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Online Trajectory Replanning for Biped Robots Based on Foot Step Position Modification

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ABSTRACT

Trajectory generation of many successful biped robots is based on the zero moment point (ZMP) criterion. Combining this criterion with a simplified model of the robot dynamics allows for an elegant formulation of the equations of motion of the robot. As solving these equations as an initial value problem does generally not lead to trajectories that allow for stable walking, the desired robot state at the end of the planning horizon has to be considered as well. The boundary value problem that arises is ill-posed and can only be solved in the general case if additional degrees of freedom are introduced. In this paper we give an overview of the real-time trajectory generation system of the humanoid robot Lola that uses the ZMP criterion and introduces additional degrees of freedom in the ZMP trajectory. We propose a modification of the foot step positions to reduce the amount of ZMP trajectory modification necessary.

1. INTRODUCTION

One of the major goals of research on humanoid robots is to create robotic systems that are capable of assisting humans in a wide variety of applications. Biped robots that exhibit a mechanical structure mimicking the human physique can utilize tools designed for humans and complete tasks in human work environments if they are capable of flexible and robust interaction with the environment. We have recently presented an event-based control system for humanoid robot walking which significantly increases the robustness of the biped walking system (Buschmann *et al*. 2012). A summary of that control framework is given in Section 2. In the control system of our humanoid robot Lola (Fig. 1), we use an online zero moment point (ZMP)-based planning method proposed by Buschmann *et al*. (2007). The basic principle of the method is presented in Section 3. One of the major challenges of planning trajectories for biped walking motion is the fact that the unilateral contacts between the feet and the ground severely limits the space of suitable trajectories. We present the method that is used in the control system of the humanoid robot Lola to obtain a solution that fulfills these constraints. In Section 4 we propose a new method of incorporating foot step modifications in the control system. Preliminary results obtained from a multibody simulation of the

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humanoid robot Lola suggest the effectiveness of our approach. In Section 5 we conclude the paper and give an outlook over future work.

2. EVENT-BASED CONTROL OF THE HUMANOID ROBOT LOLA

The humanoid robot Lola (Fig. 1) is approximately 180 cm tall, weighs about 60 kg and has 25 joints that are actuated with brushless AC motors. The control system is onboard and except for external power supply, Lola is an autonomous robotic walking system. For an overview of the mechanical design of the robot, refer to Lohmeier *et al*. (2010). Details about the sensor-controller network are presented by Favot *et al*. (2012). A comprehensive description of the robot's control system and the multibody simulation of Lola is given by Buschmann (2010).

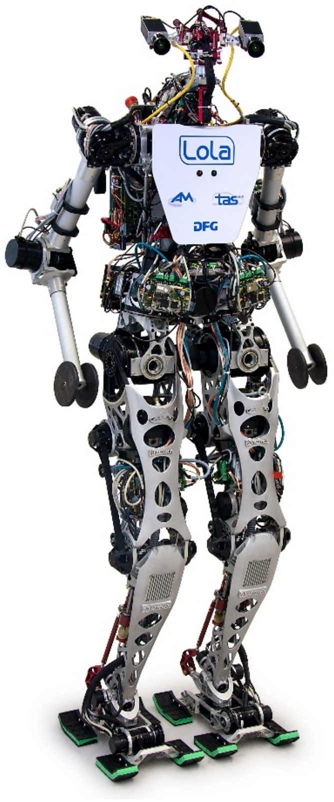

Fig. 1 Humanoid robot Lola

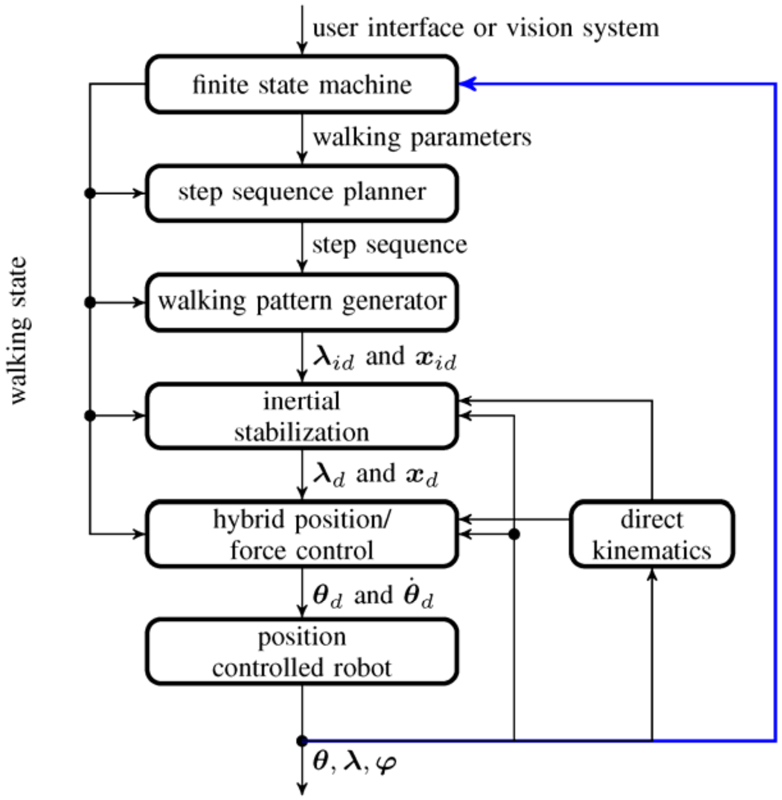

Fig. 2 Walking control system structure[1]

Fig. 2 shows the real-time walking control system that is coordinated by a finite state machine (FSM). The highest layer of states of the FSM and possible transitions between these states is illustrated in Fig. 3. Depending on the current state, a step sequence is planned based on the walking parameters provided by a human operator

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[1] Buschmann *et al*. (2012)

or an autonomous navigation system (cf. Buschmann *et al*. 2010). This sequence serves as the basis for designing suitable ideal walking trajectories $\boldsymbol{x}_{id}$ in task space and contact force trajectories $\boldsymbol{\lambda}_{id}$. These ideal contact forces are modified depending on sensor information obtained from the Inertial Measurement Unit in the robot's upper body and the information obtained from the force/torque sensors in the feet (cf. Lohmeier *et al*. 2009). The resulting desired contact forces $\boldsymbol{\lambda}_{d}$ and remaining task space dimensions are controlled using a hybrid force/position control (see Buschmann *et al*. 2009 for details).

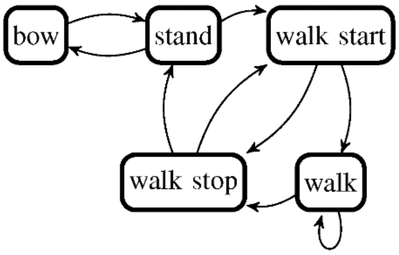

Fig. 3 Top states of the FSM[1]

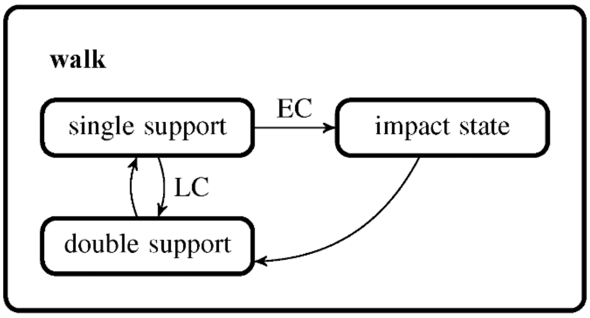

Fig. 4 Sub states of the walk state[1]

To increase the robustness of the walking system, incorporating sensor feedback not only in the three lowest control instances shown in Fig. 2 but also for trajectory planning is crucial. The blue arrow in Fig. 2 illustrates the feedback of sensor signals to the finite state machine which then initiates phase transitions if necessary. In case the robot is in the walk state (see Figs. 3 and 4), ground contact information and measurements of the inclination and angular velocity of the upper body can be utilized to determine necessary transitions between the sub states of the walking state (see Fig. 4). The walk state is subdivided into subordinate states, each representing a characteristic phase in the gait cycle where the current phase is determined from sensor information. The moment when the swing leg leaves the ground marks the transition from the double support to the single support state. We developed a stability criterion to determine the appropriate time for initiating the swing phase in Buschmann *et al*. (2012). As soon as the transition to another phase is initiated, the previously planned trajectories may no longer be suitable and trajectory replanning may be necessary.

3. REAL-TIME TRAJECTORY GENERATION

In this section we give an overview over the trajectory planning system of our humanoid robot Lola. Fig. 5 shows the consecutive steps for planning the foot and center of gravity (CoG) trajectories. The foot trajectories are calculated based on the provided walking parameters under consideration of constraints imposed from the environment. ZMP and CoG trajectories are calculated subsequently. The simplified

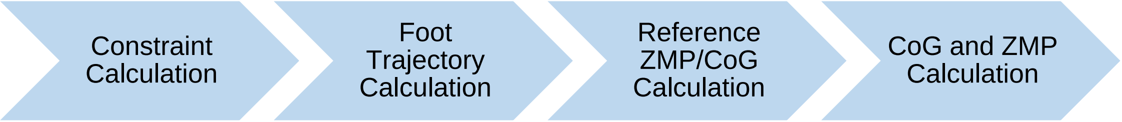

Fig. 5 Real-time trajectory generation system[2]

model we use to calculate these trajectories is derived in the following.

The principle of linear momentum of the robot with total mass m and CoG S is

$$m\ddot{\boldsymbol{r}}_S = \boldsymbol{F} + m\boldsymbol{g} \tag{1}$$

where $\boldsymbol{g}$ is the vector of gravitational acceleration and $\boldsymbol{F}$ the total ground reaction force acting at the robot. The principle of angular momentum with respect to S is

$$\dot{\boldsymbol{H}}^S = \boldsymbol{T} - \boldsymbol{r}_S \times \boldsymbol{F} \tag{2}$$

where $\boldsymbol{T}$ is the ground reaction moment acting at the robot with respect to the origin of position vector $\boldsymbol{r}_S$. By assuming constant angular momentum, i.e., $\dot{\boldsymbol{H}}^S = 0$ the well-known equations of motion

$$mz\ddot{x} - mx(\ddot{z} + g) = T_y \tag{3}$$

$$mz\ddot{y} - my(\ddot{z} + g) = -T_x \tag{4}$$

can be derived from Eqs. (1) and (2). Additionally assuming $\ddot{z} = 0$ yields the equations of the Linear Inverted Pendulum Mode (LIPM) proposed by Kajita *et al*. (2001). In this formulation, or if the CoG trajectory in the vertical direction is known, Eqs. (3) and (4) are decoupled. In the following, we only consider Eq. (3) which describes the motion of the robot in the sagittal direction.

In order to account for the dynamics of the legs, we use a simplified model of three point masses for the robot. Variables concerning the upper body are marked by the index b, those concerning the legs by the lower right index l. The upper right index of the legs makes a distinction between the left (l) and the right (r) leg. Rewriting Eq. (3) for the formulation of the three point mass model yields

$$m_b z_b \ddot{x}_b - m_b x_b(\ddot{z}_b + g) = T_y + m_l x_l^l\left(\ddot{z}_l^l + g\right) - m_l z_l^l \ddot{x}_l^l + m_l x_l^r(\ddot{z}_l^r + g) - m_l z_l^r \ddot{x}_l^r. \tag{5}$$

Defining

$$\tilde{T}_y \coloneqq T_y + m_l x_l^l\left(\ddot{z}_l^l + g\right) - m_l z_l^l \ddot{x}_l^l + m_l x_l^r(\ddot{z}_l^r + g) - m_l z_l^r \ddot{x}_l^r \tag{6}$$

and dividing (5) by $m_b z_b$ yields

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[2] Modified from Buschmann *et al*. (2007)

$$\ddot{x}_b - x_b \frac{\ddot{z}_b + g}{z_b} = \frac{\tilde{T}_y}{m_b z_b}. \tag{7}$$

The right hand side of Eq. (7) corresponds with the ZMP position x_Z. Therefore, Eq. (7) can also be written as

$$\ddot{x}_b - x_b \frac{\ddot{z}_b + g}{z_b} = \alpha\left(x_l^l, x_l^{\ddot{r}}, x_l^l, \ddot{x}_l^r, z_l^l, z_l^r, \ddot{z}_l^l\, \ddot{z}_l^r\right) x_Z. \tag{8}$$

In the control framework of the humanoid robot Lola, the planning horizon is two steps starting with each double support phase. In case of phase transitions (cf. Section 2), trajectories are replanned using the same framework. To avoid jumps in the desired joint trajectories, all planned trajectories must be C^2-continuous and connect to the trajectories of the previous planning horizon smoothly leading to the initial conditions

$$x_b(t=0) = x_{b,0} \quad \text{and} \quad \dot{x}_b(t=0) = \dot{x}_{b,0}\,. \tag{9}$$

Solving the initial value problem consisting of Eqs. (8) and (9) does generally not lead to walking trajectories that enable stable walking motion. Therefore, it is necessary to also prescribe the CoG state at the end of the planning interval, i.e., formulating the boundary conditions

$$x_b(t=T) = x_{b,T} \quad \text{and} \quad \dot{x}_b(t=T) = \dot{x}_{b,T}\,. \tag{10}$$

The CoG state at the end of the planning interval is obtained by determining a periodic solution for the walking motion based on the current walking parameters, i.e., solving the equations of motion with the conditions

$$x_b(t=T) - x_b(t=0) = \Delta x \quad \text{and} \quad \dot{x}_b(t=T) - \dot{x}_{b,T} = 0 \tag{11}$$

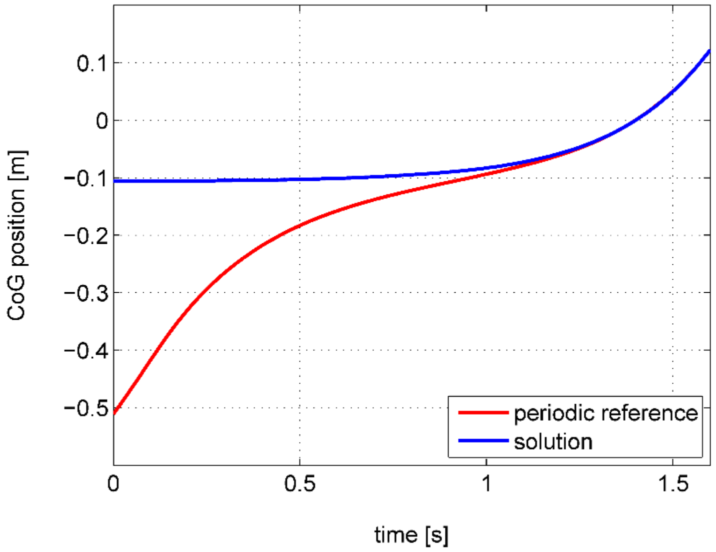

Fig. 6 CoG trajectory
The CoG trajectory converges to the periodic reference trajectory at the end of the planning horizon

where Δx is determined by the step lengths.[3] Fig. 6 shows the periodic reference trajectory obtained by solving Eq. (8) with the conditions from Eq. (11) and the solution for the CoG trajectory that starts with the prescribed initial values from Eq. (9) and converges to the reference trajectory at the end of the planning interval.

The boundary value problem that has to be solved to obtain the CoG trajectory is given by Eqs. (8)-(10). As this problem is ill-posed, two additional degrees of freedom have to be introduced to allow for a solution in the general case. In our approach, we introduce additional degrees of freedom by modifying the reference trajectory of ZMP and we rewrite (8) as

$$\ddot{x}_b - x_b \frac{\ddot{z}_b + g}{z_b} = \alpha(x_Z + \gamma_1 R_1 + \gamma_2 R_2) \tag{12}$$

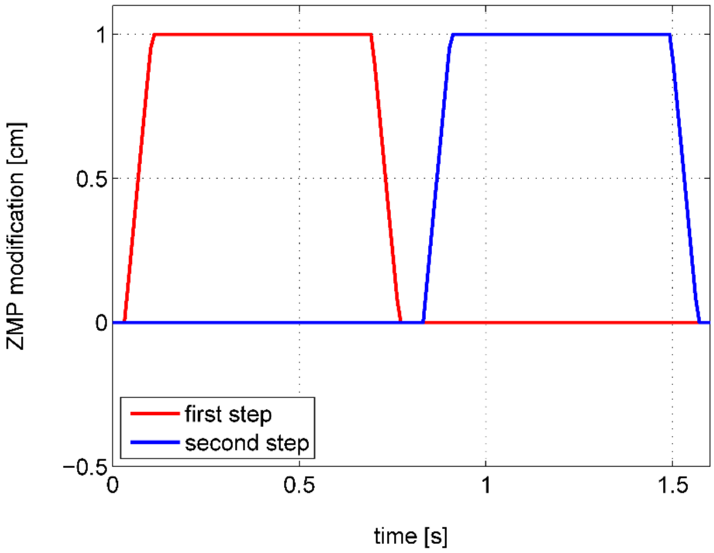

Fig. 7 Shape functions R_1 (red) and R_2 (blue) for ZMP trajectory modification

where R_1 and R_2 are shape functions and γ_1 and γ_2 are two additional free parameters. The shape functions are active during the first and the second step of the planning horizon respectively (see Fig. 7).

There are a variety of approaches to relax the given boundary value problem. As opposed to prescribing position and velocity at the end of the planning horizon, Takenaka *et al*. (2009) propose a coordinate transformation of the CoG state vector which leads to two components of CoG motion. As one of these components is inherently stable, in this approach only the divergent component is prescribed a boundary value. In the method proposed by Sugihara and Nakamura (2009), boundary conditions are relaxed by allowing deviations between the desired and actually reached states and by allowing non-continuous ZMP trajectories.

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[3] In our current implementation, the velocities of the point masses representing the legs are additionally taken into account for the velocity condition. For sake of simplicity, we omitted this detail here.

4. TRAJECTORY PLANNING WITH FOOTSTEP MODIFICATION

We propose a replanning method that utilizes a modification of the foot positions additionally to the ZMP trajectory modification. In order to improve the trajectory planning method presented in the previous section, let us consider the following case: The robot is stepping in place and receives the signal to start walking. In order to accomplish forward motion, the ZMP has to move in the opposite direction first. In case a sudden command of fast forward walking is given to the control system, the modified ZMP may not lie inside the admissible area, the support polygon. In that case, taking a little step back while stepping in place before starting to walk forward with high velocity may be the key to avoid violating these constraints.

We therefore propose to first calculate the CoG and ZMP trajectories using the method presented in the previous section. In the next step, the foot trajectories of both planned steps are modified by utilizing to the respective maximum ZMP modifications according to Eq. (12). After the foot trajectories have been modified, CoG and ZMP trajectories are calculated based on the updated foot trajectories. Fig. 8 shows the results obtained in simulation with this new method. Like in the scenario described above, the robot is stepping in place and receives the command to transition into quick walking motion.

As a result of the modification of the foot step position, the necessary modification of the ZMP trajectory is significantly reduced. Table 1 shows the difference in value for the parameters γ_1 and γ_2 according to Eq. (12).

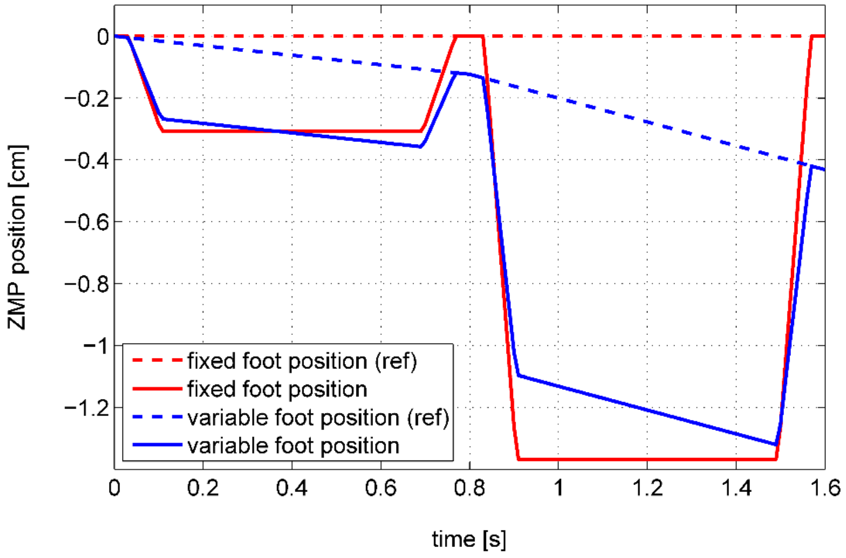

Fig. 8 Comparison of planned ZMP trajectories with or without foot step position modification

Table 1 Ramp parameters

	Fixed step position	Variable step position
γ_1	-0.31	-0.25
γ_2	-1.37	-0.93

As a response to the request of fast forward walking, the robot steps in place and modifies its foot step positions slightly backward. As a result, the ZMP trajectories are less likely to violate the constraints imposed by the unilateral ground contacts of the feet. This feature is especially useful when replanning during the single support phase as the support polygon is spanned by only one foot, further limiting the space of admissible trajectories.

5. CONCLUSIONS

We have proposed a method of modifying step positions to reduce the necessary modification of ZMP trajectories when solving the boundary value problem for CoG trajectories of biped robot walking. A series of investigations utilizing a multibody simulation of the humanoid robot Lola yielded promising results. An extension of the method to predict exactly the effect of foot step modifications on the ZMP modification is subject of our future work.

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