

## Rapid and precise control of a micro-manipulation stage combining $H^\infty$ with ILC algorithm

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### ABSTRACT

In order to achieve good performance of rapid and precise of a micro-manipulation , a feedforward-feedback controller combining iterative learning control (ILC) with  $H^\infty$  robust control is designed. Firstly, a 4th order model of the plant is obtained through system identification. Secondly, a  $H^\infty$  controller which demonstrates performance robustness of the plant is designed to reject random noise and disturbances, while the ILC algorithm is applied to reject model uncertainty and repeating disturbances. Simulation results shows the good performance of accuracy and convergence and the mapping relevance between the Q-filter bandwidth and reference signal frequency.

### 1. INTRODUCTION

Micro-manipulation is one of the key technologies in the area of manufacturing, material and bio-medical. The displacement of micro-manipulation robot is generally in the level of  $\mu\text{m}$  even nm, and the positioning accuracy or the resolution is required to achieve the level of sub-nm or nm with the response speed of several microseconds or the bandwidth of tens of Hz. For instance, it is important and essential for micro-manipulators (such as micro-grippers) used in assembly of parts and components of micro-electro-mechanic system (MEMS) to achieve the positioning accuracy of  $\mu\text{m}$  or sub- $\mu\text{m}$  level. Furthermore, requirements of rapid response and large working bandwidth in nanometer manufacturing technology are increasing. In the area of atomic force microscope (AFM)'s research and application, high precision and large bandwidth micro-manipulation platform is required to accomplish real-time scanning and imaging of the changing sample. Above all, the two main factors which are important in micro-manipulation control are the high bandwidth and precision.

To achieve high precision and rapid response in micro-manipulation, the whole control system needs to be strongly robust and stable as the existence of slight environment disturbance and system modeling error. There are mainly three kinds of control strategies, namely complete feedforward control (usually applied in open-loop), feedback control and feedforward-feedback control. (Micky 2010) studies complete open loop control of piezoelectric cantilevers by compensate errors resulted from

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hysteresis, creep and vibration in feedforward controller based on Prandtl-Ishlinskii model (PI-model) to avoid instabilities caused by feedback control. As a result, the error was reduced from 20% to less than 2.5%. (Barton 2008) proposes a cross-coupling iteration learning control (CCILC) algorithm which applies ILC algorithm into cross-coupling controller (CCC) by learn errors resulted from cross-coupling to improve signal trajectory tracking performance and to achieve precise motion control of micro-manipulation robots. A type of control strategy which combines feedforward control based on PI-model with feedback control based on internal model control (IMC) is proposed in (Micky 2011) to reject external disturbance and environmental noise and to improve system robustness. (Tan 2001) combines online adaptive learning control with PID algorithm in piezoelectric ceramics control to approximately reduce tracking error to 50% as before. Apart from this, other intelligent control algorithms, e.g.  $\mu$ -synthesis control (Micky 2006), neural network control and RobPosit algorithm based on SEM (Aude 2012), is been proposed. In summary, errors caused by external environment, system modeling and parameter uncertainties should be reduced through feedback control, such as  $H^\infty$  robust control, adaptive control and so on.

Therefore, the main motivation of this paper is to achieve high performances of precision with rapid convergence speed in micro-manipulation tasks and investigate the mapping relevance between the Q-filter bandwidth and reference signal. A feedforward-feedback control strategy combining iterative learning control (ILC) with  $H^\infty$  robust control is applied into a one-freedom micro-manipulation stage actuated by piezoelectric. This paper is organized as follows. In Section 2, we identify the plant model through step response method. Section 3 presents a  $H^\infty$  controller which demonstrates performance robustness of the stage is designed to reject noise and non-repeating disturbances. The ILC is designed to reject model uncertainty and repeating disturbances by integrating former error information into the controller for subsequent iterations in Section 4. Section 5 discusses the simulation results. Finally, this paper is ended by conclusions.

## 2. SYSTEM IDENTIFICATION

(Zadeh 1962) has defined system identification as a process of acquiring an approximate model of the system from a set of given model classes based on input and output data. The research object in this paper is a one-freedom micro-manipulation stage which is a SISO system with a voltage input signal and a displacement output signal. We chose step response method to perform system ID, and a 4<sup>th</sup> order discrete LTI model (given in Eq. (1)) is estimated.

$$G(z) = \frac{0.1085z^4 - 0.1293z^3 + 0.3616z^2 + 0.173z - 0.4264}{z^4 - 1.73z^3 + 1.834z^2 - 1.386z + 0.3686} \quad (1)$$

## 3. CONTROLLER DESIGN

The stage system is shown in Fig. 1, and its control strategy is depicted in Fig. 2.

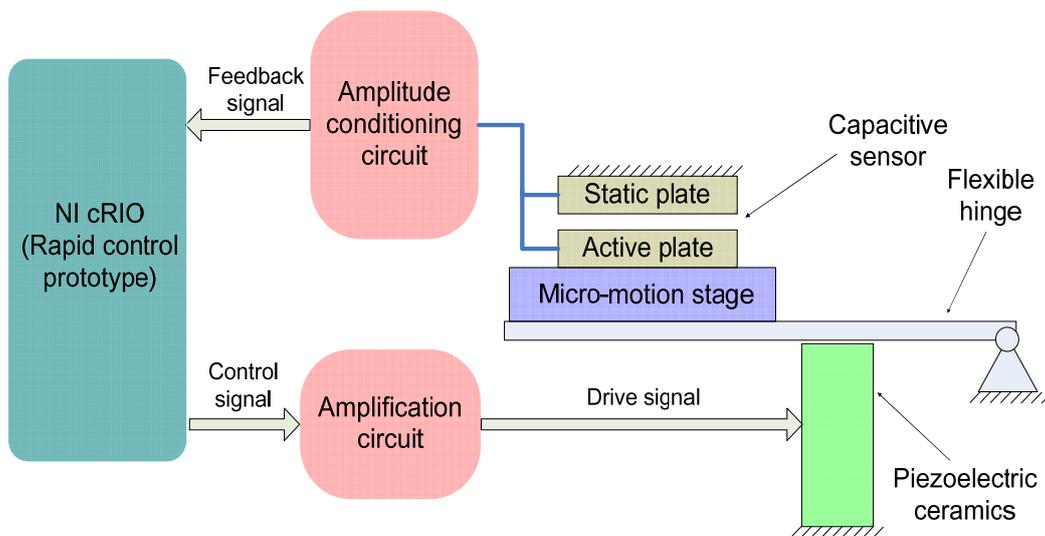


Fig. 1 Control system of the micro-manipulation stage

The control strategy consists of feedforward control (ILC) and feedback control ( $H^\infty$ ). Components of the system include reference signal  $r$ , output signal  $y_i$ , error  $e_j$ , control signal  $u_j$ , repeating external disturbance  $d_r$ , non-repeating external disturbance  $d_n$ , environmental noise  $n_j$ ,  $H^\infty$  controller  $K_d$ , plant transfer function  $G_d$  and ILC learning function  $L$ . The subscript ' $j$ ' represents each iteration and the whole process is discussed in discrete time.

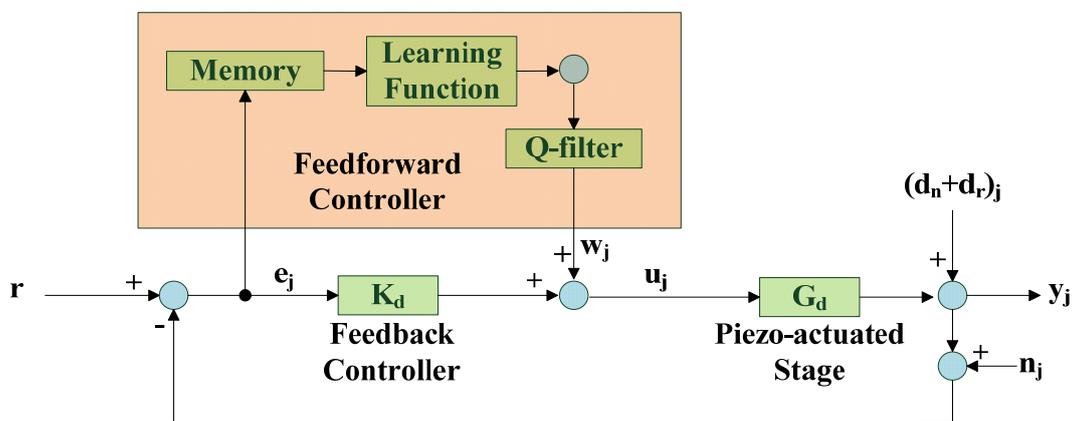


Fig. 2 Feedforward and feedback control system

### 3.1 $H^\infty$ robust control

The  $H^\infty$  controller is designed to reject non-repeating disturbance and environmental noise to track the reference signal more precisely and steadily. Compared with a classic PID controller,  $H^\infty$  has the advantage that the performance, resolution, and robustness to model uncertainty can be directly considered in the frequency domain via appropriate weighting functions (Helfrich 2008).

Define the close-loop transfer function  $CT(z)$  of the system as Eq. (2), feedback controller output-reference ratio function  $KC(z)$  as Eq. (3) in discrete time, and the sensitivity function  $S(z)$  as Eq. (4).

$$CT(z) = \frac{G(z)K(z)}{1+G(z)K(z)} \quad (2)$$

$$KC(z) = \frac{C_j(z)}{R(z)} \quad (3)$$

$$S(z) = \frac{1}{1+G(z)K(z)} \quad (4)$$

Then, the  $H^\infty$  controller transfer function  $K(z)$  can be obtain in minimizing a parameter index  $\gamma$  (given in Eq. (5)), where  $W_{CL}(z)$ ,  $W_{CT}(z)$ ,  $W_{KC}(z)$  are the weighting functions.

$$\gamma = \left\| \begin{array}{l} W_{CL}(z)S(z) \\ W_{CT}(z)CT(z) \\ W_{KC}(z)KC(z) \end{array} \right\|_\infty \quad (5)$$

The weighting function  $W_{CL}(z)$  is designed to have high gains at low frequencies and low gains at high frequencies to guarantee good tracking performance of reference signal. The weighting function  $W_{CT}(z)$  is designed to have low gains at low frequencies and high gains at high frequencies for the rapid attenuation of noise in high frequency. The weighting function  $W_{KC}(z)$  is designed to ensure that the control signals remain within limits instead of unstable. Finally, the  $H^\infty$  controller designed in this paper is described in Eq. (6). Bode plot of the controller is shown in Fig.3 in red, with the bode plot of the plant shown in blue. The  $H^\infty$  controller presents a good performance in modifying the plant bode plot to achieve a more flat and smooth curve in low frequency and a sloping curve in high frequency to guarantee a fast convergence speed.

$$Kh \inf(z) = \frac{0.1678z^{10} - 0.297z^9 + 0.02244z^8 + 0.2949z^7 - 0.3875z^6 + 0.2132z^5 + 0.1246z^4 - 0.2039z^3 + 0.07264z^2 - 0.00694z + 0.0001978}{z^{10} - 2.469z^9 + 0.5842z^8 + 2.753z^7 - 1.978z^6 - 0.4194z^5 + 0.7023z^4 - 0.1709z^3 - 0.0982z^2 + 0.1422z - 0.04606} \quad (6)$$

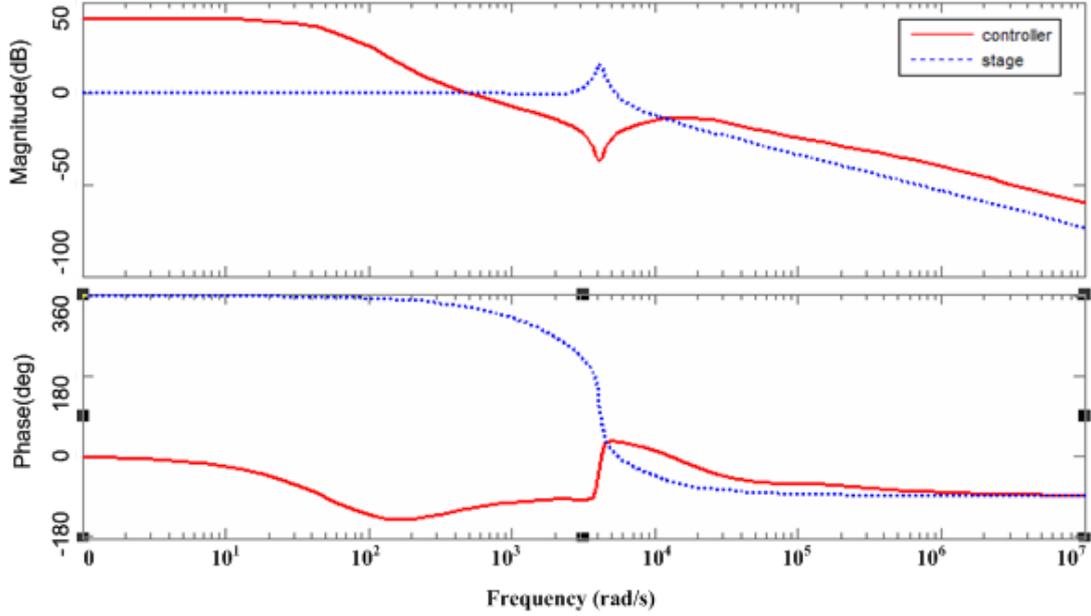


Fig. 3 Bode plots of  $H^\infty$  controller and the plant

### 3.2 ILC control

ILC is a kind of algorithm that learning former error and integrate the information into the controller for subsequent iterations, which is advantageous on rejecting model uncertainty and repeating external disturbance. It is firstly proposed in (Uchiyama 1978) and widely discussed in (Craig 1984, Kawamura 1984, Bristow 2006) and so on.

The plant is a LTI and SISO system as mentioned before, so the output  $y_j(z)$  can be obtained as Eq. (7), where  $z$  is the variable in discrete time and  $j$  is the iteration index. Here,  $d(z)$  contains  $d_n$  and  $d_j$ .  $P(z)$  is the transfer function of the plant.

$$y_j(z) = P(z)u_j(z) + d(z) \quad (7)$$

And then, error signal  $e_j$  can be written as Eq. (8).

$$e_j(z) = r(z) - y_j(z) = \frac{r(z)}{1+G(z)K(z)} - \left[ \frac{d(z)}{1+G(z)K(z)} - \frac{n(z)G(z)K(z)}{1+G(z)K(z)} \right] \quad (8)$$

For the subsequent iteration, the error information is learning by ILC controller. Eq. (9) described the theoretical control law.

$$u_{j+1}(z) = Q(z)[u_j(z) + L(z)e_j(z)] \quad (9)$$

In this equation, the filter  $Q(z)$  is used to limit the frequency range of the learning for stability and noise attenuation. Learning function  $L(z)$  need to be chosen properly for rapid convergence. A widely used ILC stability condition is shown in Eq. (10).

$$\|Q(z)(1-L(z)P(z))\|_{\infty} < 1 \quad (10)$$

We chose  $L(z)=(S(z)G(z))^{-1}$  (given in Eq. (12) in discrete time domain) for rapid convergence. Besides the learning function  $L(z)$ , the accuracy of system model  $P(z)$  is another factor to influence convergence in the process of iteration.

To establish simultaneous equations with Eqs. (4), (7) and (8), error signal  $e_{j+1}$  can be calculated in Eq. (11).

$$e_{j+1}(z) = \begin{cases} S(z)(n_{j+1}(z) - n_j(z)) - S(z)(d_j(z) + d_{j+1}(z)) & (Q = 1) \\ S(z)n_{j+1}(z) - S(z)d_{j+1} - S(z)(r(z) - dn(z)) & (Q = 0) \end{cases} \quad (11)$$

$$L(z) = \frac{1.793z^{12} - 6.707z^{11} + 7.394z^{10} + 1.94z^9 - 9.57z^8 + 5.507z^7 + 1.36z^6 - 2.408z^5 + 0.4544z^4 + 0.6231z^3 - 0.5769z^2 + 0.2262z - 0.03484}{z^{12} - 3.37z^{11} + 2.809z^{10} + 2.226z^9 - 4.459z^8 + 1.363z^7 + 1.08z^6 - 0.8038z^5 + 0.05584z^4 + 0.2307z^3 - 0.1742z^2 + 0.04151z} \quad (12)$$

#### 4. SIMULATION RESULTS

The main motivation of this paper is to design a rapid and precise controller which can be applied into controlling a piezo-actuated stage and obtaining good performance in tracking reference signal. To guarantee the performance of rapid, the control algorithm should be convergent in limit iterations, and the statistical values of tracking error need to be controlled in a valid range to achieve high precision. In addition, the relationship between the Q-filter bandwidth and reference signal frequency, which can provide suggestions for designing Q-filter in the process of semi-physical simulation and hardware in loop, is discussed according to simulation results.

We first set the bandwidth of Q-filter as 10~100 Hz with 3rd order, and input a 30 Hz sine wave. The ILC gain is set as 0.8 with 20 iterations. Results are in Fig. 4.

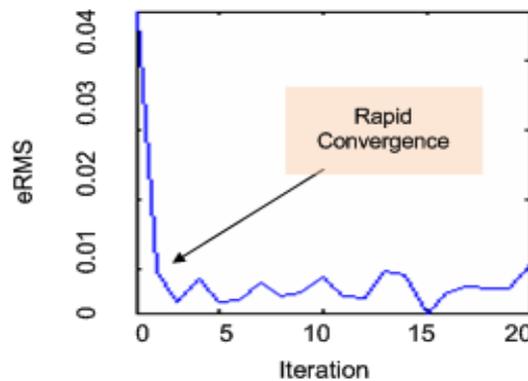


Fig. 4 Results of inputting 30 Hz sine wave

The RMS value of tracking error (eRMS in Fig. 4) is quickly convergent in less than 5 iterations and reaches a low level in the end, which proves the performance of rapid and precision of the control algorithm.

Then, we make a set of comparative simulations to investigate the inner relationship between the Q-filter bandwidth and reference signal frequency. The bandwidth is set as 10~100 Hz, 20~100 Hz and 30~100 Hz, and in each working condition, we input different reference sine waves ranging from 5Hz to 90Hz and set the ILC gain ranging from 0 to 1 (the X-axis). Results are described in Figs. 5, 6 and 7.

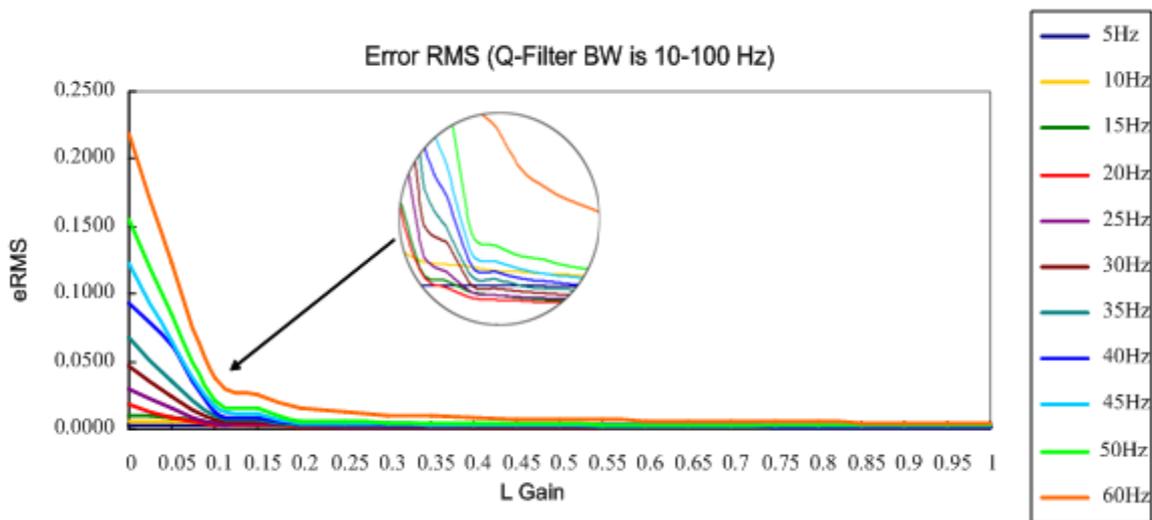


Fig. 5 Results of error RMS (set1)

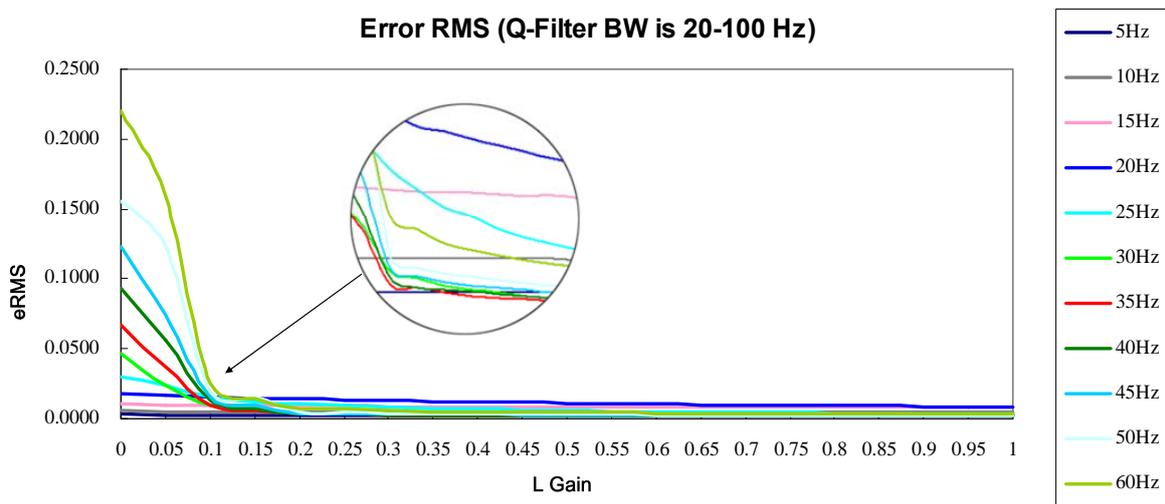


Fig. 6 Results of error RMS (set2)

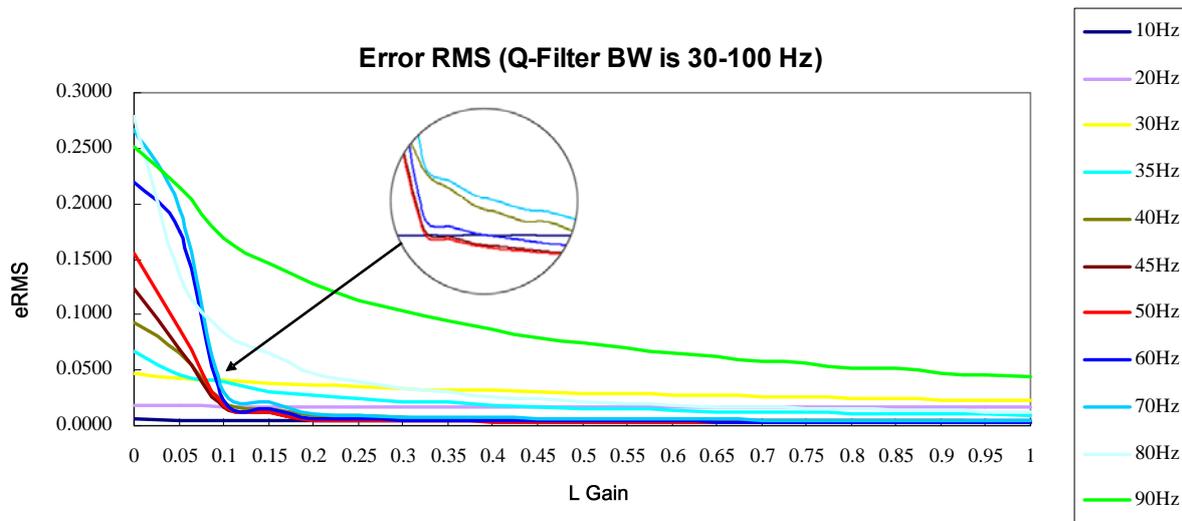


Fig. 7 Results of error RMS (set3)

Figs. 5, 6 and 7 point out that the best performance in each set can be obtained when the frequency of reference signal is respectively 20 Hz, 35 Hz and 50 Hz (marked in red in each figure) as a result of increase of start frequency of Q-filter. This point is quite beneficial for designing the Q-filter of ILC when the frequency of disturbance and noise is known. The bandwidth of Q-filter can be properly chosen according to the input signal frequency to achieve an ideal range of working bandwidth of the stage and reject the external disturbance at the same time.

## 5. CONCLUSIONS

This paper presents a control strategy of a micro-manipulation stage. In order to tracking the reference signal rapidly and precisely, a controller combines robust  $H^\infty$  control technique and feed forward iteration learning algorithm is designed. The advantage of the control strategy is in that the performance of rapid and precise can be obtained in the same time. Then, the mapping relevance between the Q-filter bandwidth and reference signal frequency is investigated. The simulation results prove that the control strategy can meet the performance requirements.

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