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# Large Amplitude Free Vibration Analysis of Functionally Graded Nano/Micro Beams on Elastic Foundation

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**ABSTRACT**

The purpose of this paper is to study the geometrically nonlinear free vibration of functionally graded nano/micro beams (FGNBs) based on the modified couple stress theory. For practical applications, some analytical expressions of nonlinear frequencies for FGNBs on the elastic foundation are developed. Hamilton's principle is employed to obtain nonlinear governing differential equations in the context of both Euler-Bernoulli and Timoshenko beam theories for a comprehensive investigation. The modified continuum theory contains one material length scale parameter to capture the size effect. The variation of two-constituent material along the thickness is modeled using Reddy's power-law. Also, the Mori-Tanaka method as an accurate homogenization technique is implemented to estimate the effective material properties of the FGNBs. The results are presented for both hinged-hinged and clamped-clamped boundary conditions. The nonlinear partial differential equations are reduced to ordinary differential equations using Galerkin method and then the powerful method of homotopy analysis is utilized to obtain the semi-analytical solutions. Eventually, the presented analytical expressions are used to examine the influences of the length scale parameter, material gradient index, slenderness ratio and elastic foundation on the nonlinear free vibration of FGNBs.

***KEYWORDS:*** Modified couple stress theory; Nonlinear vibration; Functionally graded micro-beam; Homotopy analysis method.

## 1. INTRODUCTION

Recently a new class of composite materials known as functionally graded materials (FGMs) has attracted considerable attention in many various industrial fields. These inhomogeneous composites usually are made from a mixture of metals and ceramics. In these materials, the mechanical properties change from one surface to another. The

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capability of functionally graded (FG) materials can be used in nano/microstructures by employing modern spattering machines. Meanwhile, nano/micro-beams have been widely used in biosensors, atomic force microscope and many other micro/nano-electro-mechanical systems. However, the properties of nano/micro-beams are closely related to their microstructures. To understand the mechanical behavior of such beams, it is significant to consider the size effect that resulting from their microstructures. Since the classical continuum theory could not captures the size effects, thus the non-classical theories such as classical couple stress theory (Mindlin and Tiersten 1962), the nonlocal elasticity theory (Eringen 1972), and the strain gradient theory (Lam *et al.* 2003) have been proposed.

Linear/nonlinear vibration is very common for nano/micro-beams subjected to external forces in some basic components of new nanoscale devices such as oscillators, and actuators. In this regard, some studies have been performed by employing various modified continuum theories together with different numerical or analytical solutions (Janghorban and Zare 2011, Bayat *et al.* 2013, Thai and Choi 2015, Sedighi *et al.* 2014, Malekzadeh and Shojaee 2015, Setoodeh *et al.* 2016).

However, only few researchers have paid attention to the nonlinear vibration of FG nano/micro-beams as a new potential application of nanostructures. Asghari *et al.* (2011) studied the size-dependent static and vibration behaviors of FG Euler-Bernoulli micro-beams using modified couple stress theory (MCST). Ke *et al.* (2012) investigated size effect on the dynamic stability of FG micro-beams based on MCST. They showed that the size effect on the dynamic stability is significant only when the thickness of beam has a similar value to the length scale parameter. Nateghi and Salamat-talab (2013) presented thermal effect on the size-dependent behavior of FG micro-beams in context of MCST using generalized differential quadrature. Setoodeh and Afrahim (2014) studied nonlinear vibrational behavior of FG micro-pipes conveying fluid based on strain gradient theory. They used homotopy analysis method to obtain the results. According to the available literature, no analytical expressions for the nonlinear frequencies of FG nano/micro-beams have been derived so far.

The main target of this paper is to develop size-dependent analytical expressions for the nonlinear vibration of FG nano/micro-beams using homotopy analysis method. A microstructure-dependent nonlinear Euler-Bernoulli (EBT) and Timoshenko beam (TBT) theories which account for through-thickness power-law variation of a two-constituent material are developed in the context of modified couple stress theory. The effects of nonlinear elastic foundation and boundary conditions are taken into account.

## 2. NONLINEAR SIZE-DEPENDENT EQUATIONS OF MOTION

Fig. 1 shows a FG nano/micro-beam with length $L$, width $b$, and thickness $h$ made from a mixture of ceramic and metal. In this investigation the top surface of micro-beam ($z=h/2$) is ceramic-rich and the bottom surface ($z=-h/2$) is metal-rich. The beam is resting on a nonlinear elastic foundation with linear coefficient $k_l$, nonlinear coefficient $k_{nl}$ and shear coefficient $k_G$. The effective material properties of FGNBs are estimated through the Mori-Tanaka homogenization technique as follow (Ke *et al.* 2012),