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Evaluation of initial state parameter from pressuremeter test in sand using similarity solution technique

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ABSTRACT

This study deals with a theoretical evaluation of initial state parameter from the pressuremeter test in sand using the similarity solution technique. A similarity solution for the expansion of a cylindrical cavity in sand under drained loading condition is derived by replacing partial differential equations of stress equilibrium, constitutive law, consistency condition, and continuity with first-order ordinary differential equations. The sand is modeled using a state parameter-based critical state model.
Using the soil properties of six sands, such as Monterey no. 0 sand, Hokksund sand, Kogyuk 350/2 sand, Ottawa sand, Reid Bedford sand, and Ticino sand, a theoretical correlation between initial state parameter and the loading slope of pressuremeter test is suggested.

1. INTRODUCTION

Pressuremeter tests have been used to assess the state of the soil in the field. The state variables are evaluated from the loading/unloading slope and limit pressure of the test results (Cudmani and Osinov 2001).

Assuming that the pressuremeter test can be simulated as the expansion and/or contraction of a cylindrical cavity in the soil, the relationship between the pressuremeter loading/unloading slope and the state variables can be evaluated theoretically. Hughes et al. (1977) presented a theoretical correlation between the angles of friction and dilation in sand and the loading slope. In their analysis, a closed-form small strain cavity expansion solution was developed and the sand was assumed to behave as an elastic-perfectly plastic Mohr-Coulomb material. Yu (1994) presented a theoretical correlation between the initial state parameter in sand and the loading or unloading slope. In the analysis, the sand was modeled using the state parameter-based critical state model, whereas the yield criterion proposed by Matsuoka (1976) was used. In addition, the finite element program CAVEXP was used to simulate the self-boring pressuremeter test as the expansion of a cylindrical cavity in sand. Silvestri (2001) proposed an interpretation method for the shear strength chatacteristics, volume response, and

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dilatancy properties of sand from pressuremeter expansion tests by using Rowe's (1962) stress-dilatancy theory.

The similarity solution technique has been used for the analysis of cavity expansion as well as cavity contraction (Park 2014a,b). Collins et al. (1992) presented a similarity solution for the cavity expansion in sands under drained loading conditions using the state parameter-based critical state model. They showed the application of the solution for the analysis of cone penetration test.

This study addresses a theoretical correlation of initial state parameter from the pressuremeter loading test in sand using the similarity solution technique. The large-strain solutions for stresses and displacement are derived by replacing the partial differential equations of stress equilibrium, constitutive law, consistency condition, and continuity with first-order ordinary differential equations, which are solved by the Runge-Kutta method.

2. SIMILARITY SOLUTION FOR CAVITY EXPANSION

Fig. 1 shows a cylindrical cavity being expanded in an isotropic, initially elastic soil mass subjected to a hydrostatic stress p_o. As noted by Hill (1950) and Collins and Wang (1990), the problem of cavity expansion from zero initial radius has no characteristic length and hence possesses a similarity solution, where the cavity pressure remains constant and the continuing deformation is geometrically self-similar. Four equations for stress equilibrium, constitutive law, consistency condition, and continuity are needed to describe this axisymmetric cavity expansion problem.

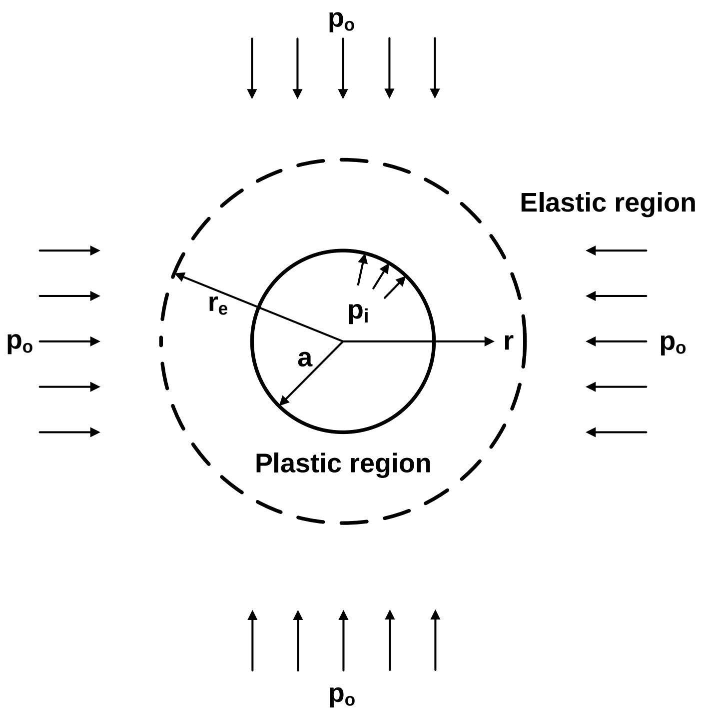

Fig. 1 A cylindrical cavity expanded in an infinite medium

2.1 Stress Equilibrium

Assuming a state of plane strain symmetry around the cylindrical cavity, the equilibrium equation in polar coordinate system is given by

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \qquad (1)$$

where σ_r is the radial stress and σ_θ is the circumferential stress.

2.2 Constitutive Law

Considering the elastic and plastic strain rates, the necessary constitutive equation is obtained as

$$A_r \hat{\sigma}_r + A_\theta \hat{\sigma}_\theta + A_w e_r = A_o e_\theta \qquad (2)$$

where

$$A_r = \frac{1}{2G}\left(1 - \nu - \frac{\nu}{d}\right);\ A_\theta = \frac{1}{2G}\left(-\nu + \frac{1-\nu}{d}\right);\ A_w = -1;\ A_o = \frac{1}{d} \qquad (3)$$

e_r, e_θ are radial and circumferential strains, G is the shear modulus, v is the Poisson's ratio, and d is a simple function of dilation angle.
The symbol (^) denotes the material time derivative associated with a given material particle and it is related to the local time derivative (˙), evaluated at fixed position r, by

$$\hat{()} = \dot{()} + w\frac{\partial ()}{\partial r} \qquad (4)$$

where w is the radial speed of a material element.

2.3 Consistency Condition

A further rate equation, valid in the plastic region, is obtained by differentiation of the yield function associated with a given solid material with respect to time, such as

$$B_r \hat{\sigma}_r + B_\theta \hat{\sigma}_\theta + B_\upsilon \hat{\upsilon} = 0 \qquad (5)$$

where

$$B_r = \frac{\partial f}{\partial \sigma_r} + \frac{\partial f}{\partial \xi}\frac{\partial \xi}{\partial p}\frac{\partial p}{\partial \sigma_r} \quad ; \quad B_\theta = \frac{\partial f}{\partial \sigma_\theta} + \frac{1}{2}\frac{\partial f}{\partial \xi}\frac{\partial \xi}{\partial p}\frac{\partial p}{\partial \sigma_\theta} \quad ; \quad B_\upsilon = \frac{\partial f}{\partial \xi}\frac{\partial \xi}{\partial \upsilon} \qquad (6)$$

f is the yield function, p is the effective stress invariant, ξ is a state parameter, and υ is the specific volume.
The state parameter is defined as the vertical distance between the current state and the critical state line in the usual υ –ln$p$ plot.

2.4 Continuity Equation

An equation can be obtained by considering the relation between volumetric change and specific volume, such as

$$\frac{\hat{\upsilon}}{\upsilon} = -(e_r + e_\theta) \qquad (7)$$

2.5 Similarity Solution

By considering the non-dimensionalized forms, such as

$$\tilde{u} = \frac{u}{r_e} \quad ; \quad \tilde{w} = \frac{w}{W} \quad ; \quad \tilde{\sigma}_r = \frac{\sigma_r}{p_1} \quad ; \quad \tilde{\sigma}_\theta = \frac{\sigma_\theta}{p_1} \tag{8}$$

the partial differential equations for stress equilibrium, constitutive law, consistency condition, and continuity can be replaced by a system of five first-order ordinary differential equations as

$$\begin{bmatrix} 1 & 0 & 0 & \dfrac{(\tilde{\sigma}_r - \tilde{\sigma}_\theta)}{(\tilde{w}-\theta)} & 0 \\ p_1\theta\tilde{A}_r & p_1\theta\tilde{A}_\theta & 0 & -\tilde{A}_w + \dfrac{\tilde{A}_o\tilde{w}}{(\tilde{w}-\theta)} & 0 \\ p_1\tilde{B}_r & p_1\tilde{B}_\theta & \tilde{B}_\upsilon & 0 & 0 \\ 0 & 0 & \theta & -\upsilon - \dfrac{\upsilon\tilde{w}}{(\tilde{w}-\theta)} & 0 \\ 0 & 0 & 0 & -\dfrac{\tilde{u}-\tilde{w}}{(\tilde{w}-\theta)} & 1 \end{bmatrix} \frac{d}{d\theta}\begin{Bmatrix} \tilde{\sigma}_r \\ \tilde{\sigma}_\theta \\ \upsilon \\ \tilde{w} \\ \tilde{u} \end{Bmatrix} = \begin{Bmatrix} \dfrac{(\tilde{\sigma}_r - \tilde{\sigma}_\theta)}{(\tilde{w}-\theta)} \\ \dfrac{\tilde{A}_o\tilde{w}}{(\tilde{w}-\theta)} \\ 0 \\ -\dfrac{\upsilon\tilde{w}}{(\tilde{w}-\theta)} \\ -\dfrac{\tilde{u}-\tilde{w}}{(\tilde{w}-\theta)} \end{Bmatrix} \tag{9}$$

where *u* is the radial displacement, r_e is the radius at the elastic-plastic interface, *W* is the speed of expansion of the elastic-plastic boundary $r=r_e$ (or ρ=1), and p_1 is a suitably chosen reference, or simply 1 kPa.
Then Eq. (9) can be solved as an initial value problem which starts from the elastic-plastic boundary ($\theta(1) = \tilde{w}(1) - 1$) to the cavity wall ($\theta$=0). For this initial value problem, the subroutine *odeint*, Runge-Kutta driver with adaptive stepsize control (Press et al. 1992) is used.

3. EVALUATION OF STATE PARAMETER

In this study, six sands (Monterey no. 0 sand, Hokksund sand, Kogyuk 350/2 sand, Ottawa sand, Reid Bedford sand, and Ticino sand), which were studied by several researchers, are chosen. Table 1 shows the critical state properties (ϕ_{cv}, λ, Γ) of sands used in this study. These values are obtained from Yu et al. (1996).

Fig. 2 shows the results of comparison. In the figure, the symbols indicate the results by the present similarity solution, while the solid line indicates the best-fit line from those results, such as

$$\xi_o = 0.67 - 2.12s \tag{10}$$

The loading slopes are obtained by considering the whole curve up to the limit pressure. The dotted line indicates the correlation by Yu (1994). The difference of the results between Yu (1994) and this study may be expected from the use of different yield criterion and plastic potential function.

Table 1. Soil properties for six sands

Sand type	λ	Γ	ϕ_{cv}(deg)	A(rad)
Monterey no. 0	0.013	1.878	32	0.83
Hokksund	0.024	1.934	32	0.80
Kogyuk	0.029	1.849	31	0.75
Ottawa	0.012	1.754	28.5	0.95
Reid Bedford	0.028	2.014	32	0.63
Ticino	0.024	1.986	31	0.60

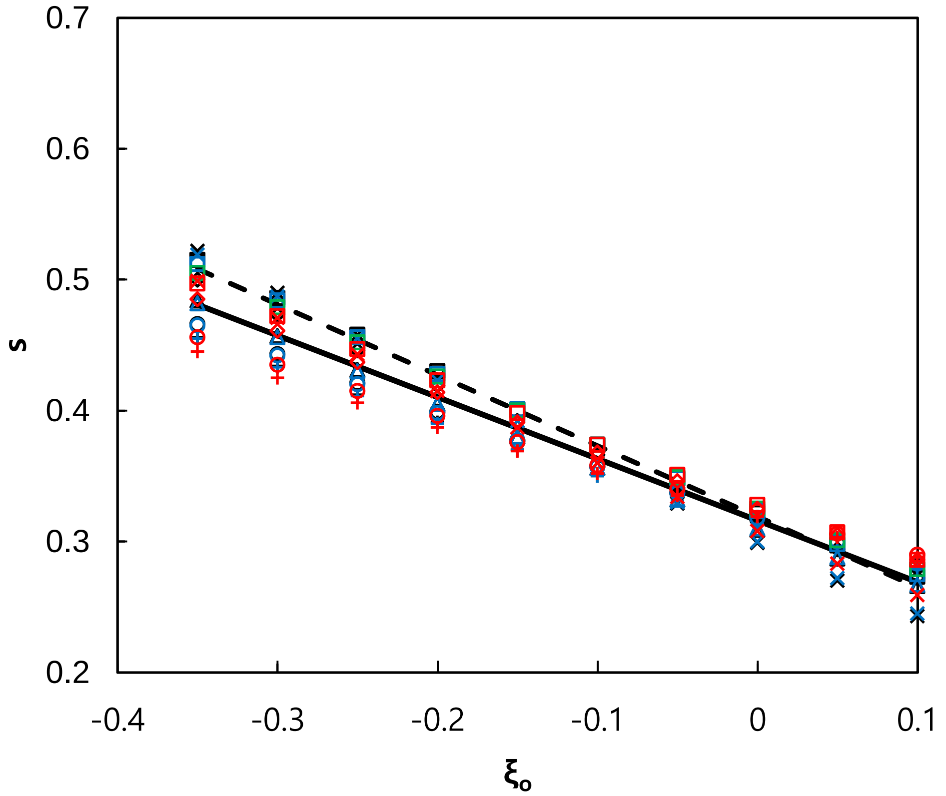

Fig. 2 Correlation between s and ξ_o

4. CONCLUSIONS

The large-strain similarity solution for a cylindrical cavity expansion, using the state parameter-based critical state soil model, has been derived to address the evaluation of initial state parameter from the pressuremeter loading test in sand. Using the present solution and dataset of six sands, a theoretical correlation between initial state parameter and the loading slope of pressuremeter test has been suggested.

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