4. BEAT FLOW FORMATION MECHANISM

A beat phenomenon or secondary frequency is observed in the symmetrically-biased and non-biased beat flows, influencing the lifts of the prisms to have a beat-like modulation. For a detailed discussion, L/W = 3.5 is taken to be an exemplification (figure 2a). It is worth viewing the representative flow structures at maximum and minimum amplitudes (associated with the secondary frequency) of the lifts. The flow structures presented in figures 2(*c*) and (*d*) correspond to the maximum and minimum C_L-amplitudes (t^* = 1836.3 and 1877.4, respectively) associated with the secondary frequency, as indicated by vertical lines in figure 2(a). Interestingly, the maximum C_L-amplitude (figure 2a) associated with the secondary frequency occurs when an inphase shedding occurs from the two sides of a gap (figure 2*c*). On the other hand, an antiphase shedding from the two sides of a gap (figure 2*d*) results in the minimum C_L-amplitude (figure 2a).

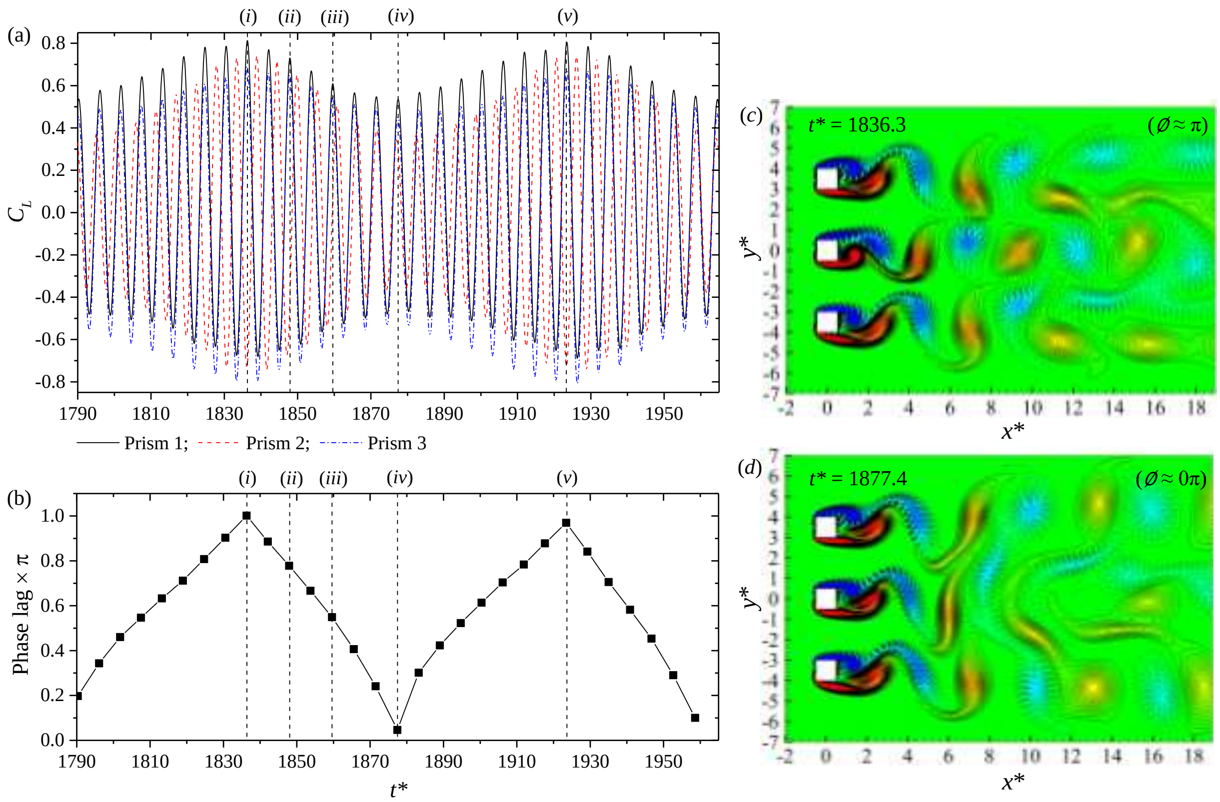

Fig. 2 (a) Time histories of lift coefficient acting on the three prisms. (b) Instantaneous phase lag between lift fluctuations of the upper and middle prisms. (c and d) Contours of instantaneous vorticity field showing vortex shedding from the prisms at t^* = 1836.3 and 1877.4, respectively. L/W = 3.5. (*i*) t^* = 1836.3, (*ii*) t^* = 1847.9, (*iii*) t^* = 1859.6, and (*iv*) t^* = 1877.4.

Both gaps simultaneously may have inphase shedding (figure 2c) or antiphase shedding (figure 2d), as the shedding phase lag between the outer prisms was a constant of ≈ 0°. C_L amplitudes of the three prisms thus reach their maxima or minima simultaneously. When the phase lag between the sheddings from the outer prisms is ≠ 0°, maximum or minimum lift amplitudes of the outer prisms do not occur simultaneously. So, the beat/secondary frequency results from a continuous change in the phase lag between the sheddings from the two sides of a gap, from inphase to antiphase, antiphase to inphase, and so on (figure 2b). Furthermore, the change in C_L amplitude of the middle prism also depends on the phase lag between sheddings from the outer prisms. Due to the different shedding frequencies from the two sides of a gap, the instantaneous phase lag changes in every primary period. It should not be confused that when the shedding frequencies are different, how can the phase lag be obtained? Here the phase lag means the phase of the longer period shedding (outer prism) with respect to that of the shorter period shedding (middle prism), i.e., considering the shorter period as a reference complete cycle period.

5. TIME-AVERAGED AND FLUCTUATING FLUID FORCES ON THE PRISMS

In this section, variation in fluid forces (time-averaged and fluctuating lift and drag coefficient, figure 3) acting on the three prisms with *L/W* will be discussed in detail and connect with the flow structures.

In regime *A*, vortex shedding only from the freestream sides of the outer prisms, and the gap flows between the prisms appear weak and dim at *L/W* ≤ 1.1 (figure $1a_1$), and become appreciably visible at 1.1 < *L/W* ≤ 1.4 (figure $1a_2$). Consequently, the pressure on the inner side surfaces of the outer prisms is negligible, compared with that of the outer side surfaces. The difference of pressure gets smaller with an increase of *L/W*, responsible for the decreasing $\overline{C_L}$ magnitude for the outer prisms ($\overline{C_L}$ ≈ 1.81 at *L/W* = 1.1, and $\overline{C_L}$ ≈ 1.04 at *L/W* = 1.3, figure 3a). Furthermore, the appreciable gap flows prolong the vortex formation and recover the convection velocity in the near wake. The negative pressure in the near wake thus gets weak, resulting in the decreasing $\overline{C_D}$ magnitude, especially for the middle prism ($\overline{C_D}$ ≈ 4.32 and 3.63 at *L/W* = 1.1 and 1.3, respectively. figure 3b). Due to the effect of gap flows, the swerving of the shear layers weakens, the C_L' magnitude of the outer prisms thus decreases sharply, from 1.33 at *L/W* = 1.1 to 0.63 at *L/W* = 1.3. Furthermore, at *L/W* = 1.2, the swerving direction of both gap-flows with very small vortices was contingent on the shedding from the freestream sides, and the gap flows are less biased with relatively large inertia at *L/W* = 1.3. The C_L' magnitude of the middle prism thus increases and decreases (C_L' ≈ 0.544, 0.675 and 0.620 for the middle prism at *L/W* = 1.1, 1.2 and 1.3, respectively. figure 3c). The vortex formation occurs very close to the base of the middle prisms at *L/W* ≤ 1.1 (figure $1a_1$), and persists behind the outer prisms and moves downward due to the effect of the gap flows at 1.1 < *L/W* ≤ 1.4 (figure $1a_2$). The C_D' magnitude of the middle prism thus decreases monotonously, while that of the outer prisms increases and decreases in the

L/W ≤ 1.1 and 1.1 < *L/W* ≤ 1.4, respectively (figure 3d).

In regime *B*, a greater flow can pass through the gaps and split the wake into three immediately downstream of the prisms. The gap flows with relatively larger vortices flip-flop randomly at different fashions (figure $1b_1$-b_4). A greater flow passing through the gaps diminishes the pressure difference between the outer and inner surface of the outer prisms, the $\overline{C_L}$ magnitude for the outer prisms thus decreases with *L/W* (figure 3a). Besides, the two shear layers of the middle prism are largely squeezed inward, associated with a smaller formation length (L_f) and wake width (ω), while both formation length and wake width get large with an increase of *L/W* because of weakened squeezing effect. Here, L_f refers to the streamwise separation between the prism's center and the points of maximum fluctuating streamwise velocity in the wake , while ω refers to the transverse separation between the two maxima in fluctuating streamwise velocity contours (Alam, Zhou & Wang 2011). Consequently, the $\overline{C_D}$ of middle prism decreases distinctly again, while that of the outer prisms also decreases, but slightly. Further, the C_L' magnitude of the middle prism is smaller than that of the outer prism, and both increases with *L/W* (figure 3c), implying that the gap flows swing with a more violent attitude. Furthermore, because the wake is chaotic, a long time statistical result gets a similar C_D' magnitude for the three prisms (≈ 0.32, figure 3d).

In regime *C*, the gap flows are symmetrically biased outward; a reversed flow region thus form near the outer side surface of the outer prisms (figure $1c_1$ and c_2), resulting in a slight increase of the $\overline{C_L}$ magnitude for the outer prisms (figure 3a). As discussed above, a substantial wide wake accompanies the middle prism ($\omega \approx 2.45W$ at $L/W = 2.5$), and a narrow wake complements each outer prism. Consequently, the negative pressure in the near wake behind the middle prism is much smaller than that of the outer prisms and thus corresponds to the smaller $\overline{C_D}$ magnitude (≈ 1.41). Further, the two shear layers of the middle prism spawn vortices almost symmetrically, not alternately, in the upper and lower wakes, resulting in the lower C_L' (≈ 0.030) and C_D' (≈ 0.094) magnitude (figure 3c and d).

In regime *D*, the gap flows are no longer biased where a single vortex street persists behind each prism (figure $1d_1$ and d_2). The magnitude of $\overline{C_L}$ for the outer prisms collapses from 0.59 (*L/W* = 2.5) to 0.18 (*L/W* = 2.7), and decreases gradually, converging towards zero (figure 2a). The magnitude of $\overline{C_D}$ for each prism is close to each other, and decreases gradually with *L/W* (figure 3b), resulting from the increasing formation length ($L_f \approx 1.74W$ at $L/W = 2.7$ and $L_f \approx 2.14W$ at $L/W = 7.0$). Besides, C_L' of the middle prism thus has a distinct jump (C_L' ≈ 0.48 at *L/W* = 2.7). The wake width of the middle prism is smaller than that of the outer prisms due to the squeezing effect from the inner shear layers of the outer prisms, while the difference of the wake width is diminishing with an increase in *L/W* ($\omega \approx 1.11W$ and $1.02W$ for the outer and middle prisms at $L/W = 2.7$, respectively, and $\omega \approx 1.03W$ and $1.0W$ at $L/W = 7.0$). The smaller wake width always corresponds to the smaller fluctuating fluid force (C_L' and C_D'), and thus the fluctuating fluid force (C_L' and C_D') for both outer and middle prisms decreases monotonously and converges toward that of an isolated prism (figure 3c and d).

In the regime *E*, the interaction between the wakes of adjacent prisms is weak. The shedding from each prism resembles that from an isolated prism, and the three vortex streets spaced sufficiently do not interact with one another (figure 1e$_1$ and e$_2$). The fluid forces ($\overline{C_L}$, C'_L, $\overline{C_D}$ and C'_D) for all three prisms are similar and close to those of an isolated prism.

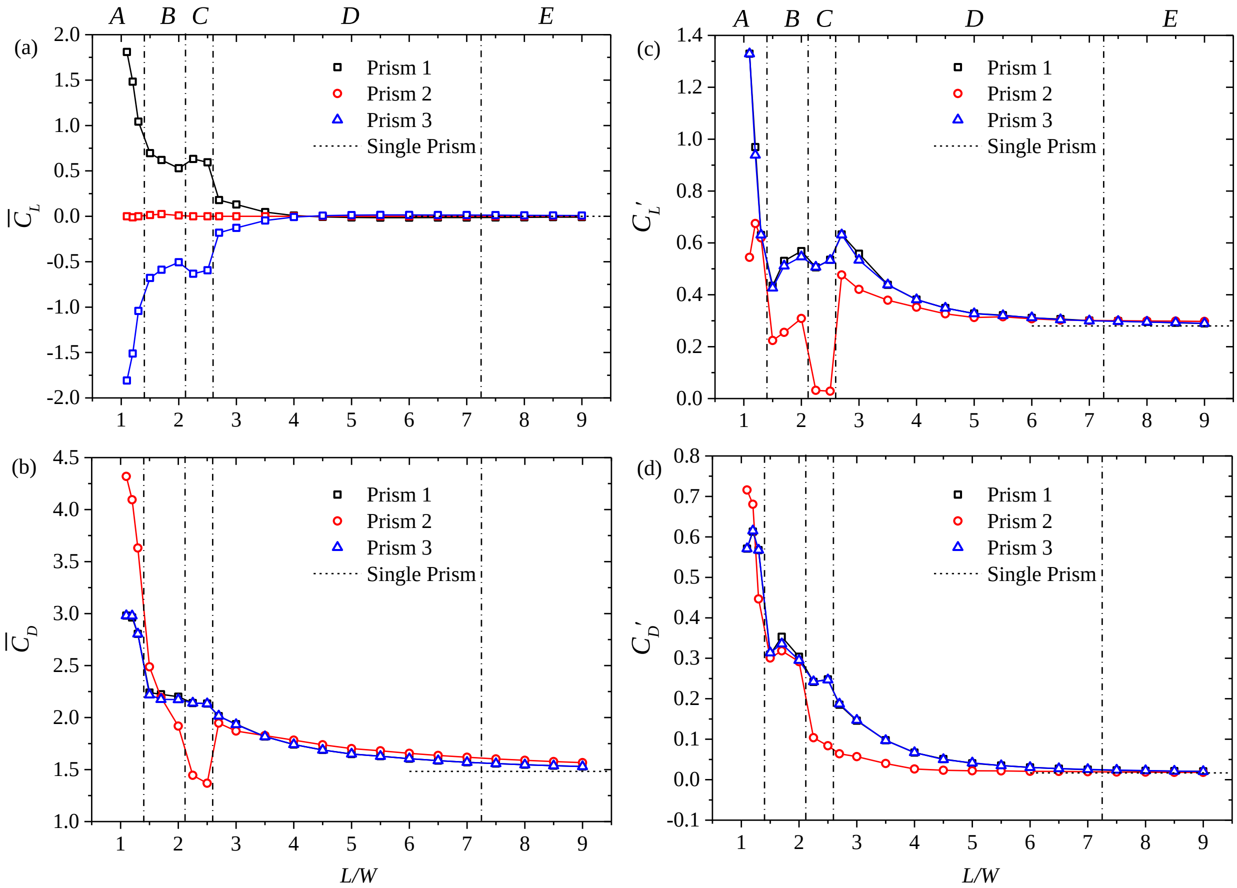

Fig. 3 Variation in (a) time-averaged lift ($\overline{C_L}$), (b) time-averaged drag ($\overline{C_D}$), (c) fluctuating lift (C'_L) and (d) fluctuating drag (C'_D) acting on the three prisms with *L/W.*

5. CONCLUSION

An investigation on the flow around three side-by-side square prisms can provide us a better understanding of complicated flow physics associated with multiple closely spaced structures where more than one gap flow is involved. A detailed study has been conducted on the wake of three side-by-side square prisms at *Re* = 150, with *L/W* = 1.1 ~ 9.0. Based on vortex structures, gap flow behaviors, shedding frequency and fluid forces, five distinct flow structures and their ranges are identified, viz., single bluff-body flow (*L/W* < 1.4), flip-flopping flow (1.4 < *L/W* < 2.1), symmetrically-biased beat flow (2.1 < *L/W* < 2.6), non-biased beat flow (2.6 < *L/W* < 7.25) and weak interaction flow (7.25 < *L/W* < 9.0). A beat phenomenon or secondary frequency is observed in the symmetrically-biased and non-biased beat flows, influencing the lifts of the prisms to have a beat-like modulation. The secondary frequency results from the periodic change in phase lag between the sheddings from two sides of a gap. The phase lag has a great impact on the lift force; the smaller the phase lag, the larger the lift amplitude. The modulation of lift amplitude stems from the phase lag change. The dependence on *L/W* of fluid forces acting on the three prisms is discussed in detail, and connected with the flow structures.

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