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Study on equivalent damping ratio of wind-induced vibration for transmission towers considering tower-line coupling effect

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ABSTRACT

The current design wind loads for transmission towers ignore the influence of the tower-line coupling effect (TCE). To make those design wind loads accurately apply to actual transmission lines, the TCE should be considered. A simplified calculation model of a tower-line system with one tower and two spans was established, and its calculation freedom degrees were reduced by assuming the mode shape of the conductors. Based on stochastic vibration theory and modal analysis method, the frequency domain solution for the resonant response of transmission towers with conductors under wind loads was derived. Comparing the resonant response formulae of the same tower with and without conductors, the equivalent damping ratio (EDR) considering the TCE was determined. Ignoring the high-order small quantities, a practical calculation formula for the EDR was obtained. Thus, a physical mathematical model considering the TCE was proposed and verified by a finite element model example. The results show that the study of the influence of conductors on the wind-induced vibration of transmission towers can be transformed into that of those towers' EDR. The proposed EDR can accurately consider the TCE. The TCE results in a significant decrease in the resonant component of the tower displacement and a significant increase in the EDR

1. INTRODUCTION

Transmission lines (TLs) are a continuous flexible combination system, mainly composed of steel towers, conductors and insulators, which are critical lifeline engineering infrastructures for a reliable electrical power network. Tower-line coupling effect (TCE) is the most significant structural property of tower-line systems and generally exists in the dynamic analysis, the structural design, and the vibration control of TLs under wind loads, even for other dynamic loads, which consecutively attracts the attention of researchers (Fu and Li, 2016; Zhang et al., 2017; Liang et al., 2020).

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Conductor-and-insulator strings affect the dynamic characteristics of transmission towers as their auxiliary structure. Early, Momomura et al. (1997) and Okamura et al. (2003) analyzed the TCE using a full-scale measurement in a mountainous area. Their research showed that the TCE has negligible influence on the tower modal shape in the frequency range below 1 Hz. Nevertheless, due to the field measurement being less conducive to parameter analysis, some other research methods are used to analyze the wind-induced vibration of transmission towers with and/or without conductors to reveal the relationship between the TCE and the structural dynamic characteristics. Through investigating wind tunnel tests of aeroelastic models, it is found that, affected by the TCE, the frequencies, especially in the along-conductor direction, of a transmission tower decrease but its damping ratios increase significantly (Liang et al., 2015), the increase of wind speed shifts the response power spectrums in a high band to lower frequency region, and increases the ratio of background or quasi-static component in the all response (Xie et al., 2017), the wind-vibration responses of super high-rise transmission towers and large long-span conductors all contain each other's response components (Zhao et al., 2018). As for numerical analyses based on the finite element modeling (FEM), the influence of the TCE on the frequency, damping and response components of transmission towers has been verified as same as the results of the tests (Hamada et al., 2017; Zhang et al., 2020), and some new findings indicate that the TCE, related to the span and tower height (We et al., 2012), change the most adverse wind direction angle of transmission towers (Fu and Li, 2016), but has a negligible influence on the conductors (Deng et al., 2004), and the higher the wind speed, the more significant the TCE (Zhang et al., 2017). On the other hand, the theoretical study concerning the TCE has also made some gains. According to an established multi-particle model, dynamic formulae for tower-line systems were derived and used to theoretically evaluates the influence of the TCE on tower frequencies (Zhou et al., 2020). Nonetheless, the effectiveness of the multi-particle model on the wind-induced vibration calculation has been not verified and, hence, it has not been promoted at present. Since the span of conductors is larger than the size of most engineering structures, the influence of the TCE needs to be considered in detail (He et al., 2019). However, the TCE is considerably complicated, as shown in Fig. 1, the study of tower dynamic characteristics affected by it under the action of wind loads is not over yet.

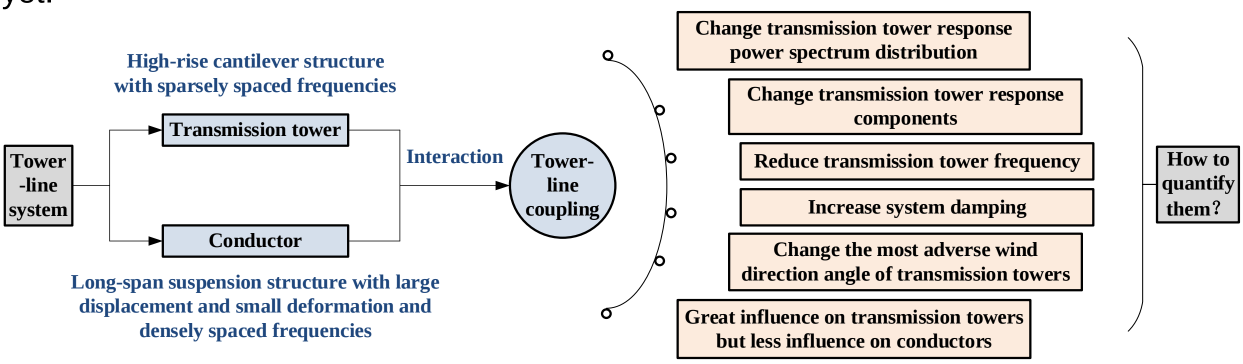

Fig. 1 The influence of tower-line coupling effect on the dynamic characteristics of transmission towers

The study results of the TCE need to be extended to the engineering practice of the wind-resistant design of TLs. A method of separating the tower and the conductor structure is widely used in wind-induced vibration analyses (Zhang et al., 2022; Zhao et al., 2022a) and in determining the design wind loads in the current applicable codes (AS/NZS 7000, 2016; ASCE 74, 2020; DL/T 5551, 2018). Essentially, this method neglects the TCE. There are three key issues still existing in the current design wind loads for TLs in these codes: One is the failure to accurately consider the influence of tower cross-arms, the second is that the concept of the ESWLs is not applicable to conductors with geometrical nonlinear wind-induced vibration, and the third is the inadequate consideration on the TCE. For the first two issues, effective solutions have been proposed (Zhao, et al, 2020; 2022b). In view of the complexity of the TCE, most of the existing studies stay at the level of qualitative analysis, do not form internationally recognized study conclusions, and also fail to solve the problem of how to apply them in the calculation and design process. For the third issue, although the necessity to take into account the TCE, especially for those towers with large long-span conductors, has been emphasized (Xue et al., 2020), no universally accepted practical calculation method for wind-induced vibration of transmission towers considering it has been put forward so far.

The quantitative analysis of the TCE is a practical yet challenging issue and also a common goal of TL wind-resistant researchers. Herein, a new practical calculation method, using the equivalent damping ratio (EDR), is proposed to quantitatively determine the influence of the TCE on transmission tower buffeting under synoptic winds. This paper firstly establishes a reduced physical model of the tower-line system with one tower and two spans, based on stochastic vibration theory and modal analysis method, derives the frequency-domain dynamics formulae for the model due to the tower wind loads, and then proposes an EDR expression considering the influence of the conductor-and-insulator strings on the dynamic characteristics of the tower. Eventually, the methods proposed in this paper are verified by a FEM. The present study aims to establish a quantitative calculation framework for wind-induced vibration of TLs that decouples the TCE.

2. Wind-induced vibration calculation for transmission towers with conductors

2.1 Simplified calculation model

A simplified model for the tower-line system with one tower and two spans, as shown in Fig. 2, is established to analyze the TCE. Here, H is the tower total height; l_{ca} is the cantilever length of the cross-arm; l_{in} is the suspension insulator string length; L is the conductor span. Wherein, the transmission tower is a dense structure with a square variable section, decreasing in size from bottom to top, and a constant cross-section for the cross-arms, and there is no height difference between the hanging points at both ends of the conductor. Without loss of generality, the methodology proposed in this section can be extended to TLs with multiple spans and towers.

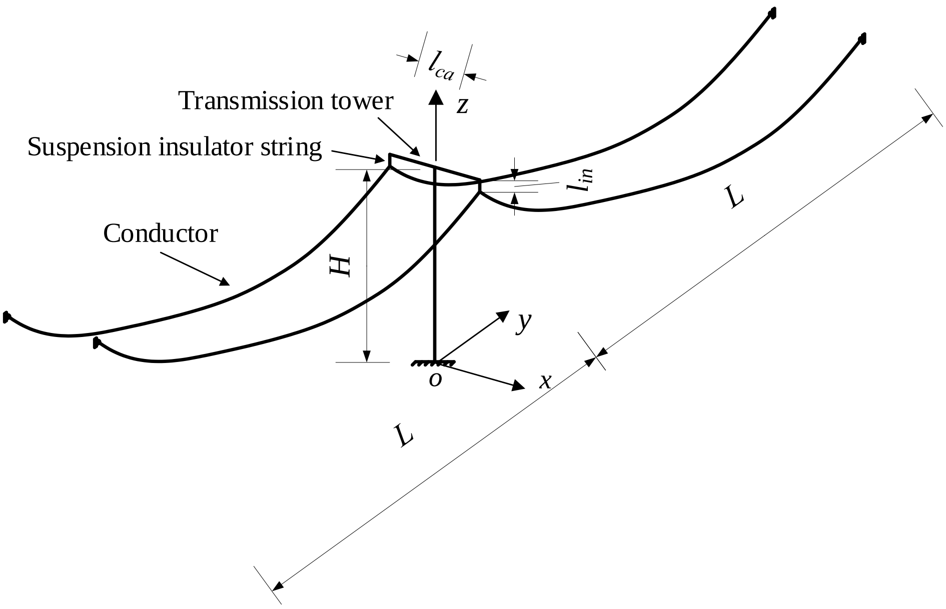

Fig. 2 Simplified calculation model for the transmission tower-line system

2.2 Wind-induced vibration calculation formula derivation

Two assumptions are introduced before formula derivation: The wind-induced vibrations of the conductors on the windward and leeward sides are synchronized; Since the insulator has a little influence on the mechanical characteristics of the cable structure composed of the conductors and the suspension insulator strings, to simplify the calculation, the frequency and damping ratio values of the insulator are consistent with those of the conductors. A tower-line system is composed of the cable structure and tower structure. By giving the conductor mode shape excited by tower vibration, the number of the degree-of-freedom of the cable structure can be significantly reduced. Considering the cable structure as a series system of the conductors and the suspension insulator strings, there is a relationship: $M^*_{ci}=M^*_c+M^*_{in}$, $K^*_{ci}=K^*_c+K^*_{in}$ and $C^*_{ci}=C^*_c+C^*_{in}$, where M^*, K^* and C^* are the generalized mass, generalized stiffness and generalized damping, respectively; The subscript '*ci*', '*c*' and '*in*' indicate the cable structure, the conductors and the suspension insulator strings, respectively. Herein, we only focus on the influence of the conductor structure on the wind-induced vibration of the transmission tower, not the wind loads acting on the conductors, because the latter belongs to the research category of the equivalent static wind loads. Thus, the expression for wind-induced vibration of the transmission tower with conductors in Fig. 2 excited by the tower wind loads is:

$$\mathbf{M}_t\ddot{\mathbf{u}}_t+\mathbf{C}_t\dot{\mathbf{u}}_t+\mathbf{K}_t\mathbf{u}_t=\mathbf{F}_t+\mathbf{F}_{ci}, \tag{1}$$

where the subscript '*t*' indicates the transmission tower; $\mathbf{M}_t$, $\mathbf{K}_t$ and $\mathbf{C}_t$ are the mass matrix, stiffness matrix and damping matrix of the transmission tower, respectively; $\mathbf{F}_t$ is the wind loads acting on the transmission tower; $\mathbf{F}_{ci}$ is the force of the cable structure on the transmission tower; $\mathbf{u}_t$ is the displacement of the transmission tower relative to the ground.

The motion expression of the cable structure in Fig. 2 due to the wind-induced vibration of the transmission tower is

$$M_{ci}^{*}\ddot{u}_{ci}+C_{ci}^{*}\left[\dot{u}_{ci}-\dot{u}_t\left(z_j\right)\right]+K_{ci}^{*}\left[u_{ci}-u_t\left(z_j\right)\right]=0\,, \tag{2}$$

where z_j is the height of the connection point between the transmission tower and the cable structure, and $z_j=H$ in Fig. 2.

Letting $u_d=u_{ci}-u_t(z_j)$, where u_d is the relative displacement of the cable structure to the transmission tower, and using the mode decomposition method for Eq. (1), the following is

$$\ddot{q}_t+2\omega_t\zeta_t\dot{q}_t+\omega_t^2 q_t=F_t^{*}/M_t^{*}+\mu\phi_t\left(z_j\right)\left(2\omega_{ci}\zeta_{ci}\dot{u}_d+\omega_{ci}^2\dot{u}_d\right), \tag{3}$$

where q_t is the generalized coordinate; ω_t is the undamped natural circular frequency; ζ_t is the total damping ratio, including structural damping and aerodynamic damping; F_t^{*} is the generalized wind loads; $\phi_t(z)$ is the mode shape for the first mode, and $\phi_t(z)=(z/H)^2$, $0\le z\le H$; μ is the generalized mass ratio of the cable structure to the transmission tower, and $\mu=M_{ci}^{*}/M_t^{*}$.

Similarly, using the mode decomposition method for Eq. (2), one obtains

$$\phi_t\left(z_j\right)\ddot{q}_t+\ddot{u}_{ci}+2\omega_{ci}\zeta_{ci}\dot{u}_d+\omega_{ci}^2 u_d=0\,, \tag{4}$$

where, according to the previous assumptions, ζ_{ci} and ω_{ci} are approximately taken as ζ_c and ω_c, respectively. Let $F_t^{*}/M_t^{*}=\mathrm{e}^{\mathrm{i}\omega t}$ in Eq. (3), and then there are relations $q_t=H_{tl}(\omega)\mathrm{e}^{\mathrm{i}\omega t}$ and $u_d=H_d(\omega)\mathrm{e}^{\mathrm{i}\omega t}$. By substituting these two equations into Eq. (3) and Eq. (4), one obtains

$$\left|H_{tl}\left(\omega\right)\right|^2=\frac{1}{\omega_t^4\left(E_1+4F_1\right)}, \tag{5}$$

$$E_1=\left\{1-\left[1+\mu\phi_t^2\left(z_j\right)\right]\frac{\omega^2}{\omega_t^2}-\mu\phi_t^2\left(z_j\right)\frac{\omega^2}{\omega_t^2}\frac{\dfrac{\omega_{ci}}{\omega}-1}{\left(\dfrac{\omega_{ci}^2}{\omega^2}-1\right)^2+4\zeta_{ci}\dfrac{\omega_{ci}^2}{\omega^2}}\right\}^2, \tag{6}$$

$$F_1=\left[\zeta_t\frac{\omega}{\omega_t}+\mu\phi_t^2\left(z_j\right)\frac{\omega^2}{\omega_t^2}\frac{\zeta_{ci}\dfrac{\omega_{ci}}{\omega}}{\left(\dfrac{\omega_{ci}^2}{\omega^2}-1\right)^2+4\zeta_{ci}\dfrac{\omega_{ci}^2}{\omega^2}}\right]^2, \tag{7}$$

Adopting the height-independent fluctuating wind velocity power spectrum function $S_f(n)$ proposed by Davenport (1965) and the frequency-independent correlation function $coh(z_1,z_2)$ proposed by Shiotani and Arai (1967), the mean square value of the along-wind displacement resonant component of the transmission tower with conductors is:

$$\sigma_{ut,r}^2(z)=\phi_t^2(z)\frac{1}{M_t^{*2}}\int_0^H\int_0^H\phi_t(z_1)\phi_t(z_2)\sigma_f(z_1)\sigma_f(z_2)\, coh(z_1,z_2)\mathrm{d}z_1\mathrm{d}z_2\int_0^\infty\left|H_{tl}(n)\right|^2\mathrm{d}nS_f(n_t), \tag{8}$$

$$\int_0^\infty\left|H_{tl}(n)\right|^2\mathrm{d}n=\frac{1}{4\omega_t^3}C(\mu,\lambda_n,\zeta_t,\zeta_{ci}), \tag{9}$$

where λ_n is the frequency ratio of the cable structure to the transmission tower.

For the same tower without conductors, the integral expression of its mechanical admittance function square is

$$\int_0^\infty\left|H_t(n)\right|^2\mathrm{d}n=\frac{1}{8\omega_t^3\zeta_t}. \tag{10}$$

Comparing Eq. (9) and (10), the effect of adding conductors is equivalent to changing the damping ratio of the tower. At this time, the EDR of the tower considering the influence of adding conductors is

$$\zeta_e=\frac{1}{2C(\mu,\lambda_n,\zeta_t,\zeta_{ci})}. \tag{11}$$

By ignoring the high-order small quantities of ζ_e, a practical calculation formula for the EDR is obtained, given by:

$$\zeta_e\approx\zeta_t+\mu\lambda_n\zeta_{ci}. \tag{12}$$

2.3 Verification of the derived formula

The accuracy of the derived formula is verified by an example calculation. Firstly, a TL (as shown in Fig. 2) is modeled and analyzed by ANSYS software. The calculation parameters of the TL are: The total height of the transmission tower is H=90 m. The unit height mass of the tower body is $m(z)$=1072.096$(1\text{-}0.5\frac{z}{H})^2$ kg/m. The unit height windshielding area of the tower body is $b_s(z)$=1.658$(1\text{-}0.5\frac{z}{H})$ m. The cross-arm mass is M_{ca}=5514.522 kg. The cross-arm windshielding area is $A_{s,ca}$=0.372 m^2. The mode shape of the transmission tower is $\phi_t(z)=\left(\frac{z}{H}\right)^2$. The drag coefficient of the transmission tower is $\mu_s(z)$=1. The length of the suspension insulator string is l_{in}=10 m. The span of the conductors is L=700 m. The number of bundled conductors is N_c=6. The mass per unit length of the conductor m_c=1.917 kg/m. The conductor diameter is D_c=0.032 m. The drag coefficient of the conductor is μ_{sc}=1. The initial horizontal conductor tension is T_0=70500.446 N. The mass per unit length of the suspension insulator string is m_{in}=123.808 kg/m. The suspension insulator string diameter is D_{in}=0.36 m. The category B ground roughness (the terrain type of open country and towns with sparse houses) is used for the calculations, the design average wind speed at 10 m height is 30 m/s and the oncoming wind direction is perpendicular to the span direction.

The time domain results of the tower top displacement calculated by ANSYS software are decomposed. Since the wind-induced vibration of the transmission tower is a steady random process under the action of synoptic winds, the average response component can be obtained by averaging the time-varying displacement. The fluctuating displacement is determined by subtracting the average displacement from the total displacement response. After identifying the frequency range of the resonant displacement, the band-pass filtering technique can be used to extract the resonant displacement from the fluctuating displacement. The background response is then determined by subtracting the resonant displacement from the fluctuating displacement.

We use two methods to determine the frequency range of resonant displacement. The first method (Hamada et al., 2017) is to select the two lower points on either side of the power spectrum peak in the displacement power spectrum density diagram, and take the frequency difference between these two points as the bandwidth of the resonant displacement. The second method (Elawady et al., 2017) is to determine the slope threshold value for judging the resonant displacement by repeated trial calculation. When the slopes of displacement power spectral density in the logarithmic coordinate graph exceeds a given threshold value, these displacements are regarded as the resonant displacements, and their frequency values are used to determine the resonant displacement bandwidth. Generally, the frequency ranges of the resonant responses determined by these two methods are different but close, and the error of the time-history statistics of the resonant responses determined by them is small, so the extraction results of the two methods are reliable. These two methods are used to extract the resonant component of the tower top displacement in Fig. 2 and the results are shown in Table 1.

Table 1
Resonant component parameters of the tower top wind-induced displacement

Calculation method category	Transmission tower fundamental frequency (Hz)	Resonant response frequency range (Hz)	Resonant response root-mean-square (mm)	Relative error (%)
The first method	1.522	1.417~1.628	3.57	-2.46
The second method		1.379~1.601	3.66	

Since the resonant response frequency range determined by these two methods can effectively contain the main resonant energy components near the fundamental frequency of the transmission tower, the results calculated by these two methods are representative. The first method is used to determine the frequency range of the tower top resonant displacement component, and the time history value of the component is obtained by filtering the original signal. The calculation results are shown in Fig. 3. The displacement response time history data in Fig. 3(d) is statistically analyzed, and its root-mean-square (RMS) value is calculated to be 3.57 mm. Using the Eq. (8) proposed in this paper, the displacement RMS value of the example is calculated to be 3.70 mm. The relative error of the displacement RMS value calculated from the time history data compared to it obtained from the method proposed in this paper is 3.51%. The calculation results of the two methods are in good agreement. The Eq. (11) proposed in this paper is further used to calculate ζ_e=0.052, which is significantly larger than the structural damping ratio of 0.01.

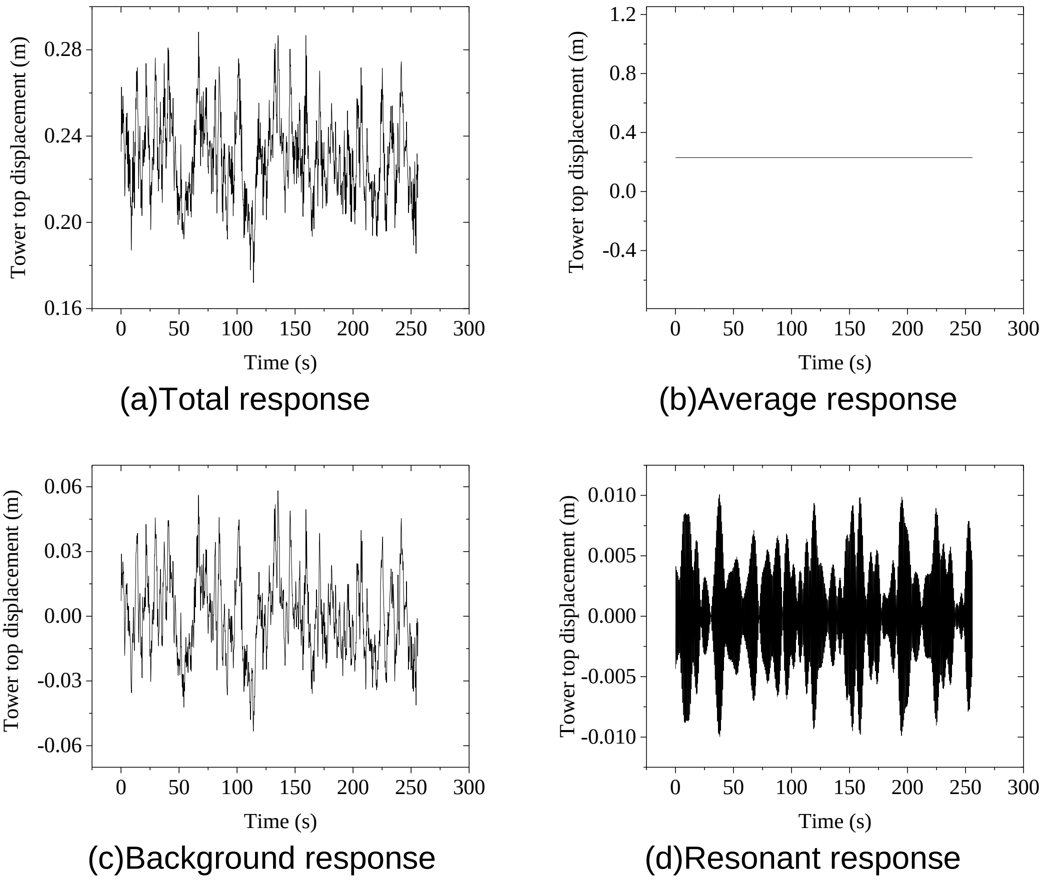

Fig. 3 Time history value of tower top displacement and its component

5. Conclusions

- Through establishing a simplified calculation model of the tower-line system, giving the conductor mode shape excited by tower vibration, and treating the conductors and suspension insulator strings as a series cable structure, based on stochastic vibration theory and modal analysis method, the root-mean-square (RMS) expression of the wind-induced resonant response of the transmission tower considering the influence of tower-line coupling effect is derived. The expression of equivalent damping ratio (EDR) of the transmission tower considering the TCE is derived, and a practical calculation expression is obtained by ignoring the high-order small quantities of the EDR. The study of the influence of conductors on wind-induced vibration of towers can be transformed into that of the EDR. A transmission tower after adding conductors is equivalent to the same tower with a larger damping ratio.
- An example is calculated by the time domain method and the theoretical expression proposed in this paper. The results show that the wind-induced resonant displacements of the transmission tower with conductors determined by the two methods agree well, which verifies the correctness of the proposed calculation expression considering the influence of the TCE. Due to the influence of adding

conductors, compared with the case of a transmission tower without conductors, the resonant response component of the same tower with conductors is significantly reduced, but its total damping ratio is significantly increased.

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