individual fitness. Based upon Roulette Principle, the individuals with better fitness are retained for the next iteration calculation. The population is updated and iterated through selection, crossover and mutation, the iteration process is shown in Fig. 3.

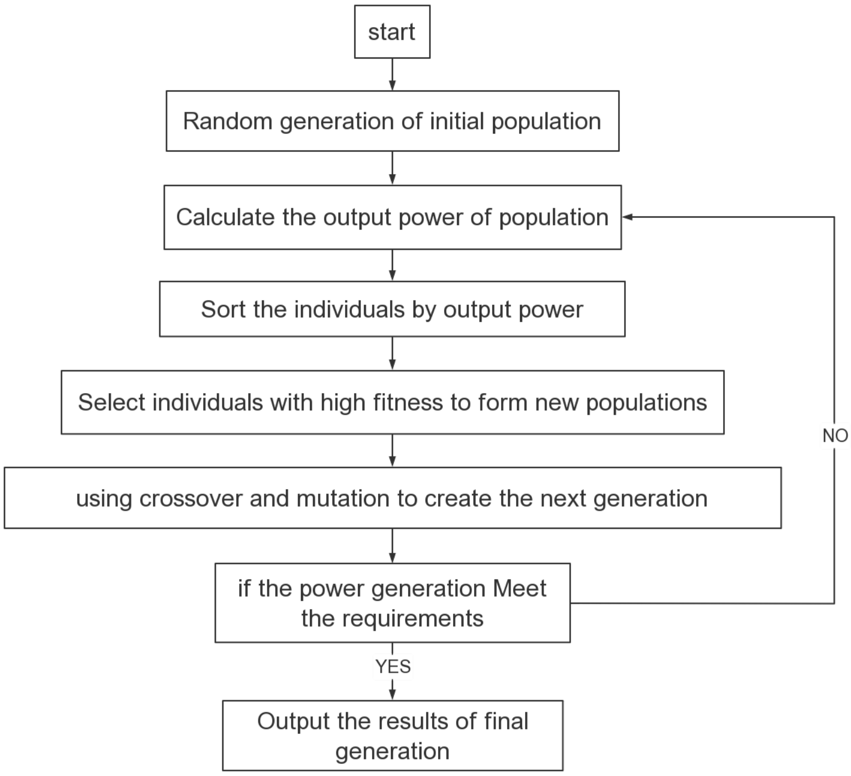


Fig. 3 Schematic diagram of genetic algorithm

3.2 Real-coded

Among genetic algorithms, binary-coded is the most widely used coding method, where each chromosome is a binary string consisting of 0 and 1. For this optimization problem, the variable is too lengthy, the computational domain is too large, which increases the workload. Therefore, the present study adopts real-coded to represent the optimization variables, which overcomes the deficiency of binary coding and improves the efficiency and accuracy of the algorithm. In this study, the optimization variables are the Cartesian coordinates of the wind turbines in the wind farm.

In the present study, each individual to be optimized is represented by a string composed of real values, as shown in Fig. 4, where (x_i, y_i) is the real-coded GA of *i*th turbine.

x_1	$\cdots$	x_i	$\cdots$	x_N	y_1	$\cdots$	y_i	$\cdots$	y_N

Fig. 4 Real-coded method

3.3 Objective function

During optimization, boundary conditions and constraints between wind turbines should be taken into consideration, as shown in Fig. 5:

$$\begin{cases} D_{ij} \geq D_s, i, j = 1,2, \dots, N, i \neq j \\ x_i \in [0,2000] \\ y_i \in [0,2000] \\ \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \geq 1.9D \\ x_i = x_j, ith, jth\ turbines\ in\ the\ same\ column \\ y_i = y_j, ith, jth\ turbines\ in\ the\ same\ row \end{cases} \tag{8}$$

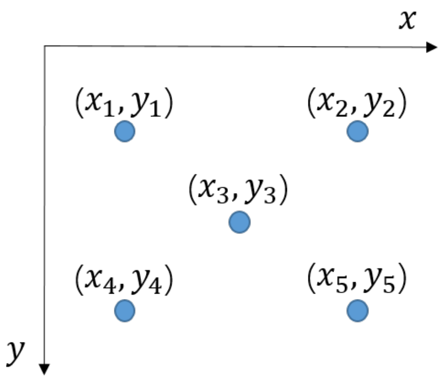


Fig. 5 Wind farm layout

where D_{ij} is the distance between *i*th and *j*th turbines; D_s is the minimum safe distance between wind turbines; x_i is the abscissa of *i*th turbine, and y_i is the ordinate of *i*th turbine.

The objective function of optimization is the total output power of the wind farm, which can be roughly simplified as (Grady 2005):

$$P = \sum_{i=1}^{N} 0.3u_i^3 \tag{9}$$

Where N represents the total number of wind turbines in the wind farm; u_i is the wind speed of the *i*th turbine; *P* is the total power output of the wind farm. The proportion of wake attenuation loss in the wind farm can be expressed by the efficiency of the wind farm as follows (18 Shor-cut design of wind farms) ：

$$\eta = \frac{P}{P_0} \tag{10}$$

where P_0 is the ideal total output power of the wind farm without the influence of tail flow.

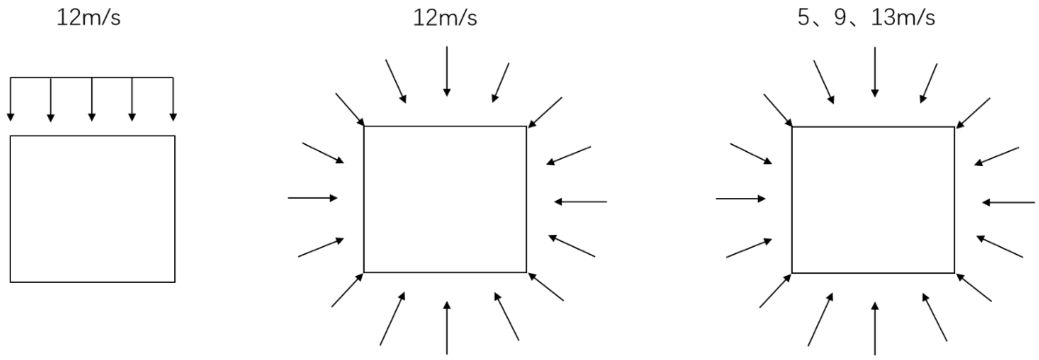
4. CASE STUDY

For a 2km×2km classic wind farm, on the basis of Grady's optimized layout, the present study takes optimal power output into consideration to further optimize the layout, and then compares the optimization result with that based on Jensen Wake Model. Table 1 lists information of the wind turbines and wind field used in this study.

Table 1 Wind turbine parameters

Hub height H	Diameter D	Surface roughness Z_0	Thrust coefficient C_T
60m	40m	0.3	0.88

Case a: constant wind speed and wind direction (12m/s);
Case b: constant wind speed (12m/s) and variable wind direction (36 directions);
Case c: variable wind speed and wind direction (36 directions);
Schematic diagrams of Case a, b and c are shown in Fig. 6.



a. Case a b. Case b c. Case c

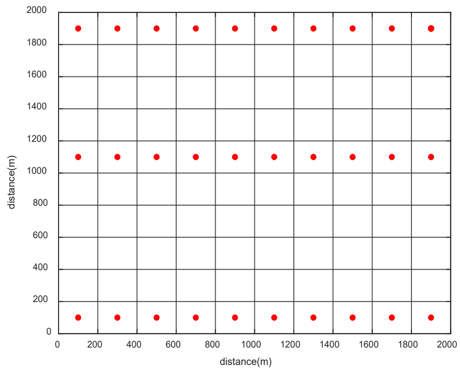
Fig. 6 Wind condition

4.1 Case a: constant wind speed and wind direction

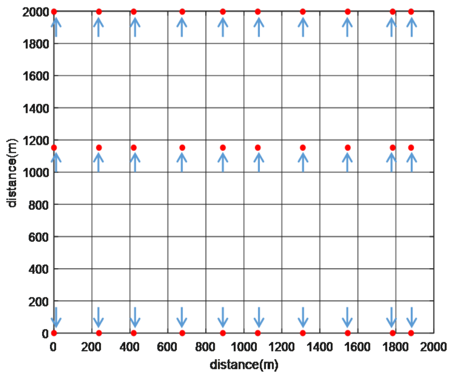
The wind farm includes 30 wind turbines, and the location of the wind turbines is further optimized. See Fig. 7 for the optimized layout and Table 2 for the optimized results. Fig. 7 shows that, in the optimized layout, the wind turbines around the wind farm are offset in the forward and backward directions in rows, and in the left and right directions in columns. The position of the wind turbines in the middle is slightly offset to the downwind.

Table 2 Case a optimization results

	Three dimensional Gaussian wake model		Jensen wake model	
	P(kW)	η(%)	P(kW)	η(%)
Grady's layout	14646	94.17	14297	91.9
present layout	14781	95.04	14497	93.3
Improvement	0.93%		1.4%	



a. Grady's layout



b. Present layout

Fig. 7 Wind farm layout (Cace a)

4.2 Case b: constant wind speed and variable wind direction

For complex wind conditions with multiple wind directions, Cartesian coordinate transformation is required. The wind direction angle in case a is set to 0°, and when the wind direction angle between the incoming wind and the wind turbine is θ, the coordinates of *i*th turbine relative to the wind direction angle θ are calculated by Eq. (11) (Parada 2017)：

$$(X_i, Y_i) = (x_i, y_i) \cdot \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} \quad (11)$$

where: (x_i, y_i) is the coordinates of the original *i*th turbine.

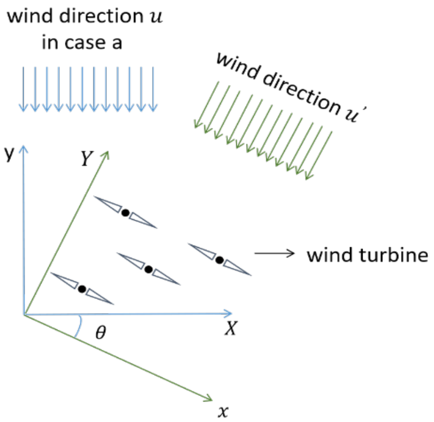
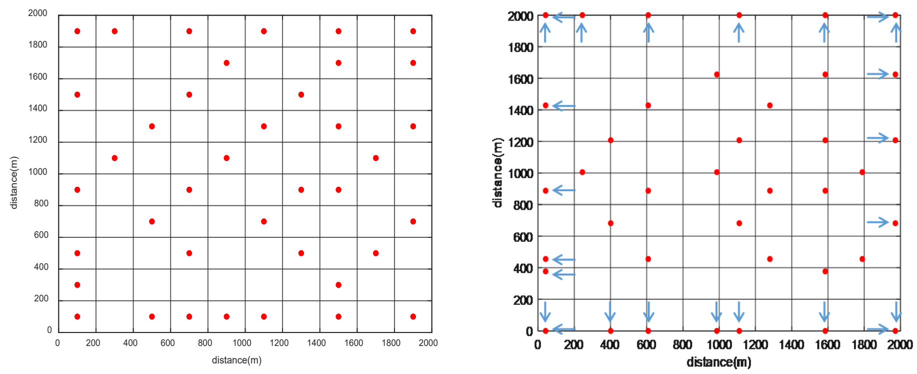


Fig. 8 Schematic diagram of coordinate conversion

On this basis, the follow-up optimization work is carried out. The wind farm includes 39 wind turbines, and the optimized layout and results are shown in Fig. 9 and Table 3 respectively. As can be seen from Fig. 9, in the optimized layout, the wind turbines around the wind farm are scattered to the boundary of the wind farm, which increases the distance between the peripheral wind turbines and the internal ones, and the turbines are offset to the left and right directions of the X axis in columns, thus reducing the wake effect between the wind turbines.



a. Grady's layout b. Present layout

Fig. 9 Wind farm layout (Case b)

Table 3 Case b optimization results

	Three dimensional Gaussian wake model		Jensen wake model	
	P(kW)	η(%)	P(kW)	η(%)
Grady's layout	18551	91.76	16949	83.83
present layout	18723	92.61	17415	86.14
Improvement	0.92%		2.7%	

4.3 Case c: variable wind speed and wind direction

In classic wind farms, the wind speed in Case c was relatively high, reaching 14m/s, which was quite different from the actual wind conditions. Therefore, the probability distribution of wind speed and direction was improved in the present study. The improved wind speed distribution is more consistent with the wind conditions of the actual wind field, and the average wind speed is about 7.5m/s. The improved probability distribution of wind speed and direction is shown in Fig. 10. The layout of the classic wind farm is optimized in Case c. See Fig. 11 for the optimized layout and Table 4 for the results. From Fig. 11, it can be seen that the trend of peripheral wind turbines shifting around the wind farm is the same as that of Case b, and more wind turbines are arranged in the wind direction with high probability of wind speed. The results in Table 4 show that the optimization based on Gaussian Wake Model increases the power output by 1.05%, and that based on Jensen Wake Model increases by 2.19%.

From the comparison of the above optimization results, it can be seen that in three Case, the output power increased by 0.9%-1% based on the three-dimensional Gaussian Wake Model, and the output power increased by 1.4% in Case a, and increased by more than 2% in Case b and c based on Jensen Wake Model. From the increase range of output power, the optimization based on Jensen Wake Model is higher than that based on three-dimensional Ishihara Gaussian Wake Model. However, since Jensen Wake Model overestimates wake effect and velocity-deficit, the optimization space seems to be larger, which leads to false improvement of higher power output. The three-dimensional Gaussian Wake Model is more accurate in predicting the velocity-deficit, and the calculated output power is better than that calculated by Jensen model

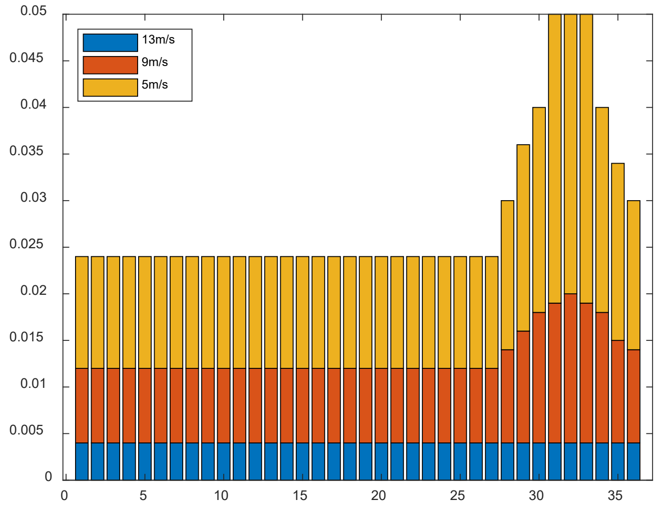
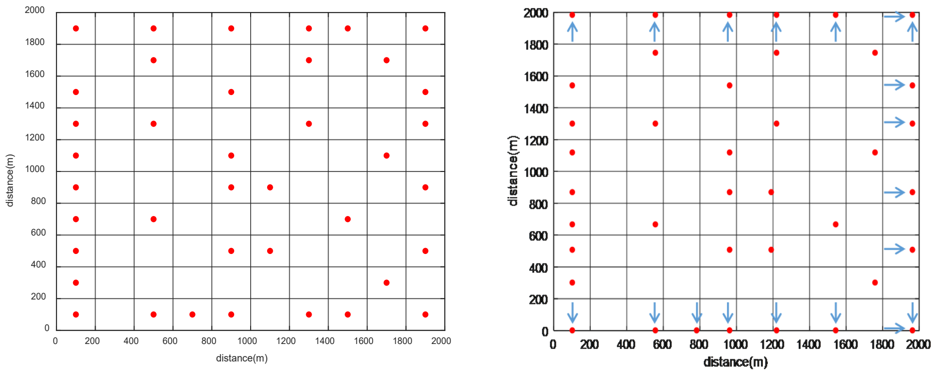


Fig. 10 Probability distribution of wind direction and speed



a. Grady's layout b. Present layout

Fig. 11 Wind farm layout (Case c)

Table 4 Case c optimization results

	Three dimensional Gaussian wake model		Jensen wake model	
	P(kW)	η(%)	P(kW)	η(%)
Grady's layout	6789	92.77	6024	82.31
present layout	6860	93.74	6156	84.12
Improvement	1.05%		2.19%	

5. CONCLUSION

Based on the "Three-Dimensional Ishihara Gaussian Wake Model", the present study proposes a method to optimize the layout of wind farms by adjusting the coordinates of wind turbines in a row or column at the same time, and increases output power. With the real-coded genetic algorithm as the optimization objective, the optimized layout given by Grady for a 2km×2km classic wind farm under three different wind conditions is re-optimized, which solves the problems of scattered layout in current wind farm layout optimization schemes, which is inconsistent with the neat arrangement requirements of the actual wind farm installation, and improves output power of the wind farm at the same time. This study finds that the increase range of wind output power based upon Jensen Wake Model optimization is obviously higher than that based on Gaussian Wake Model optimization. However, because of its serious error and low accuracy in wind speed estimation, the overall effect of Gaussian Wake Model optimization scheme is better.

The innovation of this study is adjusting the position of one row or column of wind turbines in the wind farm at the same time, optimizing the layout, making it meet the requirements of regular layout, and improving output power. Under three working conditions, the optimized output power increased by about 1%. The effectiveness of the wind turbine layout optimization method proposed in the present study is verified, which provides a new feasible method for wind farm installation layout.

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