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Monopile Head Stiffness and Natural Frequency Assessment of Some Installed OWTs using a Pseudo 3D Nonlinear FE Model

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ABSTRACT

The monopiles which consist of large diameter ($> 4m$) open-ended steel deep foundation constitute the support structures of the vast majority of Offshore Wind Turbines (OWTs) constructed to date. These slender structures installed in sites of harsh environment are dynamically sensitive which makes the determination of the dynamic characteristics of paramount importance. Among these characteristics, the natural frequency is the most significant parameter whose accurate value allows to perform a safe design by keeping the natural frequency far away from the excitation frequencies, and consequently avoiding resonance and reducing the fatigue damage accumulation during the life time.

By modeling soil/monopile interaction by three springs which are respectively: lateral stiffness K_L, rotational stiffness K_R and cross-coupling stiffness K_{LR}, the offshore turbine natural frequency has been found to be a function of these stiffnesses and hence their accurate estimation is significantly important.

In this paper, a Computer program NAMPULAL which has been written on the basis of a nonlinear pseudo 3D computational method, combining the Finite Element and the Finite Difference procedures, is employed for the analysis of monopiles under axial, lateral and moment loading in a medium being characterized by the hyperbolic model as a criterion of behavior. The lateral behavior of monopiles supporting OWTs of different wind farm sites is considered through the monopiles head movements (displacements and rotations) where K_L, K_R and K_{LR} are obtained and then substituted in the analytical expression of natural frequency.

The numerical computations allowed to find monopile head stiffnesses and then natural frequencies of each OWT considered. The comparison between the computed and the measured natural frequencies was in excellent agreement on one hand, and the comparison between results of the present study and those of other published works based on empirical data showed slight differences on the other hand. Comments about these facts are given at the end of the paper.

Keywords: Finite element analysis; Vertical slices; Nonlinear analysis; Hyperbolic model; Monopiles; Offshore Wind Turbines; Natural frequency.

1. INTRODUCTION

Wind energy generation by means of wind turbines is one of the most promising and the fastest growing renewable energy sources in the world. Since winds are steadier and stronger in remote

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offshore locations, a great number of offshore wind farms are constructed or are under construction.

Installing wind turbines in these harsh offshore sites, where water depth may range between 30 m and 100 m or more is a challenging task. It involves indeed, high stakes and high expenses in addition to great deal of high-tech equipment.

Over the last two decades, several different foundation solutions corresponding to various sea depths and soil conditions have been proposed for the offshore wind turbines (Abed et al. 2016; Arshad and O'Kelly 2016 and Leblanc 2009). The most widely employed foundation for water depths up to 30 m, one specific foundation type has proven its effectiveness regarding its structural simplicity, manufacturing and low rate of its installation expenses. This type of foundation known as monopile foundation, is a large diameter (3 to 6 m) hollow open ended steel pile with length-to-diameter ratio usually less than 10 (Achmus et al. 2009; Aissa et al. 2016; Arany et al. 2015 and Damgaard et al. 2014). The dominating loads to which the monopiles are subjected are large horizontal forces from wind and waves acting above the monopile heads resulting thus in huge overturning moments. These monopile foundations are traditionally designed based on the p-y curves method where the soil resistance is usually modeled by the subgrade reaction method (Winkler type approach). The commonly used p-y approach described in the offshore guidelines (API 2011 and DNV 2011) is generally assumed to be sufficiently accurate for pile diameters D ≤ 2 m. However, several investigations (Kallehave et al. 2012; Sorensen 2012 and Otsmane and Amar-Bouzid 2016) indicate that the pile deflections of large diameter monopiles are underestimated for service loads and overestimated for small operational loads. The main shortcoming stems from the fact that the p-y curves recommended in offshore design regulations were developed for piles with diameters up to approximately 2.0 m and are based on a very limited number of tests. Hence, the method has not been validated for piles with diameters of 4 to 6 m.

As far as the accurate estimation of the dynamic characteristics of an OWT is concerned, the finite element method is naturally the obvious method that can satisfy the key requirements for an OWT safe design. In this paper a Computer program **NAMPULAL** (**N**onlinear **A**nalysis of **M**onopiles **U**nder **L**ateral and **A**xial **L**oading) representing the encoding of a nonlinear pseudo 3D computational method, combining the Finite Element and the Finite Difference procedures, is employed for the analysis of monopiles under axial, lateral and moment loading in a medium being characterized by the hyperbolic model as a criterion of behavior. **NAMPULAL** is used to study the lateral behavior of three monopiles supporting OWTs from three different wind farms installed in Europe, in the aim to accurately estimate the monopiles head movements (displacements and rotations) and consequently, the lateral stiffness K_L, the rotational stiffness K_R and the cross-coupling stiffness K_{LR}. Since the natural frequency is a function of monopile/subsoil interaction, the stiffness values are soon substituted in the analytical expression of natural frequency. In general, the results of comparison between computed and measured natural frequencies showed a good agreement.

2. APPROPRIATE OWT MODELING FOR DYNAMIC ANALYSIS

OWT foundation systems are permanently subjected to large horizontal loads and resulting moments in comparison with axial loads which are generally excluded from consideration since the OWT is sufficiently stiff in the vertical direction. Hence, the foundation response under lateral loads is a major consideration, with the foundation design dominated by considerations of the dynamic and fatigue responses under working loads rather than the ultimate load-carrying capacity.

The design of the structure and the substructure of an OWT should be carried out in such a way that it sustains the permanent dynamic forces induced by vibrations during its operational life. These forces with the combination of the operating frequency could potentially trigger the resonance phenomenon. This can have catastrophic consequences and needs to be avoided at all costs. In order to perform a proper design, an accurate OWT mechanical model that accounts properly for the monopile/subsoil interaction should be considered.

2.1 Mechanical model representing the OWT and its interaction with subsea soil

The main components of an OWT are: the nacelle-rotor assembly, the tower, the substructure (transition piece + monopile overhang) and the embedded part which is the foundation (the monopile in this paper) (Figure 1a). The rotor-nacelle assembly containing the key electro-mechanical components including the gearbox and generator, is the part allowing the transfer of mechanical energy produced by the rotation of blades into electric energy. The tower of length L_T accounting for the distance from the rotor-nacelle assembly to the top of the transition piece, has a varying bending stiffness $(EI)_T$, a thickness t_T and top and bottom diameters which are respectively: D_t and D_b. The monopile overhang and transition piece welded together are assumed to constitute one element, called the substructure. The latter has a length of L_s which is defined as the distance from the mudline (seabed) to the bottom of the tower. The diameter D_s and the thickness t_s of the substructure are assumed to have the same values of those of the mobopile on which the substructure is founded. Consequently the bending stiffness of the substructure is the same as that of the monopile $(EI)_p$.

In order to obtain a mechanical model allowing to incorporate the soil/structure interaction in the study, it is useful to replace the subsoil/monopile system by a set of springs through which the tower is connected to the subsoil as shown in Figure 1b. This Figure illustrates a mechanical model, in which the interaction subsoil/monopile is represented by four springs, a lateral, a rocking, a cross-coupling and a vertical spring whose stiffnesses are respectively K_L, K_R, K_{LR} and K_V. Most authors disregarded the axial vibrations since the wind turbines are very stiff vertically.

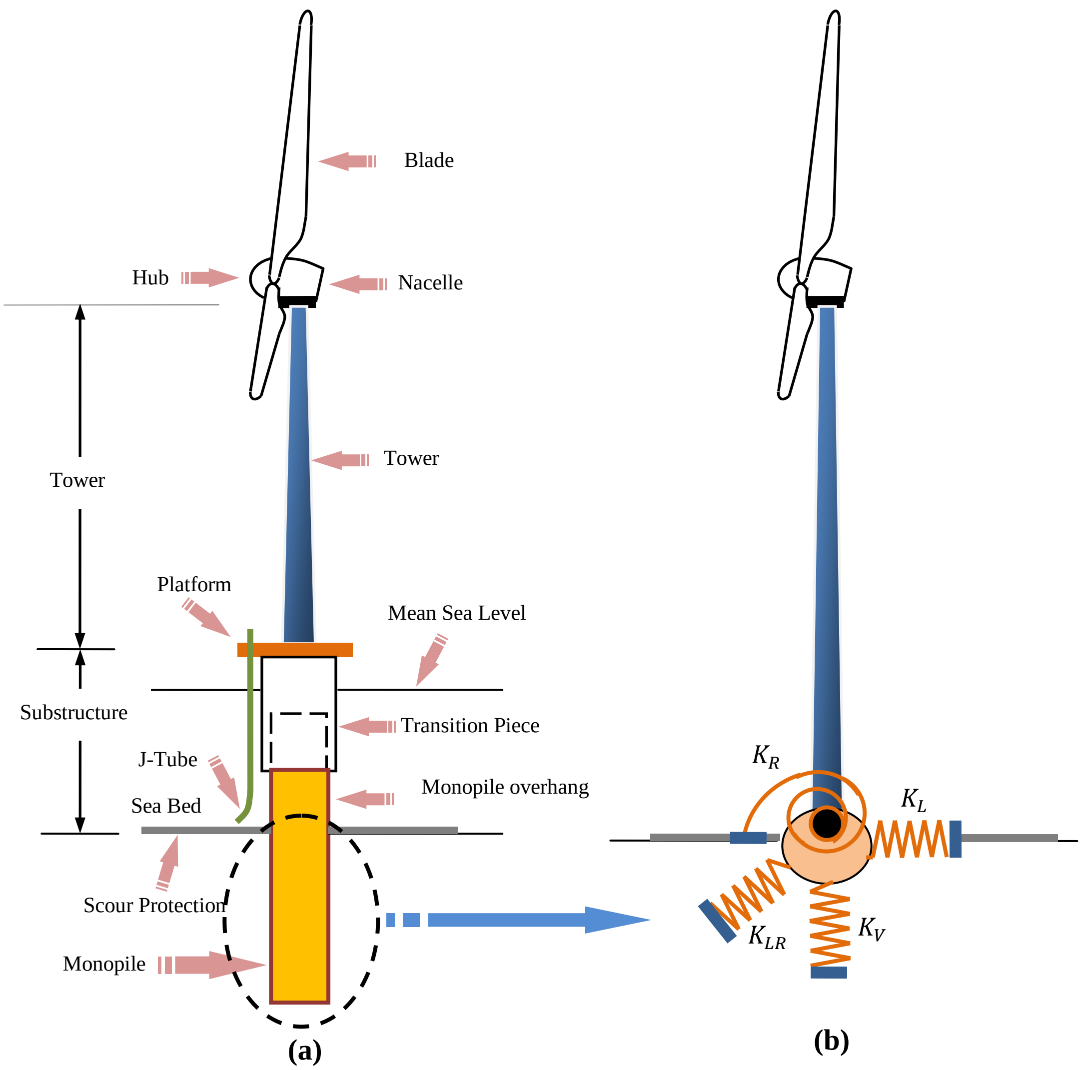

Fig. 1. The OWT model used: (a) Principal components and (b) Model counting for SSI through monopile head springs

2.2 Proposed expressions for natural frequency

The first eigenfrequency of a wind turbine is highly dependent on the material properties used in the construction on one hand and is significantly affected by the stiffness of the soil surrounding the monopile on the other hand. As such, the accurate assessment of this stiffness in terms of modeling for the design process is very important in ensuring that the system frequency can be reliably estimated.

In the computation of eigenfrequency f_1 most authors (Yi et al. 2015 and Prendergast et al. 2015) simply replaced the soil with a medium having an infinite stiffness. In this context, Blevins (2001) used a model in which the wind turbine is considered as an inverted pendulum having a flexural rigidity EI, a tower mass per meter m_{tower} and a top mass m_{top}. His expression for the first eigenfrequency is:

$$f_1 \cong \sqrt{\frac{3\,EI}{(m_{top}+(33/140)\,m_{tower}L)4\pi^2L^3}} \tag{1}$$

Expression in which, L is the tower height. The natural frequency expressed by eq. (1) is based on a uniform tower cross-section

Since dynamic analysis of tower/monopile/soil as one system is hard to perform, other authors tried to find a natural frequency expression counting for soil stiffness (Luqing et al. 2014, Prendergast et al. 2015 and Zaaijer 2005). On the basis of a numerical solution of transcendental frequency equation, Adhikari and Bhattacharya (2011, 2012) proposed an exact approach where only lateral and rotational stiffnesses have been included. Furthermore and in order to improve the first natural frequency equation, Arany et al. (2014, 2015), derived expressions of natural frequency of offshore wind turbines on three-spring flexible foundations by means of two beam models: Bernoulli-Euler and Timoshenko. The natural frequency in both cases has been obtained numerically from the resulting transcendental equations. They proposed a closed form expression containing, in addition to K_L, K_R, the cross-coupling stiffness K_{LR} of the monopile. Their equation for the natural frequency is:

$$f_\eta = C_R C_L f_{FB} \tag{2}$$

Where f_{FB} is called fixed base frequency which is simply identical to the equation (1). The factors C_R and C_L account for the stiffness provided by the monopile, and are functions of tower's geometrical properties. Their analytical expressions are given by:

$$C_R(\eta_L, \eta_R, \eta_{LR}) = 1 - \frac{1}{1+a\left(\eta_R - \frac{\eta_{LR}^2}{\eta_L}\right)} \tag{3}$$

$$C_L(\eta_L, \eta_R, \eta_{LR}) = 1 - \frac{1}{1+b\left(\eta_L - \frac{\eta_{LR}^2}{\eta_R}\right)} \tag{4}$$

Table 1 gives the parameters involved in the expressions (3) and (4).

Table 1. Structural expressions required for C_R and C_L.

$\boldsymbol{\eta_L}$	$\boldsymbol{\eta_R}$	$\boldsymbol{\eta_{LR}}$	$\boldsymbol{a}$	$\boldsymbol{b}$
$K_L L_{tower}^3 / EI_\eta$	$K_R L_{tower} / EI_\eta$	$K_{LR} L_{tower}^2 / EI_\eta$	0.6	0.5

Where a and b are empirical constants, which have been obtained by fitting closed form curves. The applicability of equations (3) and (4) is conditioned by:

$$\eta_R > 1.2\,\frac{\eta_{LR}^2}{\eta_L}, \qquad \eta_L > 1.2\,\frac{\eta_{LR}^2}{\eta_R} \tag{5}$$

The equation (2) is very important in the sense that, the two first parameters, account for the interaction between the soil and the monopile through the spring stiffnesses. Moreover, they incorporate terms related to the equivalent tower stiffness which should be evaluated properly.

2.3 Targeted frequency interval for safe design

The main sources of excitation are wind and waves as an offshore wind turbine is in a permanent interaction with two media: air and water. The excitations generated by wind are the frequencies that are close to the rotational frequencies of the rotor $1P$ and the blade passing frequency $3P$ (The blade/tower interaction), since most turbines existing in the market have 3 blades.

The Figure 2, illustrates the power spectral density in function of frequency intervals. Furthermore, the wind turbulence causing excitations are also plotted in the Figure.

The region before the $1P$ is called the “Soft–Soft” region while the region after the $3P$ is known as “Stiff–Stiff” region. If the natural frequency of the design lies in the Soft–Soft interval it will be too flexible while in the Stiff–Stiff region it will be too rigid (Heavy/Expensive), making it inappropriate for the design. As evident from Figure 2 the “Soft–Soft” usually contains the wave and wind turbulence excitation frequencies this is another important reason why this frequency region is usually avoided.

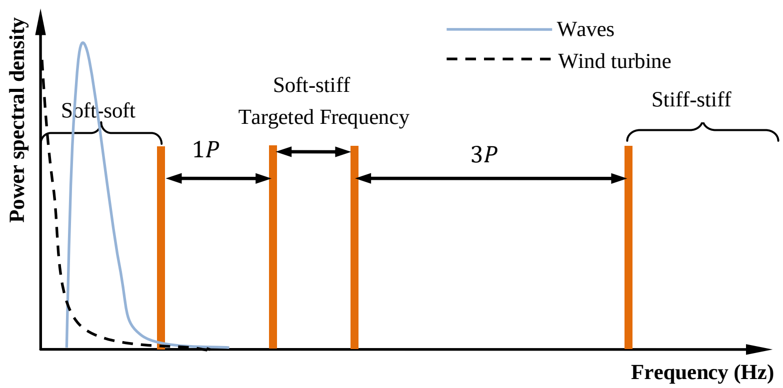

Fig. 2 Excitation intervals of a three bladed offshore wind turbine.

To avoid resonance which may lead to the failure, the OWT should be designed such that the first natural frequency should lie between turbine and blade passing frequencies, i.e, in the interval corresponding to the “Soft-Stiff ” interval in Figure 2. So, the targeted region is the one illustrated in the Figure 2.

3. THE PSEUDO 3D FE METHODOLOGY: COMPUTER PROGRAM NAMPULAL

A pseudo 3D finite element model has been performed to analyze soil/structure interaction problems in non-linear media. This numerical technique, which has been called Nonlinear Finite Element Vertical Slices Model (NFEVSM), is based on the discretization of the 3D soil/structure medium into a series of vertical slices, each one represented by a 2D boundary value problem.

The procedure involves the combination of the finite element (FE) method and the finite difference (FD) method for analyzing the embedded structure and the surrounding soil sub-structured. The soil in this approach, was considered to obey the hyperbolic model proposed by Duncan and Chang (1970).

As more details of about the theoretical developments of this numerical procedure have been given and thoroughly explained in another paper (Otsmane and Amar-Bouzid, 2016), only a short description will be given here.

The three-dimensional aspect of the problem, which consists of several vertical panels (slices) on different thicknesses is accounted for by coupling the shear forces between the slices. The three-dimensional soil/structure problem portrayed in Figure 3 shows a soil/ structure interaction problem example (Figure 3(a)) and the vertical slices model where the different slices are acted upon by external forces and body forces (Figure 3(b)).

The response of an individual slice to an external loading is governed by the following equilibrium equations:

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}+b_x=0, \qquad \frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\sigma_y}{\partial y}+b_y=0 \tag{6}$$

The terms b_x and b_y appearing in the equations (6) are considered to be the key elements in the development of the FE vertical slices model. These physical parameters termed body forces in this model have been interpreted as fictitious forces transmitted to the slice under consideration by shear forces applied from slices at the left and at the right. For a given slice i, this can be mathematically expressed as:

$$b_x=\frac{\tau^l_{zx_i}-\tau^r_{zx_i}}{t_i},\; b_y=\frac{\tau^l_{zy_i}-\tau^r_{zy_i}}{t_i} \tag{7}$$

Where $\tau^l_{zx_i}$ and $\tau^l_{zy_i}$ are shear stresses acting at the left interface of slice i, whereas $\tau^r_{zx_i}$ and $\tau^r_{zy_i}$ are shear stresses acting at the right interface of slice i. t_i is the slice thickness.

The displacements and consequently, the stresses and deformations in each slice are determined by the conventional finite element method, using 2D finite elements. According to the standard formulation in the displacement based finite element method, the element stiffness matrix in slice i can be written as:

$$\int_v (\boldsymbol{B}^t\boldsymbol{D}\,\boldsymbol{B}+\boldsymbol{N}^t\boldsymbol{L}^{pc}\boldsymbol{N})\,\boldsymbol{a}_i dv=\int_v (\boldsymbol{N}^t\boldsymbol{L}^{pr}\boldsymbol{N})\,\boldsymbol{a}_{i-1}dv+\int_v \left(\boldsymbol{N}^t\boldsymbol{L}^{fl}\boldsymbol{N}\right)\boldsymbol{a}_{i+1}dv+\boldsymbol{p}_i \tag{8}$$

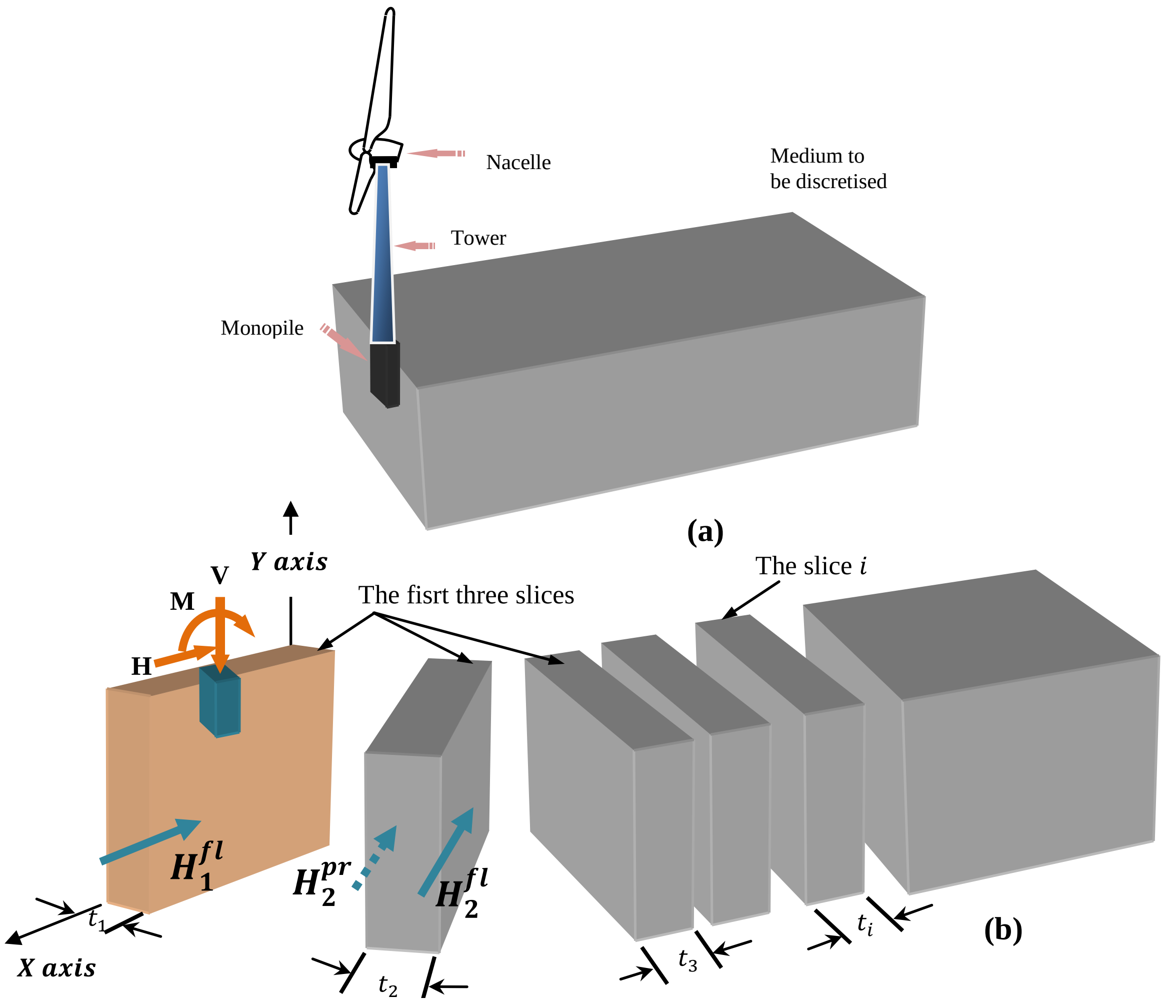

Fig. 3. (a) Real-world soil structure interaction problem, (b) The Vertical Slices Model showing the interacting slices subjected to external and body forces.

$\boldsymbol{p}_i$ in equation (8), is the external forces vector to which the slice i is subjected and $\boldsymbol{a}_{i-1}$, $\boldsymbol{a}_i$ and $\boldsymbol{a}_{i+1}$ are the element nodal displacement vectors of slices $i-1$, i and $i+1$ respectively. $\boldsymbol{I}$ is the identity matrix and $\boldsymbol{D}$ is the matrix corresponding to a problem of plane stresses.

The other parameters are:

$$\boldsymbol{L}^{pc} = l_i^{pc}\boldsymbol{I}\ ,\ \boldsymbol{L}^{pr} = l_i^{pr}\boldsymbol{I} \text{ and } \boldsymbol{L}^{fl} = l_i^{fl}\boldsymbol{I} \tag{9}$$

with $$l_i^{pr} = \frac{2\,G_{i-1}G_i}{t_i(G_{i-1}t_i + G_i t_{i-1})}\ ,\quad l_i^{fl} = \frac{2\,G_i G_{i+1}}{t_i(G_i t_{i+1} + G_{i+1}t_i)} \quad \text{and} \quad l_i^{pc} = l_i^{pr} + l_i^{fl} \tag{10}$$

Expression (8) may be re-written in a compact form as:

$$\boldsymbol{S}_i\boldsymbol{a}_i = \boldsymbol{H}_i^{pr} + \boldsymbol{H}_i^{fl} + \boldsymbol{p}_i \tag{11}$$

This equation cannot be resolved straight-fully, since the right hand terms are not available explicitly at the same time. Thus, it must be resolved according to an updating iterative process:

$$\boldsymbol{S}_i^j\boldsymbol{a}_i^j = \boldsymbol{H}_i^{pr^j} + \boldsymbol{H}_i^{fl^{j-1}} + \boldsymbol{p}_i \qquad \text{for } j = 1,2,\ldots\ldots\ldots,j_{max} \tag{12}$$

Where, j denotes for the iteration number and j_{max} is determined by a certain convergence criterion. The nonlinearity in VS Model stems from the implementation the hyperbolic model proposed by Duncan and Chang (1970) for modeling the soil. In fact, they found out that both tangential modulus E_i and ultimate stress deviator $(\sigma_1 - \sigma_3)_{ult}$ are dependent on the minor principal stress σ_3. More precisely they suggested for the initial tangent modulus the following formula:

$$E_i = KP_a \left(\frac{\sigma_3}{p_a}\right)^n \tag{13}$$

Where K is dimensionless factor termed 'modulus number', n is a dimensionless parameter called 'modulus exponent' and p_a is the atmospheric pressure used to make K and n non-dimensional.

The ultimate stress difference $(\sigma_1 - \sigma_3)_{ult}$ is defined in terms of the actual failure stress difference by another parameter called failure ratio R_f which is given by:

$$R_f = \frac{(\sigma_1 - \sigma_3)_f}{(\sigma_1 - \sigma_3)_{ult}} \tag{14}$$

Using Mohr-Coulomb failure criterion, where the envelope is considered as a straight line, the principal stress difference at failure is related to the confining pressure σ_3 as:

$$(\sigma_1 - \sigma_3)_f = \frac{2\,C\cos\phi + 2\sigma_3\sin\phi}{1 - \sin\phi} \tag{15}$$

Where C is the cohesion intercept and ϕ the friction angle.

The tangent modulus E_t is given by:

$$E_t = \left[1 - \frac{R_f(1-\sin\phi)(\sigma_1 - \sigma_3)}{2c\cos\phi + 2\sigma_3\sin\phi}\right]^2 Kp_a \left(\frac{\sigma_3}{p_a}\right)^n \tag{16}$$

For unloading and reloading cycles Duncan and Chang proposed the following expression:

$$E_{ur} = K_{ur}P_a \left(\frac{\sigma_3}{p_a}\right)^n \tag{17}$$

Where, E_{ur} is the loading-unloading modulus and K_{ur} is the corresponding modulus number.

In the Duncan-Chang's basic model the Poisson's ratio ν_s was assumed to be constant throughout the whole process.

On the basis of what has been presented before, a computer program **NAMPULAL** (**N**onlinear **A**nalysis of **M**onopiles **U**nder **L**ateral and **A**xial **L**oding**)** has been written specifically to deal with monopiles under horizontal and vertical loading as well as an overturning moment.

Although approximate, the computer code **NAMPULAL** exhibits many advantages over other numerical codes in dealing with nonlinear soil/structure interaction problems for many reasons. Firstly, the soil nonlinearity is accounted for through the hyperbolic model, which is a very strong soil failure criterion, especially for sandy soils. Secondly, **NAMPULAL** is based on the combination of two methods among the most powerful ones: the FE and FD methods. Thirdly, as most rigorous packages using the finite element method require 3D modeling, in the NFEVSM only 2D discretisation is needed. The 3D aspect of the problem is taken into account by the iterative nature of the process which sweeps all the slices constituting the considered medium. Hence, large amounts of computer resources and human effort are significantly reduced. Fourthly, the implemented hyperbolic model requires only two iterations, which may result in a significant reduction in CPU time.

The performances of this computer code have been assessed against analysis of the behavior of OWT monopiles where several powerful packages were used (Otsmane and Amar-Bouzid, 2016). The results were in excellent agreement with those of other rigorous procedures, although the soil data were not totally available.

This computer code is employed to determine the monopile head stiffnesses for the OWTs studied in this paper.

4. ANALYSIS OF THREE DIFFERENT OFFSHORE WIND TURBINES

Three (03) Offshore Wind Turbines have been selected from three (03) wind farm sites. These are: Lely A2 (UK), Irene Vorrink (The Netherlands) and Kentish Flats (UK). These wind turbines have been chosen for the full availability of their data, especially the measured first natural frequencies. Soil conditions at the site and sources from which the OWT data are withdrawn are summarized in Table 2.

Table 2. Soil conditions at Lely, Irene Vorrink and Kentish Flats wind farm sites.

Wind farm name	**Country**	**Soil conditions at the site**	**Sources providing data and measured natural frequencies**
Lely Offshore Wind Farm	UK	Soft clay in the uppermost layer to dense and very dense sand layers below	Arany et al. 2016, and Zaaijer 2002
Irene Vorrink Offshore Wind Farm	Netherlands	Soft layers of silt and clayin the upper seabed to dense sand and very dense sand below	Arany et al. 2016, and Zaaijer 2002
Kentish Flats Offshore Wind Farm	UK	Layers of dense sand and firm clay	Arany et al. 2016

The site geotechnical investigations indicate that almost all the OWTs chosen in this paper are installed through deep layers of dense sand. The details of structural data are summarized in Table 3.

A comprehensive mesh study has been performed to find the optimal FE mesh that captures the behavior of monopiles under lateral loading in a non-linear medium characterized by the hyperbolic model as a yield criterion. A mesh of 20 monopile diameter D_p in both sides of the monopile and one monopile length L_p under the monopile tip has been adopted for the study of all OWTs considered here. Furthermore, 35 Finite Elements in both sides of the monopile and 36 Finite Elements in vertical direction as well as 21 slices have been chosen to analyze the Pseudo 3D medium under consideration.

Table 3. Input parameters for the three OWTs chosen for this study.

OWT component dimension	Symbol (unit)	Lely A2	Irene Vorrink	Kentish Flats
Tower height	L_T (m)	37.9	44.5	60.06
Substructure height	L_s (m)	12.1	5.2/6	16.0
Structure height	$L(m)$	50.0	49.7/50.5	76.06
Tower top diameter	D_t (m)	1.90	1.7	2.3
Tower bottom diameter	D_b (m)	3.20	3.5	4.45
Tower wall thickness	t_T (mm)	13.0	13.0	22.0
Substructure diameter	D_s (m)	3.2	3.5	4.3
Substructure wall thickness	t_s (mm)	35	28	45
Tower material Young's modulus	E_T (Gpa)	210.0	210.0	210.0
Tower mass	m_T (ton)	31.44	37.0	108.0
Top mass	m_{RNA} (ton)	32.0	35.5.0	130.8
Monopile diameter	D_p (m)	3.2	3.5	4.3
Monopile Wall Thickness	t_p (mm)	35	28	45
Monopile material Young's modulus	$E_p(Gpa)$	210.0	210.0	210.0
Monopile Depth	L_p (m)	13.5	19.0	29.5
Shear modulus of the soil	$G_s(Mpa)$	140.0	55.0	60.0
Poisson's ratio of the soil	v_s	0.4	0.5	0.4
Soil's Young's modulus	$E_s(Mpa)$	392.0	165.0	168.0
Measured frequency	(Hz)	0.634	0.546/ 0.563	0.339

3.1 Computed monopile head stiffnesses and comparison with those of other procedures

The best way to evaluate the stiffness coefficients K_L, K_R, and K_{LR} is to compute the initial stiffnesses (tangential values at the origin) of the monopile head-deformations curves resulting from the study of the monopile in interaction with subsoil considered a s a non-linear material.

If we assume that the monopile head movements and applied efforts (force and moment) are related through flexibility coefficients, one can write the relationships in a matrix form as:

$$\begin{Bmatrix} u_L \\ \theta_R \end{Bmatrix} = \begin{bmatrix} I_L & I_{LR} \\ I_{RL} & I_R \end{bmatrix} \begin{Bmatrix} H \\ M \end{Bmatrix} \tag{18}$$

Where, H and M are respectively the shear force and the overturning moment applied at the monopile head and u_L and θ_R are respectively the lateral displacement and rotation of the monopile head.

Inverting equation (18), it easy to obtain:

$$\begin{Bmatrix} H \\ M \end{Bmatrix} = \begin{bmatrix} K_L & K_{LR} \\ K_{RL} & K_R \end{bmatrix} \begin{Bmatrix} u_L \\ \theta_R \end{Bmatrix} \tag{19}$$

The stiffness coefficients are related to flexibility ones by the following terms:

$$K_L = \frac{I_R}{I_L I_R - I_{LR}^2} \quad K_R = \frac{I_L}{I_L I_R - I_{LR}^2}, \quad K_{LR} = \frac{I_{LR}}{I_L I_R - I_{LR}^2} \tag{20}$$

In the finite element analyses controlled by forces, the values of K_L, K_R and K_{LR} cannot be found in a straightforward way. The flexibility coefficients are determined first, and then inverted to obtain the stiffness coefficients of equation (20).

In order to determine I_L and I_{LR}, an arbitrary pure horizontal load ($H \neq 0$ and $M = 0$) is applied at the the monopile head at the groundline level. The parameter $1/I_L$ is then obtained by simply computing the slope of the resulting curve at the origin. The parameter $1/I_{LR}$ is computed from the curve giving the variation of H in function of rotation θ issued from the same analysis.

As the rocking flexibility factor I_R, needs a pure bending, the monopile/soil system is analyzed under an arbitrary overturning moment ($M \neq 0$ and $H = 0$) applied at the top of the pile at the mudline level. From the curve portraying the increasing values of M against the obtained rotations θ, the reciprocal of flexibility factor $1/I_R$ is evaluated by simply computing the slope of curve tangent at the origin.

As the monopile head stiffness does not depend on the loading level, a horizontal load H of $1000\ kN$ in magnitude has been applied in ten (10) increments at the top of each monopile in the three wind farms considered, in the aim to compute the monopile head flexibility factors I_L and I_{LR}.

For the computation of I_R, an applied moment M at the top of monopile of $20000\ kN\ m$ in magnitude has been considered.

The almost linear relationships obtained between H and u and H and θ on one hand and between M and θ on the other hand somewhat easen the task to compute I_L ,I_{LR} and I_R which can be performed by simply inverting the slopes of their corresponding load-deformation curves. Then by using equations (20), the stiffness coefficients are obtained. The stiffness coefficients are shown in Table 4 for all turbines considered in this paper. The stiffness coefficients K_L**,** K_R and K_{LR} are respectively in units of (GN/m)**,** $(GN.m/rad)$ and (GN).

Table 4. Stiffness coefficients K_L, K_{LR} and K_R characterizing monopiles in Lely A2, Irene Vorrink and Kentish Flats wind farms.

	Lely A2			**Irene Vorrink**			**Kentish Flats**		
	K_L	K_R	K_{LR}	K_L	K_R	K_{LR}	K_L	K_R	K_{LR}
PRSENT STUDY	**0.339**	**17.049**	**-1.682**	**0.321**	**12.169**	**-1.213**	**0.472**	**28.975**	**-2.278**
ARANY ET AL. 2016	**0.520**	**23.630**	**-2.740**	**0.580**	**29.67**	**-3.25**	**0.820**	**58.770**	**-5.42**

As the monopile head stiffness coefficients play an important role in the correct assessment of the natural frequency, which is in turn a significant parameter in the design of any OWT, it is useful to compare the values from the present study shown in Table 4 with other methods. Indeed, and on the basis of the formulae developed by Poulos and Davis (1980), Randolph (1981) and Carter and Kulhawy (1992), Arany et al. (2016) determined the values of the monopile stiffness coefficients which are added to the Table 4 for comparison.

The close examination of Table 4, allows the reader to note one important point. NAMPULAL's results are approximately half those given by (Arany et al. 2016) for the OWTs whose supporting monopiles are driven in dense sand. We believe here, that the Nonlinear FE vertical slices model results are more accurate as they have been obtained using the hyperbolic model as soil behavior which is an excellent model for this kind of soil strata, while Arany et al. used empirical data from works dedicated especially for slender piles.

3.2 Computed natural frequencies

The expression (1) is employed here to give the fixed base natural frequency. This expression which depends only of the OWT structure properties, gives the values of the fixed base natural frequencies for the different turbines as shown in Table 5.

Table 5. Fixed base natural frequencies for the different OWTs.

Wind farm	Lely A2	Irene Vorrink	Kentish Flats
$\boldsymbol{f_{FB}(Hz)}$	0.719	0.659-0.669	0.368

The left-hand coefficients of the eqs. (3) and (4) depend on values of η_L, η_R and η_{LR}. Table 6 shows the values of C_R and C_L for the three OWTs considered in this paper.

Table 6. C_R and C_L for the OWTs considered in the current study.

Offshore Wind Farm	Lely A2	Irene Vorrink	Kentish Flats
$\boldsymbol{C_R}$	0.867	0.867	0.857
$\boldsymbol{C_L}$	0.996	0.997	0.998

The values present in Table 6, make it quite clear that the coefficient C_R is the dominant factor that can bring the value of the fixed base frequency to the measured one. However, the coefficient C_L is very close to unity, and hence its influence in changing the value of f_{FB} is very small. This has been also noticed by Arany et al. (2016).

The natural frequency which is simply obtained by multiplying the flexibility coefficients (shown in Table 6) by the fixed base frequency for each OWT is given in Table 7. Also shown are errors between the measured and the computed natural frequencies.

Table 7. Predicted and measured natural frequencies of all OWTs.

	Predicted frequency $\boldsymbol{f_\eta = C_R C_L f_{FB}}$	Measured frequency	Error (%)

Lely A2	**0.621**	**0.634**	**-2.006**
Irene Vorrink	**0.570-0.579**	**0.546-0.563**	**1.277-2.918**
Kentish Flats	**0.315**	**0.339**	**-7.682**

4. CONCLUSIONS

Large diameter monopile foundations for offshore wind turbines which are currently in the range of 04 to 06 m, with the potential to become larger in the near future, are subjected to large horizontal forces resulting in huge overturning moments. In current practice, OWT monopile foundations are usually designed using general geotechnical standards in combination with more specific guidelines and semi-empirical formulas that have been largely developed in the sector of offshore oil/gas industry. While the methods on which the current design codes have been built, are theoretically rigorous, the input p-y curves, are based upon very limited field data and hence they have not been validated for large diameter monopiles.

Offshore wind turbines can be considered as high slenderness low stiffness dynamical system involving complex interaction between the wind, wave and the soil. Consequently, the behavior of an offshore wind turbine as well as its substructure are significantly affected by the determination of the first natural frequency.

As far as the accurate determination of the natural frequency is concerned the finite element method is the most powerful numerical method that may be used in dealing with such a problem. Indeed, a computer program **NAMPULAL** has been used to study the lateral behavior of three monopiles supporting OWTs from three different wind farms installed in Europe.

The computed values of K_L, K_R and K_{LR} were not far from those of other well-known method in the literature. Since the obtained values of natural frequency were in excellent agreement with those of the measured ones, we believe that results of the present study are reliable as the computer program **NAMPULAL** has been written on the basis of a strong background.

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