Where, j denotes for the iteration number and j_{max} is determined by a certain convergence criterion. The nonlinearity in VS Model stems from the implementation the hyperbolic model proposed by Duncan and Chang (1970) for modeling the soil. In fact, they found out that both tangential modulus E_i and ultimate stress deviator $(\sigma_1 - \sigma_3)_{ult}$ are dependent on the minor principal stress σ_3. More precisely they suggested for the initial tangent modulus the following formula:

$$E_i = KP_a \left(\frac{\sigma_3}{p_a}\right)^n \tag{13}$$

Where K is dimensionless factor termed 'modulus number', n is a dimensionless parameter called 'modulus exponent' and p_a is the atmospheric pressure used to make K and n non-dimensional.

The ultimate stress difference $(\sigma_1 - \sigma_3)_{ult}$ is defined in terms of the actual failure stress difference by another parameter called failure ratio R_f which is given by:

$$R_f = \frac{(\sigma_1-\sigma_3)_f}{(\sigma_1-\sigma_3)_{ult}} \tag{14}$$

Using Mohr-Coulomb failure criterion, where the envelope is considered as a straight line, the principal stress difference at failure is related to the confining pressure σ_3 as:

$$(\sigma_1 - \sigma_3)_f = \frac{2\,C\cos\phi + 2\sigma_3\sin\phi}{1-\sin\phi} \tag{15}$$

Where C is the cohesion intercept and ϕ the friction angle.

The tangent modulus E_t is given by:

$$E_t = \left[1 - \frac{R_f(1-\sin\phi)(\sigma_1-\sigma_3)}{2c\cos\phi + 2\sigma_3\sin\phi}\right]^2 Kp_a \left(\frac{\sigma_3}{p_a}\right)^n \tag{16}$$

For unloading and reloading cycles Duncan and Chang proposed the following expression:

$$E_{ur} = K_{ur}P_a \left(\frac{\sigma_3}{p_a}\right)^n \tag{17}$$

Where, E_{ur} is the loading-unloading modulus and K_{ur} is the corresponding modulus number.

In the Duncan-Chang's basic model the Poisson's ratio ν_s was assumed to be constant throughout the whole process.

On the basis of what has been presented before, a computer program **NAMPULAL** (**N**onlinear **A**nalysis of **M**onopiles **U**nder **L**ateral and **A**xial **L**oding**)** has been written specifically to deal with monopiles under horizontal and vertical loading as well as an overturning moment.

Although approximate, the computer code **NAMPULAL** exhibits many advantages over other numerical codes in dealing with nonlinear soil/structure interaction problems for many reasons. Firstly, the soil nonlinearity is accounted for through the hyperbolic model, which is a very strong soil failure criterion, especially for sandy soils. Secondly, **NAMPULAL** is based on the combination of two methods among the most powerful ones: the FE and FD methods. Thirdly, as most rigorous packages using the finite element method require 3D modeling, in the NFEVSM only 2D discretisation is needed. The 3D aspect of the problem is taken into account by the iterative nature of the process which sweeps all the slices constituting the considered medium. Hence, large amounts of computer resources and human effort are significantly reduced. Fourthly, the implemented hyperbolic model requires only two iterations, which may result in a significant reduction in CPU time.

The performances of this computer code have been assessed against analysis of the behavior of OWT monopiles where several powerful packages were used (Otsmane and Amar-Bouzid, 2016). The results were in excellent agreement with those of other rigorous procedures, although the soil data were not totally available.

This computer code is employed to determine the monopile head stiffnesses for the OWTs studied in this paper.

4. ANALYSIS OF THREE DIFFERENT OFFSHORE WIND TURBINES

Three (03) Offshore Wind Turbines have been selected from three (03) wind farm sites. These are: Lely A2 (UK), Irene Vorrink (The Netherlands) and Kentish Flats (UK). These wind turbines have been chosen for the full availability of their data, especially the measured first natural frequencies. Soil conditions at the site and sources from which the OWT data are withdrawn are summarized in Table 2.

Table 2. Soil conditions at Lely, Irene Vorrink and Kentish Flats wind farm sites.

Wind farm name	**Country**	**Soil conditions at the site**	**Sources providing data and measured natural frequencies**
Lely Offshore Wind Farm	UK	Soft clay in the uppermost layer to dense and very dense sand layers below	Arany et al. 2016, and Zaaijer 2002
Irene Vorrink Offshore Wind Farm	Netherlands	Soft layers of silt and clayin the upper seabed to dense sand and very dense sand below	Arany et al. 2016, and Zaaijer 2002
Kentish Flats Offshore Wind Farm	UK	Layers of dense sand and firm clay	Arany et al. 2016

The site geotechnical investigations indicate that almost all the OWTs chosen in this paper are installed through deep layers of dense sand. The details of structural data are summarized in Table 3.

A comprehensive mesh study has been performed to find the optimal FE mesh that captures the behavior of monopiles under lateral loading in a non-linear medium characterized by the hyperbolic model as a yield criterion. A mesh of 20 monopile diameter D_p in both sides of the monopile and one monopile length L_p under the monopile tip has been adopted for the study of all OWTs considered here. Furthermore, 35 Finite Elements in both sides of the monopile and 36 Finite Elements in vertical direction as well as 21 slices have been chosen to analyze the Pseudo 3D medium under consideration.

Table 3. Input parameters for the three OWTs chosen for this study.

OWT component dimension	Symbol (unit)	Lely A2	Irene Vorrink	Kentis h Flats
Tower height	L_T (m)	37.9	44.5	60.06
Substructure height	L_s (m)	12.1	5.2/6	16.0
Structure height	$L(m)$	50.0	49.7/50.5	76.06
Tower top diameter	D_t (m)	1.90	1.7	2.3
Tower bottom diameter	D_b (m)	3.20	3.5	4.45
Tower wall thickness	t_T (mm)	13.0	13.0	22.0
Substructure diameter	D_s (m)	3.2	3.5	4.3
Substructure wall thickness	t_s (mm)	35	28	45
Tower material Young's modulus	E_T (Gpa)	210.0	210.0	210.0
Tower mass	m_T (ton)	31.44	37.0	108.0
Top mass	m_{RNA} (ton)	32.0	35.5.0	130.8
Monopile diameter	D_p (m)	3.2	3.5	4.3
Monopile Wall Thickness	t_p (mm)	35	28	45
Monopile material Young's modulus	$E_p(Gpa)$	210.0	210.0	210.0
Monopile Depth	L_p (m)	13.5	19.0	29.5
Shear modulus of the soil	$G_s(Mpa)$	140.0	55.0	60.0
Poisson's ratio of the soil	ν_s	0.4	0.5	0.4
Soil's Young's modulus	$E_s(Mpa)$	392.0	165.0	168.0
Measured frequency	(Hz)	0.634	0.546/ 0.563	0.339

3.1 Computed monopile head stiffnesses and comparison with those of other procedures

The best way to evaluate the stiffness coefficients K_L, K_R, and K_{LR} is to compute the initial stiffnesses (tangential values at the origin) of the monopile head-deformations curves resulting from the study of the monopile in interaction with subsoil considered a s a non-linear material.

If we assume that the monopile head movements and applied efforts (force and moment) are related through flexibility coefficients, one can write the relationships in a matrix form as:

$$\begin{Bmatrix} u_L \\ \theta_R \end{Bmatrix} = \begin{bmatrix} I_L & I_{LR} \\ I_{RL} & I_R \end{bmatrix} \begin{Bmatrix} H \\ M \end{Bmatrix} \tag{18}$$

Where, H and M are respectively the shear force and the overturning moment applied at the monopile head and u_L and θ_R are respectively the lateral displacement and rotation of the monopile head.

Inverting equation (18), it easy to obtain:

$$\begin{Bmatrix} H \\ M \end{Bmatrix} = \begin{bmatrix} K_L & K_{LR} \\ K_{RL} & K_R \end{bmatrix} \begin{Bmatrix} u_L \\ \theta_R \end{Bmatrix} \tag{19}$$

The stiffness coefficients are related to flexibility ones by the following terms:

$$K_L = \frac{I_R}{I_L I_R - I_{LR}^2} \quad K_R = \frac{I_L}{I_L I_R - I_{LR}^2}, \quad K_{LR} = \frac{I_{LR}}{I_L I_R - I_{LR}^2} \tag{20}$$

In the finite element analyses controlled by forces, the values of K_L, K_R and K_{LR} cannot be found in a straightforward way. The flexibility coefficients are determined first, and then inverted to obtain the stiffness coefficients of equation (20).

In order to determine I_L and I_{LR}, an arbitrary pure horizontal load ($H \neq 0$ and $M = 0$) is applied at the the monopile head at the groundline level. The parameter $1/I_L$ is then obtained by simply computing the slope of the resulting curve at the origin. The parameter $1/I_{LR}$ is computed from the curve giving the variation of H in function of rotation θ issued from the same analysis.

As the rocking flexibility factor I_R, needs a pure bending, the monopile/soil system is analyzed under an arbitrary overturning moment ($M \neq 0$ and $H = 0$) applied at the top of the pile at the mudline level. From the curve portraying the increasing values of M against the obtained rotations θ, the reciprocal of flexibility factor $1/I_R$ is evaluated by simply computing the slope of curve tangent at the origin.

As the monopile head stiffness does not depend on the loading level, a horizontal load H of $1000\ kN$ in magnitude has been applied in ten (10) increments at the top of each monopile in the three wind farms considered, in the aim to compute the monopile head flexibility factors I_L and I_{LR}.

For the computation of I_R, an applied moment M at the top of monopile of $20000\ kN\ m$ in magnitude has been considered.

The almost linear relationships obtained between H and u and H and θ on one hand and between M and θ on the other hand somewhat easen the task to compute I_L ,I_{LR} and I_R which can be performed by simply inverting the slopes of their corresponding load-deformation curves. Then by using equations (20), the stiffness coefficients are obtained. The stiffness coefficients are shown in Table 4 for all turbines considered in this paper. The stiffness coefficients K_L, K_R and K_{LR} are respectively in units of (GN/m), $(GN.m/rad)$ and (GN).

Table 4. Stiffness coefficients K_L, K_{LR} and K_R characterizing monopiles in Lely A2, Irene Vorrink and Kentish Flats wind farms.

	Lely A2			**Irene Vorrink**			**Kentish Flats**		
	K_L	K_R	K_{LR}	K_L	K_R	K_{LR}	K_L	K_R	K_{LR}
PRSENT STUDY	**0.339**	**17.049**	**-1.682**	**0.321**	**12.169**	**-1.213**	**0.472**	**28.975**	**-2.278**
ARANY ET AL. 2016	**0.520**	**23.630**	**-2.740**	**0.580**	**29.67**	**-3.25**	**0.820**	**58.770**	**-5.42**

As the monopile head stiffness coefficients play an important role in the correct assessment of the natural frequency, which is in turn a significant parameter in the design of any OWT, it is useful to compare the values from the present study shown in Table 4 with other methods. Indeed, and on the basis of the formulae developed by Poulos and Davis (1980), Randolph (1981) and Carter and Kulhawy (1992), Arany et al. (2016) determined the values of the monopile stiffness coefficients which are added to the Table 4 for comparison.

The close examination of Table 4, allows the reader to note one important point. NAMPULAL's results are approximately half those given by (Arany et al. 2016) for the OWTs whose supporting monopiles are driven in dense sand. We believe here, that the Nonlinear FE vertical slices model results are more accurate as they have been obtained using the hyperbolic model as soil behavior which is an excellent model for this kind of soil strata, while Arany et al. used empirical data from works dedicated especially for slender piles.

3.2 Computed natural frequencies

The expression (1) is employed here to give the fixed base natural frequency. This expression which depends only of the OWT structure properties, gives the values of the fixed base natural frequencies for the different turbines as shown in Table 5.

Table 5. Fixed base natural frequencies for the different OWTs.

Wind farm	**Lely A2**	**Irene Vorrink**	**Kentish Flats**
$\boldsymbol{f_{FB}(Hz)}$	**0.719**	**0.659-0.669**	**0.368**

The left-hand coefficients of the eqs. (3) and (4) depend on values of η_L, η_R and η_{LR}. Table 6 shows the values of C_R and C_L for the three OWTs considered in this paper.

Table 6. C_R and C_L for the OWTs considered in the current study.

Offshore Wind Farm	**Lely A2**	**Irene Vorrink**	**Kentish Flats**
$\boldsymbol{C_R}$	**0.867**	**0.867**	**0.857**
$\boldsymbol{C_L}$	**0.996**	**0.997**	**0.998**

The values present in Table 6, make it quite clear that the coefficient C_R is the dominant factor that can bring the value of the fixed base frequency to the measured one. However, the coefficient C_L is very close to unity, and hence its influence in changing the value of f_{FB} is very small. This has been also noticed by Arany et al. (2016).

The natural frequency which is simply obtained by multiplying the flexibility coefficients (shown in Table 6) by the fixed base frequency for each OWT is given in Table 7. Also shown are errors between the measured and the computed natural frequencies.

Table 7. Predicted and measured natural frequencies of all OWTs.

	Predicted frequency $\boldsymbol{f_\eta = C_R C_L f_{FB}}$	**Measured frequency**	**Error (%)**

Lely A2	**0.621**	**0.634**	**-2.006**
Irene Vorrink	**0.570-0.579**	**0.546-0.563**	**1.277-2.918**
Kentish Flats	**0.315**	**0.339**	**-7.682**

4. CONCLUSIONS

Large diameter monopile foundations for offshore wind turbines which are currently in the range of 04 to 06 m, with the potential to become larger in the near future, are subjected to large horizontal forces resulting in huge overturning moments. In current practice, OWT monopile foundations are usually designed using general geotechnical standards in combination with more specific guidelines and semi-empirical formulas that have been largely developed in the sector of offshore oil/gas industry. While the methods on which the current design codes have been built, are theoretically rigorous, the input p-y curves, are based upon very limited field data and hence they have not been validated for large diameter monopiles.

Offshore wind turbines can be considered as high slenderness low stiffness dynamical system involving complex interaction between the wind, wave and the soil. Consequently, the behavior of an offshore wind turbine as well as its substructure are significantly affected by the determination of the first natural frequency.

As far as the accurate determination of the natural frequency is concerned the finite element method is the most powerful numerical method that may be used in dealing with such a problem. Indeed, a computer program **NAMPULAL** has been used to study the lateral behavior of three monopiles supporting OWTs from three different wind farms installed in Europe.

The computed values of K_L, K_R and K_{LR} were not far from those of other well-known method in the literature. Since the obtained values of natural frequency were in excellent agreement with those of the measured ones, we believe that results of the present study are reliable as the computer program **NAMPULAL** has been written on the basis of a strong background.

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