of lateral position, (Yu. 2003), as shown in Fig.5. In it, x direction means the direction along the rail, and z direction means the direction along the wind, also y direction is the direction of gravity. This direction setting apply to this paper.

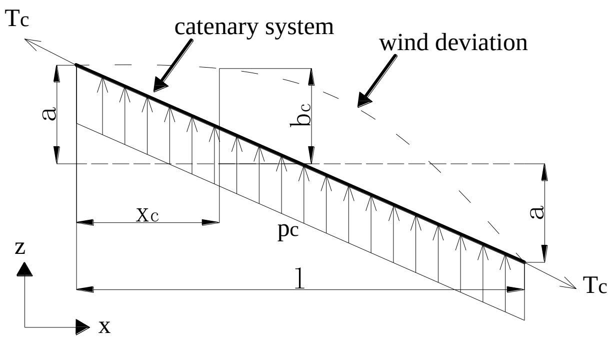

Fig.5 General view and stress state of contact wire

From this figure, it can be seen the stress state, including tensions on contact wire, T_c, and the wind load along the wire, p_c. So the basic calculation method of contact wire can be deduced from this state by force and geometrical analyzing (Shen and Xu et al. 1997), with following expression:

$$b_c = \frac{p_c l^2}{8T_c} + \frac{2T_c a^2}{p_c l^2} \quad (1)$$

This deviation happens at $x_c = \frac{l}{2} - \frac{2aF_c}{p_c l}$, which takes the lateral position into consideration.

There exists a limitation that x_c should in the range of span, so $0 \le x_c = \frac{l}{2} - \frac{2aF_c}{p_c l} \le l$.

It is known that the parameters in $\frac{2aF_c}{p_c l}$ are all positive numbers, so the limitation only needs to meet the requirement of $\frac{F_c}{p_c} \le \frac{l^2}{4a}$

This is the basic equation to calculate the wind deviation of contact wire. But it is widely known that contact wire is just a part of catenary system, so its deviation should be take the function of whole system into consideration.

3.1 Chinese Method

There are two methods to calculate the deviation in China, the equivalent method and the mean value method.

The equivalent method regards the catenary system as a whole, introducing a parameter m to revise the basic equation into a new one with the following expression:

$$b_{c,c1} = \frac{mp_c l^2}{8T_c} + \frac{2T_c a^2}{mp_c l^2} \tag{2}$$

Eq.2 neglects the complex function in the system while only introduces m, which is less than 1, as an equivalent parameter to be timed by wind load. Since it is regarded that contact wire is restrained by messenger wire through droppers. The value of m is influenced by its physical characteristics, 0.9 for copper wires and 0.85 for steel-aluminum wire which is determined by experience.

The difference of mean value method is that it takes the function of messenger wire into consideration. In order to study the contribution of messenger wire to the contact wire, the wind deviations of both wires are calculated respectively first. Then their effects are decided by their ratios of wind load to tension on wire. Finally the deviation is turned out as follows:

$$b_{c,c2} = \frac{1}{2}\left(\frac{p_c}{T_c} + \frac{p_m}{T_m}\right)\frac{l^2}{8} + \frac{2a^2}{\frac{1}{2}\left(\frac{p_c}{T_c} + \frac{p_m}{T_m}\right)l^2} \tag{3}$$

These two methods can calculate the wind deviation of contact wire. The former one needs less information thus being the more convenient and less accurate one by only considering contact wire. Compared to the equivalent method, the mean value method shows a more complex equation to take the messenger wire into account. While both of them neglect the function of droppers, which are important factors.

3.2 German Method

Wind deviations of contact wire and messenger are different when winds impose on them, thus making the droppers deliver forces between them. This method is to determine the interaction force *ΔF* first and then figure out the offset deviation *Δb* by its geometrical analysis (Friedrish. 2001), as shown in Fig.6. In this process, it regards the shortest dropper l_d as 2/3 times the height of catenary system H_c. Finally the wind deviation is shown as below:

$$b_{c,g} = b_c - \Delta b \tag{4}$$

and

$$\Delta b = \frac{2H_c \cdot \Delta F}{3G_c} = \frac{2H_c}{3G_c} \cdot \frac{\frac{p_c}{T_c} - \frac{p_m}{T_m}}{\frac{16H_c}{3G_c \cdot l^2} + \frac{1}{T_c} + \frac{1}{T_m}} \tag{5}$$

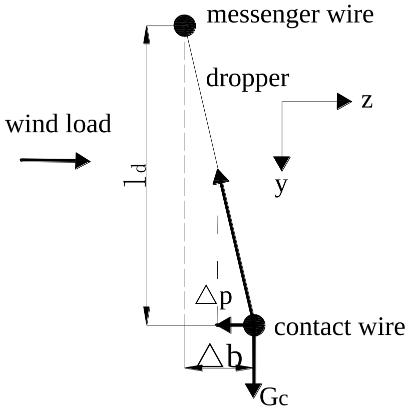

Fig.6. General view of droppers' function

There exists a problem the *Δb* is in the middle span of the catenary system, while b_c takes the situation at x_c. So this method just combines two figures at different places and assume that it nearly equals the maximum displacement.

3.3 Soviet Method

Soviet method also takes the function of droppers into consideration. Compared with the German one, it calculate the offset wind load *Δp* on contact wire caused by droppers and messenger instead of the offset deviation (Walther. 1986). There goes the equation:

$$b_{c,s} = \frac{(p_c - \Delta p)l^2}{8T_c} \tag{6}$$

and

$$\Delta p = \frac{\frac{p_c}{F_c} - \frac{p_m}{F_m}}{\frac{8l_d}{G_c \cdot l^2} + \frac{1}{F_c} + \frac{1}{F_m}} \tag{7}$$

In this process, it can be seen clearly that it ignores the function of lateral positon, and just thinks the maximum deviation happened in the middle of the span.

4. WIND DEVIATION PROPOSAL

After studying the methods in calculating deviation of different countries, a new equation is proposed by building hypothesizes forwardly:

a. Droppers meet Hooke's low, with their internal forces linear increase with their growth, thus regarding them as springs which can only bearing tensions.

b. The forces between contact wire and messenger wire are created by droppers, and compared with the wind load, they are relatively small. So the wires can still meet the assumption of parabola.

c. Since length of lateral position is particularly small compared with the span, the force created by droppers is in the same direction with wind load in top view.

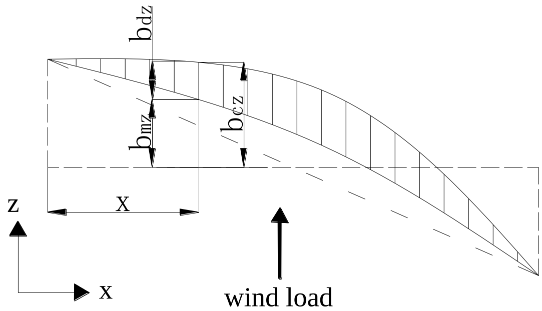

Fig.7 Top view of catenary system under wind load

Here start to analysis the contact wire and messenger wire in top view, shown in Fig.7. The deviation of contact wire b_{cz} and the deviation of messenger wire b_{mz} can be determined respectively as follows:

$$b_{cz} = \frac{p_c x(l-x)}{2T_c} + \frac{a(l-2x)}{l} \tag{8}$$

$$b_{mz} = \frac{p_m x(l-x)}{2T_m} + \frac{a(l-2x)}{l} \tag{9}$$

So the deviation of droppers b_{dz} can got by Eq.8-Eq.9:

$$b_{dz} = \frac{p_c x(l-x)}{2T_c} - \frac{p_m x(l-x)}{2T_m} = \frac{1}{2}\left(\frac{p_c}{T_c} - \frac{p_m}{T_m}\right)x(l-x) \tag{10}$$

thus forces on droppers p_{dx} can be deduced as follows:

$$p_{dx} = \frac{1}{2}k\left(\frac{p_c}{T_c} - \frac{p_m}{T_m}\right)x(l-x) \tag{11}$$

where k is the stiffness of droppers. This equation shows the fact that the forces between contact wire and messenger wire created by droppers meet the rule of parabolic distribution, with *k* and ($\frac{p_c}{T_c} - \frac{p_m}{T_m}$) are constant. The maximum figure of p_{dx} can refer to *Δb* in Eq.7, so the dropper forces can be revised as follows:

$$p_{dx} = \Delta p \frac{4}{l^2} x(l-x) = \frac{\frac{p_c}{F_c} - \frac{p_m}{F_m}}{\frac{8l_d}{G_c \cdot l^2} + \frac{1}{F_c} + \frac{1}{F_m}} \cdot \frac{4}{l^2} \cdot x \cdot (l-x) \tag{12}$$

Based on the hypothesis c., p_{dx} has the same direction as p_c. Then the wind deviation of contact wire can be deduced as following appearance:

$$b_{cx,n} = \frac{(p_c - p_{dx})x(l-x)}{2T_c} + \frac{a(l-2x)}{l} = -\frac{2\Delta p}{T_c l^2} x^4 + \frac{4\Delta p}{T_c l} x^3 - (\frac{p_c}{2T_c} + \frac{2\Delta p}{T_c}) x^2 + (\frac{p_c l}{2T_c} - \frac{2a}{l}) x + a \tag{13}$$

so the position of maximum deviation can be calculated as follows:

$$\frac{\mathrm{d}(b_{cx,n})}{\mathrm{d}x} = -\frac{8\Delta p}{T_c l^2} x^3 + \frac{12\Delta p}{T_c l} x^2 - (\frac{p_c}{T_c} + \frac{4\Delta p}{T_c}) x + \frac{p_c l}{2T_c} - \frac{2a}{l} = 0 \tag{14}$$

Compared with the former methods, the new one is a quartic equation instead of a quadratic equation. Though it has the complex expression, it has more accurate result to give consult in the design process. And it can also be solved quickly by computational technics.

5. FINITE ELEMENT ANALYSIS

In order to verify the correction of the new method, a finite element analysis has been studied on the catenary system. A model has been established with contact wire, messenger wire and droppers with link elements, which cannot be subjected to bending moment. The ends of wires are hinge joints to simulate the connection in reality with the way of pulleys, which can only shift in one direction. At the meantime, tensions on the wires are transfer to prestressing force and keep a balance with their own weights to reach the shape of original sag. Equivalent static wind loads are exert to wires in uniform distribution so as to calculate the performance of contact wire. The model are shown as follows in Fig.8.

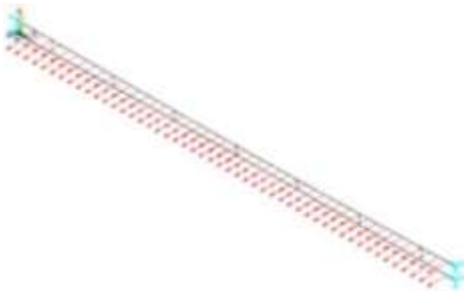

Fig.8 Finite element model of catenary system

6. CASE STUDY

In order to compare the difference in theoretical calculation methods mentioned above and verify the correction of new method by using finite element method, a case study is taken.

Since the limitation mentioned above, the wind speed of 35, 40, 45, 50 and 55m/s have been chosen to make calculation. Wind deviation can be figured out from wind pressure on catenary system which are convert from wind speed.

The results of wind deviation are shown in Table 2.

Table 2. Wind deviation of different methods

Wind speed /(m/s)	$b_{c,c1}$/m	$b_{c,c2}$/m	$b_{c,g}$/m	$b_{c,s}$/m	$b_{c,n}$/m	FEM/m
35	0.300	0.305	0.321	0.170	0.302	0.303
40	0.311	0.331	0.346	0.222	0.322	0.323
45	0.338	0.374	0.388	0.280	0.360	0.357
50	0.379	0.430	0.444	0.346	0.410	0.400
55	0.431	0.497	0.509	0.420	0.473	0.448

The figures of new method are between those of the equivalent method and the mean method and with the increasing of wind speed, the differences between them are also increasing. This phenomenon is due to the fact that simplified calculation of Chinese method bring errors while the new method has been covered the insufficiencies of those two methods so as to figure out a more accurate result. The figures of new method is smaller than those of the German method, for the reason of the new method fixing the confusing in two kinds of displacements and combining it to the correct answer. Compared with the Soviet method, the figures of the new one are larger, since it takes the displacement of lateral position into consideration. It is just the displacement in the mid of span. But the wind speed increasing, the difference with other methods are decreasing, so it shows the fact that, the point of deviation is shifting to the mid of span.

Verified by the finite element method, it can be seen that the result of the new method is close to the finite element method. When the wind speed is lower than 45m/s, the errors between them are less than 1%. While with the increase of wind speed, their errors are also increasing. This phenomenon may due to the different material and joint condition.

The new method details the stress distribution of droppers, it may be identified as the most accurate one. In the process of designing, the new method of deviation calculation may bring more convinced situation to guarantee the safer operation. So it is suggested that the new method may give more accurate consult to the rail design.

7. CONCLUSION

Wind tunnel test will bring the most convinced figure and intuitive result, showing the fact that the displacement of contact wire is increasing nonlinearly with wind speed. While wind tunnel test would cost a lot and be limited by experimental site, measuring equipment and many uncontrollable factors.

Methods in three countries have been studied and compared the differences, including two Chinese methods, a German one and a Soviet one. Inappropriate points in them have been figured out, including neglecting the function of droppers and length of lateral position or just calculating the deviation approximately.

A new method is proposed to take these questions into consideration. Compared with the former methods, the new one is a quartic equation instead of a quadratic equation. Though it has the complex expression, it has more accurate result to give consult in the design process with finite element software proved.

The case study shows the difference of theoretical methods mention above by comparing with finite element method. Finally the correction of the new method is proved and is proved that it is an appropriate and convenient method to calculate the deviation in strong wind with incorrect points in other methods recovered.

This work can give reference to the design of catenary system, since it offers a convenient way to calculate the wind deviation which is a vital measuring indicator.

REFERENCES

Friedrish K. (2001), “Contact lines for electric railways,” *Publics corporate publishing*.

Park, T.J., Kim, B.J., Wang, Y.Y. and Han, C.S. (2002), "A catenary system analysis for studying the dynamic characteristics of a high speed rail pantograph", *Journal of Mechanical Science & Technology*., **16**(4), 436-447.

Shen SZ, Xu CB, and Z C. (1997), "Suspended cable structure design", *China building industry press.*

Vickerman, R. (1997), "High-speed rail in Europe: experience and issues for future development", *Annals of Regional Science*., **31**(1), 21-38.

Wan JY. (2003), “Catenary of high speed rail,” *Southwest Jiaotong university press*.

Walther M. (1986), “Catenary,” *China railway publishing house.*

Xie, Q. Zhi, X. Sun, QG. and Xiao, YW. (2015), " Wind Tunnel Test on Aeroelastic Model of Catenary System for High Speed Railway", *China Railway Science*., **36**(3), 73-80.