

Simplified representation of 3D angular position with least singularity for an anatomical movement

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ABSTRACT

In this study, a novel method is proposed to represent the three dimensional angular position of the body segment in space with least singularity adopting the new definition of the axial rotation. Firstly, the orientation of the axis of the body segment is defined by the geographical coordinate system (ISO 31-11). Secondly, the axial rotation of the body segment is defined by the rotation offset from the geodesic movement of the body segment from the zero position. To minimize the possibility of the singularity, the posture in the womb is preferred as the zero position of the human body in contrast to the clinical definition of the standing posture. In addition, the North Pole is defined along the z-axis to match to the clinical definition. However, it would be better if the Pole be taken along the y-axis to avoid the singularity at the Pole.

In this way of defining the three dimensional position of the human body, we can reduce singularities in an anatomical movement. Also, the angular position becomes vector additive as the representation of the position is path independent by the introduction of the new axial rotation definition.

1. INTRODUCTION

There are many conventions to define the three dimensional rotational position in space. For example, Euler angles are the most popular method to define the rotation in space (Chao, 1980). However, the angles are rotational sequence dependent and non-additive along with singularity problems. To avoid singularity problems, Euler parameters or quaternions are introduced with fourth parameter even though three parameters are necessary and sufficient (Haug, 1992). Another method is to use the projection angles to the rectangular coordinate planes along with axial rotation (Yoon, 2013).

There are numerous articles published about the three dimensional rotation of the limb segments. Chao (1980) proposed a tri-axial goniometer to measure the joint angles by matching yaw, pitch, and roll angles to flexion-extension, abduction-adduction, and axial rotation angles respectively and the joint coordinate system by Grood and Suntay (1983) agrees well to this system. However, this definition of three

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dimensional rotation produces the pseudo-axial rotation angle resulting in slightly different value from the one based on the integration of the roll angular velocity (Ishida 1990) and could not explain Codman's paradox (Codman 1934). Cheng *et al.* (2000) proposed a new definition of axial rotation while Masuda *et al.* (2008) proposed a remedy to the pseudo-rotation adjusting with the azimuth angle influence. Yoon (2013) recently proposed a new definition of the axial rotation with clear explanation of Codman's paradox.

If we could agree with the new definition of the axial rotation, we may use the projection angles or geographical coordinate to define the other two parameters describing the three dimensional rotation of a rigid body in space. If we use the projection angles, we have to choose which two of the three orthogonal planes to use for the projections depending on the orientation of the body segment.

This study is proposing a new definition of the three dimensional rotation using the geographical coordinate system to define the orientation of the axis along with the definition of axial rotation that we may maintain the vector additive characteristics as well as simplicity as the coordinate is well defined in ISO 31-11.

2. Angular Position

A three dimensional angular position may be represented either by the quaternion, screw-axis rotation or Euler angles as shown in Fig. 1. In more intuitive way, we can use the axial rotation combined with its axis orientation which can be specified either by the projection angles or the position on the unit spherical surface denoted by the longitude and latitude. The last method is named as the geographical method and proposed in this study: the longitude and latitude plus the axial rotation. One advantage of using the geographical method is the definitions of the longitude and latitude are well established by ISO 31-11 along with the axial rotation recently defined as the deviation from the geodesic basic movement (Yoon, 2013). Another advantage of the proposed method is that the angles defined in this way are vector additive. For example, consider a case in which there is a consecutive rotational motion from the starting point to the position 1 then to the position 2. The first and the second motions can be represented as

$$M_{01} = (\alpha_1, \beta_1, \delta_1) \quad (1)$$

$$M_{12} = (\alpha_2, \beta_2, \delta_2) \quad (2)$$

where α_i is the longitude, β_i is the latitude, and δ_i is the axial rotation. Then, the combined motion from the starting position to the position 2 can be represented as

$$M_{02} = (\alpha_1 + \alpha_2, \beta_1 + \beta_2, \delta_1 + \delta_2) \quad (3)$$

With regards to the axial rotation, the y-axis rotation of the gimbal mechanism shown in Fig. 1 was commonly used. However, Ishida (1990) noted that the gimbal axis rotation is a pseudo-axial rotation and Masuda *et al.* (2008) proposed a method to

obtain the true axial rotation by adding the contribution by the longitudinal rotation to the pseudo-axial rotation. In more simple way, Cheng *et al.* (2000) proposed a method of determining the axial rotation. Recently, Yoon (2013) re-interpreted this method by defining the axial rotation as the angular difference between the actual position and reference position defined by the basic movement. The basic movement here is obtained by the geodesic path from the starting point to the target point with no axial rotation on the unit spherical surface.

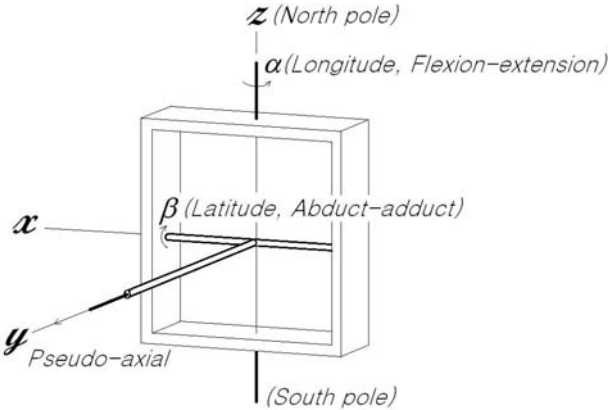


Fig. 1 Conventional definition of joint rotation at hip

3. Coordinate System

Fig. 2 shows an example of the coordinate system recommended by Wu *et al.* (2002) of the international society of biomechanics showing the y-axis along the long axis of the femur and x-axis perpendicular to the femoral plane defined on the femur.

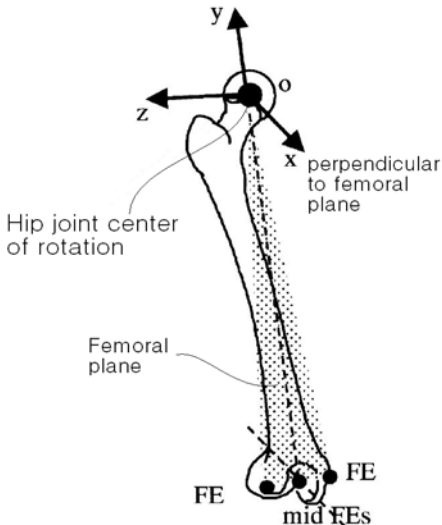


Fig. 2 Coordinate system recommended by ISB at hip

They also recommended us for the initial body orientation to use the anatomical standing posture as shown in Fig. 3. However, the gimbal mechanism commonly used as shown in Fig. 1 has gimbal lock at the North and South poles. Thus, this coordinate system may have singularity if the femur has 90 degrees abduction which may occur in some activities.

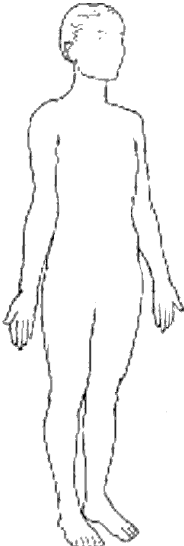


Fig. 3 Anatomical standing posture

To avoid those singularities, in the geographical method, the y-axis is taken along those poles as shown in Fig. 4.

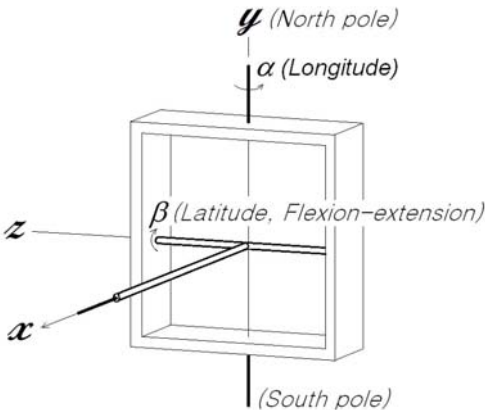


Fig. 4 Modified definition of joint rotation using geographical coordinate at hip

We could select for the y-axis to pass through the center of the joint cartilage contact surface, which can be achieved naturally by selecting the fetus posture as our new reference position as shown in Fig. 5. In that case, we may have singularity at the initial posture. But we may define the zero position at this fetus posture that we may avoid

singularity even at this reference posture. In this geographical coordinate system, the axial rotation can represent the abduction/adduction.



Fig. 5 Fetus position at womb

The angular velocities for this geographical coordinate system in Fig. 4 can be obtained as

$$\omega_x = \dot{\beta} \sin \alpha + \dot{\delta} \cos \beta \cos \alpha \quad (4)$$

$$\omega_y = \dot{\alpha} + \dot{\delta} \sin \beta \quad (5)$$

$$\omega_z = \dot{\beta} \cos \alpha - \dot{\delta} \cos \beta \sin \alpha \quad (6)$$

where δ is the axial rotation different from the pseudo-rotation.

4. DISCUSSIONS

Since Euler angles were introduced 200 years ago to describe the three dimensional rotation of a rigid body in space, we had to suffer from the peculiar characteristics of Euler parameters: no vector additiveness and sequence dependency as well as gimbal lock. Anyhow, using this Euler angles, Chao (1980) proposed a triaxial goniometer to measure the joint angles, which is later employed to define a joint coordinate system by Grood and Suntay (1983). However, the pseudo-axial rotation from this system can not explain Codman's paradox (1934).

Yoon (2013) suggested to use the projection angles in place for Euler angles. However, there are three orthogonal planes and we have to determine which two of the three planes to use depending on the orientation of the body segment while the geographical coordinate is well defined by ISO 31-11.

Another problem in the biomechanics application is that we used to take the standing posture of the human body as the initial or zero position as suggested by Wu *et al.* (2002) of the international society of biomechanics. In that situation, we may expect

singularities if the hip joint is abducted 90 degrees with the gimbal mechanism shown in Fig. 1. To avoid those singularities, we better orient the poles to pass through the center of the cartilage contact surface of articular joint and take the initial orientation at the South Pole, which is quite close to the fetus posture in the womb as shown in Fig. 5.

5. CONCLUSIONS

In this study, new method of describing the three dimensional rotation in space is proposed with several advantages over the conventional Euler angles: The angular parameters are vector additive and have least singularity when we take the polar axis of the coordinate to pass through the center of the cartilage contact surface for a limb. Also, the geographical coordinate system is well defined by ISO 31-11 that we don't have to worry about the rotational sequence as in the case of using Euler angles.

This method can be applied even to the conventional kinematics and dynamics of the multi-body mechanical systems.

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