

## **Non-linear Controllers for Non-linear Model of Hovering Autonomous Underwater Vehicles**

\*Boris Braginsky<sup>1)</sup>, Ilan Zohar<sup>2)</sup> and Hugo Guterman<sup>3)</sup>

<sup>1), 2), 3)</sup> *Department of Electrical & Computer Engineering,  
Ben Gurion University of the Negev, Beer Sheva, Israel*  
[\\*borisbr@bgu.ac.il](mailto:borisbr@bgu.ac.il)

### **ABSTRACT**

Non-linear controllers for the attitude, altitude and position of the Hovering Autonomous Underwater Vehicle (HAUV) non-linear model are presented. These controllers allow the vehicle to reach any desired point in the horizontal and vertical planes. The controller was applied to a small-sized, torpedo-shaped HAUV with six degrees of freedom. The coupling between attitude and position makes the trajectory tracking control problem particularly hard. However, at least in the case of set-point control, it is possible to adopt a two decouple control strategy. A non-linear controller is applied for achieving attitude regulation, while a non-linear control strategy is applied for achieving smooth motion back to the desired position coordinates. The performance of the proposed control scheme is evaluated numerically.

**Keywords-**: HAUV; Non-linear control

### **1. INTRODUCTION**

The field of Unmanned Underwater Vehicles (UUVs) is of increasing interest of the scientific community due to its many interesting applications. These vehicles are capable of performing complex missions in spite of the many limitations of embedded sensors, processing and control (Gianluc 2006). These vehicles can be divided into two groups: Remotely Operated Vehicles (ROVs), which are underwater vehicles that are physically linked, via a tether, to an operator and Autonomous Underwater Vehicles (AUVs), which navigate fully autonomously. These vehicles have civilian and military applications and perform specific tasks such as search and rescue in high risk areas, autonomous sensing for weather forecasting, maintenance and fault detection of marine platforms and pipelines (oil and gas), underwater archeology and many more (Desa 2006). Generally, locomotion of the AUV is linear, while the ROV has hovering capabilities. Lately, Hovering AUVs (HAUV) have been proposed. The HAUV has several advantages over regular AUVs, such as maneuverability (it is possible move in any direction) and hovering (Torres 2012, Ferreira 2012 and Maalouf 2012).

In this study, non-linear controllers for the attitude and position of the HAUV non-linear model are presented. These controllers allow the vehicle to reach any desired point in the horizontal and the vertical planes. The control algorithms were applied to HAUV design and a model was built at the Laboratory of Autonomous Robotics (LAR) to participate in the AUVSI RoboSub competition (Hydro Camel Team, Fig. 1). The

platform is equipped with state of the art sensors and embedded processors, which are used to control the HAUV. Propulsion is generated by six brushless DC motors, two operating in each direction (Fig. 2). The HAUV dimensions are 150 centimeters in length and 30 centimeters in diameter, and the air weight is about 40 kg, while the water weight is almost neutral. The six thrusters placed on the hull provide six degrees of freedom (DOFs).



Fig. 1 Torpedo-shaped HAUV

The coupling between the attitude and the position makes the trajectory tracking control problem particularly hard. However, at least in the case of set-point control, it is possible to adopt a two decouple control strategy. A non-linear controller is applied for achieving attitude regulation and an independent non-linear control strategy is applied for achieving smooth motion back to the desired position coordinates. The proposed control scheme performance is evaluated numerically.

## 2. MODELING

In this section, the dynamic equations of the AUV's motion are outlined. In general, any movement of a vehicle in a 3D space involves 6 degrees of freedom (DOFs). It is convenient to define two coordinate frames, as shown in Fig. 2.

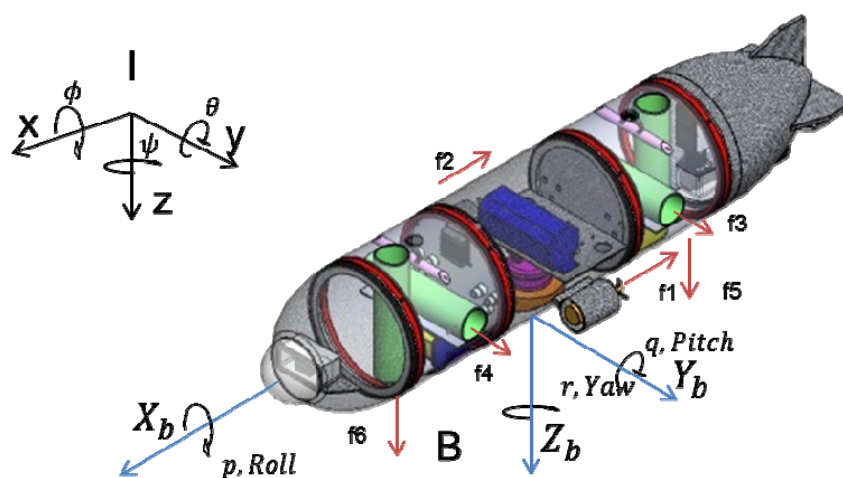


Fig. 2 The HAUV body frame and the motor thrust forces

The body frame mechanical system motion is given by Eq. (1).

$$\begin{aligned} m\dot{v}_b + \omega_b \times mv_b &= F_b \\ J\dot{\omega}_b + \omega_b \times J\omega_b &= M_b \end{aligned} \quad (1)$$

where  $J$  is the inertia tensor with respect to the body frame.

The above equations describe how the forces and moments affect the translational and rotational velocity of the rigid body. The kinematic equations that will allow relating quantities defined in terms of the body coordinate system to quantities in the inertial system, and vice versa, must be stated. Roll, pitch and yaw are the rotations of the body about the x, y, and z axes, respectively. The transformation  $R_i^b$  (described in Eq. (2)) was calculated by cascading the 3 separate angular transformation matrices. The order of the rotations from the inertial frame of reference was: rotation about the z-axis with the yaw angle  $\psi$ , then rotation about the y-axis with the pitch angle  $\theta$ , and, finally, rotation about the x-axis with the roll angle  $\phi$ . The following coordinate transform relates a vector in body-fixed coordinates with a vector in inertial or earth-fixed coordinates (Etkin 1982).

$$R_i^b = \begin{bmatrix} c\theta c\psi & c\psi s\theta s\phi - c\phi s\psi & c\psi s\theta c\phi + s\phi s\psi \\ c\theta s\psi & c\phi c\psi + s\theta s\phi s\psi & -s\phi c\psi + s\theta c\phi s\psi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \quad (2)$$

where  $s\alpha \doteq \sin \alpha$  and  $c\alpha \doteq \cos \alpha$ .

Let the Euler angles vector be  $\zeta \doteq [\phi, \theta, \psi]^T$ , the rotation speed vector be  $\omega_b \doteq [p, q, r]^T$  and the linear velocity in body axes be  $v_b = [u, v, w]^T$ . We wish to express the relationship between the body angular velocity  $\omega_b$  and the Euler vector rate of change  $\dot{\zeta} = [\dot{\phi}, \dot{\theta}, \dot{\psi}]^T$ . Assuming that  $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ , Eq. (3) is obtained and since  $\det L_{ib} = 1/\cos \theta$  is the relationship between the angular velocity and the Euler angle, rates may be inverted provided that  $\theta \neq \pi/2$ . Assuming that this is the case, one has

$$\dot{\zeta} = L_{ib}(\zeta)\omega_b \quad (3)$$

where

$$L_{ib} = \begin{bmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi \sec \theta & \cos \phi \sec \theta \end{bmatrix} \quad (4)$$

This is not a problem, as the vehicle motion does not normally approach the singularity condition. If this situation were to occur, then it would become necessary to model the vehicle motion using extreme pitch angles, and the analysis could then resort to an alternative kinematics representation, such as quaternions.

Applying Eqs. (1) and (3), the motion of the vehicle is described by the following Eq. (5).

$$\begin{aligned} \dot{v}_b &= -\omega_b \times v_b + \frac{F_b}{m} \\ \dot{\zeta} &= L(\zeta)\omega_b \end{aligned} \quad (5)$$

$$\dot{\omega}_b = -J^{-1}\omega_b \times J\omega_b + J^{-1}M_b$$

Since the water vehicle is considered a rigid body, the force  $F_b$  and the moment  $M_b$  are due to the action of the hydrodynamic, propulsive, gravitational and buoyancy field forces. In the current analysis we neglect the gravitational and buoyancy forces, as the vehicle is almost neutral in the water. The result of the combined external forces and moments is described as follows (McEwen 2006)

$$F_b = [\vec{F}_{bodylift} + \vec{F}_{bodydrag} + \vec{F}_{thrust}] \quad (6)$$

$$M_b = [\vec{M}_{bodylift} + \vec{M}_{bodydrag} + \vec{M}_{thrust}]$$

In what follows, the cross product for  $c, d \in R^3$  is expressed as a matrix operator, that is  $S(c)d = c \times d = -d \times c$  where  $S(\cdot)$  is a  $3 \times 3$  skew symmetric matrix. Applying the rotation matrix  $R_i^b$  in Eq. (2), the following state-space model of the considering system is obtained

$$\begin{aligned} \dot{\chi}_1 &= \chi_2 \\ \dot{\chi}_2 &= \frac{R_b^i(\zeta)F_b}{m} \\ \dot{\zeta} &= L_{ib}(\zeta)\omega_b \end{aligned} \quad (7)$$

$$\dot{\omega}_b = J^{-1}S(J\omega_b)\omega_B + J^{-1}M_b$$

where  $\chi_1 = [x, y, z]^T$  is the position vector of the vehicle center of mass in terms of the inertial frame,  $\chi_2 = [\dot{x}, \dot{y}, \dot{z}]^T$ .

With regard to the thrust produced by the 6 motors, the force and torque vectors of thrust in Eq. (6) are given by

$$M_{thrust} = [\alpha(F5 + F6), (1 - p)r_z(F5 - F6), r_x(F1 - F2) + r_y(F3 - F4)] \quad (8)$$

$$F_{thrust} = [b(F1 + F2), b(F3 + F4), b(F5 + F6)]$$

where  $r_x, r_y$  and  $r_z$  are the distances from the motors to the body center of mass,  $\alpha$  is

the angle of flipper for controlling the roll and  $b > 0$  is the thrust factor.

Vehicle body lift results from the vehicle moving through the water at an angle of attack, causing flow separation and a subsequent drop in pressure along the aft, upper section of the vehicle hull. This pressure drop is modeled as a point force applied at the center of pressure. As this center of pressure does not line up with the origin of the vehicle-fixed coordinate system, this force also leads to a pitching moment about the origin. In our model, Hoerner's estimate of body lift was used (Hoerner 1985). In vector form, the HAUV lift force and lift moments are as follows

$$F_{bodylift} = [0, Y_{uv}uv, Z_{uw}uw]^T \quad (9)$$

$$M_{bodylift} = [0, M_{uw}uw, N_{uv}uv]^T$$

In vector form, the HAUV damping forces and moments are as follows (Rentschler 2003)

$$F_{bodydrag} = [X_{u|u}|u|u|, Y_{v|v}|v|v| + Y_{r|r}|r|r|, Z_{w|w}|w|w| + Z_{q|q}|q|q|]$$

$$M_{bodydrag} = [K_{p|p}|p|p|, M_{w|w}|w|w| + M_{q|q}|q|q|, N_{v|v}| + N_{r|r}|r|r|]$$
(10)

The variables from Eqs. (9) and (10) described in Table 1.

Table 1 Non linear forces and moment coefficients

$X_{u u }$	Axial drag [kg/m]	$K_{p p }$	Rolling Resistance [ $\text{kg} \cdot \text{m}^2 / \text{rad}^2$ ]
$Y_{v v }$	Cross flow Drag [kg/m]	$M_{w w }$	Cross flow Drag [kg]
$Y_{r r }$	Cross flow Drag [ $\text{kg} \cdot \text{m} / \text{rad}^2$ ]	$M_{q q }$	Cross flow Drag [ $\text{kg} \cdot \text{m}^2 / \text{rad}^2$ ]
$Y_{uv}$	Body lift Force [kg/m]	$M_{uw}$	Body lift Moment [kg]
$Z_{q q }$	Cross flow Drag [ $\text{kg} \cdot \text{m} / \text{rad}^2$ ]	$N_{v v }$	Cross flow Drag [kg]
$Z_{w w }$	Cross flow Drag [kg/m]	$N_{r r }$	Cross flow Drag [ $\text{kg} \cdot \text{m}^2 / \text{rad}^2$ ]
$Z_{uw}$	Body lift Force [kg/m]	$N_{uv}$	Body lift Moment [kg]

### 3. STABILIZING CONTROLLER FOR THE ATTITUDE SUBSYSTEM

#### 3.1 Attitude regulation

In the controller design it is assumed that all state variables are measured. It is important to state that both the  $\zeta$  and  $\omega_b$  subsystems in Eq. (7) are independent of the position and velocity vectors  $\chi_1, \chi_2$ , respectively, while the attitude  $\zeta$  vectors are highly coupled with the vector  $\omega_b$ .

We concentrate now on the attitude subsystem, defined by

$$\dot{\zeta} = L_{ib}(\zeta)\omega_b \quad (11)$$

$$\dot{\omega}_b = J^{-1}S(J\omega_b)\omega_B + J^{-1}M_b$$

Following the approach and results in (Zohar 2012) the following attitude controller is defined

$$M_{thrust} = -(L_{ib}(\zeta)K\zeta + B\omega_b + \vec{M}_{bodylift} + \vec{M}_{bodydrag}) \quad (12)$$

where  $K = K^T, B = B^T > 0$  are arbitrarily selected constant matrices. Using Eq. (11) and the Lyapunov condition, the stability of the system can be proved.

$$V(\zeta, \omega_b) = \frac{1}{2} [\zeta^T K \zeta + \omega_b^T J \omega_b] \quad (13)$$

Substituting  $M_{thrust}$  Eq. (12) into Eq. (11), the derivative of  $V$  along the trajectories of the resulting closed-loop system, namely,  $\dot{V} = \zeta^T K \dot{\zeta} + \omega_b^T J \dot{\omega}_b$ , satisfies

$$\dot{V} = \zeta^T K L_{ib}(\zeta) \omega_b + \omega_b^T S(J \omega_b) \omega_b + \omega_b^T (\vec{M}_{bodylift} + \vec{M}_{bodydrag} + \vec{M}_{thrust}) \quad (14)$$

When  $S(J \omega_b)$  is skew symmetric, then  $\omega_b^T S(J \omega_b) \omega_b = 0$  and  $\zeta^T K L_{ib}(\zeta) \omega_b = \omega_b^T L_{ib}^T K \omega_b$ . Therefore Eq. (14) can be written as follows

$$\begin{aligned} \dot{V} &= \zeta^T K L_{ib}(\zeta) \omega_b + \omega_b^T (\vec{M}_{bodylift} + \vec{M}_{bodydrag}) \\ &- \omega_b^T (L_{ib}(\zeta) K \zeta + B \omega_b + \vec{M}_{bodylift} + \vec{M}_{bodydrag}) = -\omega_b^T B \omega_b < 0 \end{aligned} \quad (15)$$

### 3.2 Position regulation

Due to the non-linear coupling between the attitude and the position variables, the process of attitude regulation is associated with drift in the  $\chi_1 = [x, y, z]^T$  coordinates. To reduce the resulting drift, an additional non-linear controller for position regulation is proposed.

The position subsystem is defined by the following equations

$$\begin{aligned} \dot{\chi}_1 &= \chi_2 \\ \dot{\chi}_2 &= \frac{R_b^i(\zeta) F_b}{m} \end{aligned} \quad (16)$$

In order to stabilize the position subsystem,  $F_{thrust}$  is defined as

$$F_{thrust} = [-F_{bodylift} - F_{bodydrag} - R_i^b(\zeta)^{-1}(\chi_1 m + \chi_2)] \quad (17)$$

where  $A = A^T > 0$  are arbitrarily selected constant matrices. When  $\det(R_i^b(\zeta)) = 1$  for all  $\theta$  and  $\psi$ , then  $R_i^b(\zeta)^{-1}$  will always exist. Using the Lyapunov stability condition we

can prove stability of the system. The following Lyapunov function is proposed

$$V_2 = \frac{1}{2}(\chi_1^T A \chi_2 + \chi_2^T A \chi_2) \quad (18)$$

Substituting  $F_{thrust}$  from Eq. (17) into Eq. (16), the derivative of  $V_2$  along the trajectories of the resulting closed-loop system, namely,  $\dot{V}_2 = \chi_1^T A \dot{\chi}_1 + \chi_2^T A \dot{\chi}_2$  satisfies

$$\dot{V}_2 = \chi_1^T A \chi_2 + \frac{\chi_2^T A R_i^b(\zeta) F_b}{m} = \chi_1^T A \chi_2 + \frac{\chi_2^T A (-\chi_1 m - \chi_2)}{m} = -\chi_2^T A \chi_2 < 0$$

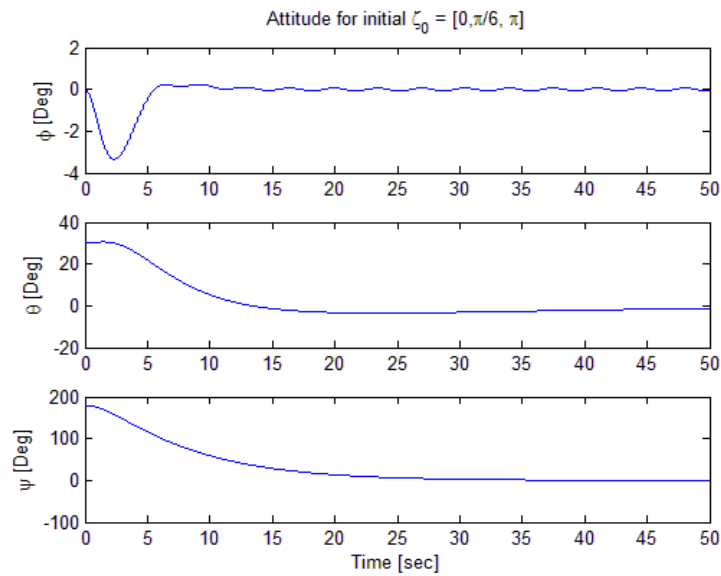


Fig. 3 Attitude set point tracking

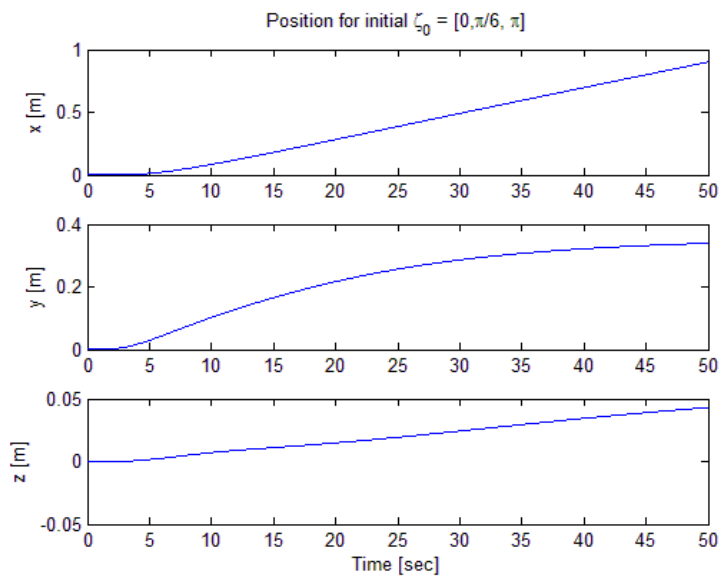


Fig. 4 Position drift during the attitude regulation

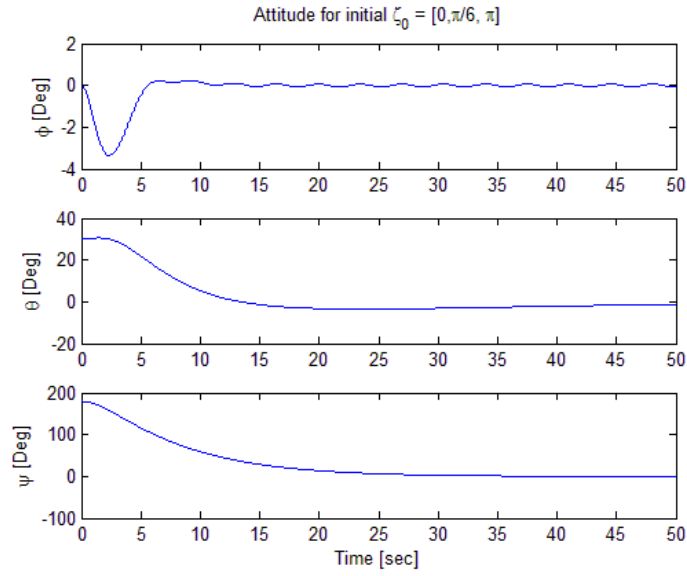


Fig. 5 Attitude set point tracking

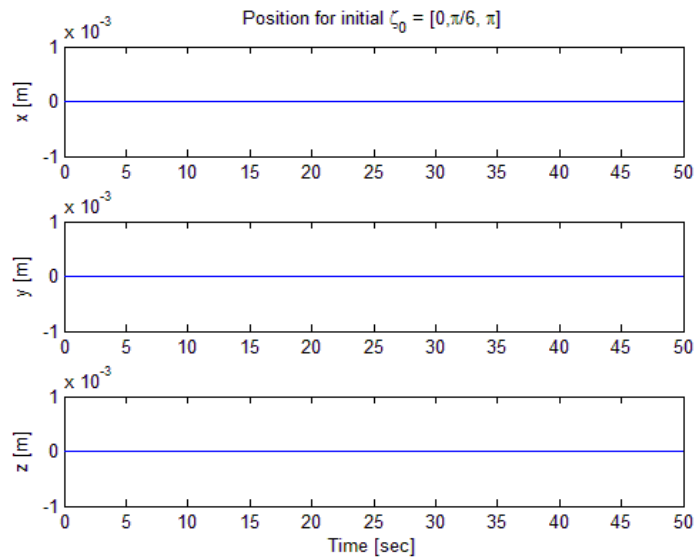


Fig. 6 Position drift during the attitude and position regulation

#### 4. SIMULATION RESULTS

*Example 1:* In this case, the HAUV control objective is to stabilize the attitude. The considered initial condition is  $\zeta(0) = [0, \frac{\pi}{6}, \pi]^T$  (angles in radians) and the initial position is  $\chi_1 = [0,0,0]$ . Figs. 3 and 4 demonstrate attitude set-point tracking and position drift during attitude regulation.

*Example 2:* This example demonstrates the action of the two controllers (attitude and position). The attitude non-linear controller tries to achieve zero attitude and



position. The non-linear controller objective is to keep the HAUV in its initial position ( $\chi_2(0) = [0,0,0]$ ). The results of the attitude controller are shown in Fig. 5, while the behavior of the position controller appears in Fig. 6.

## 5. CONCLUSIONS

The main goal of this research was to develop non-linear controllers for stabilizing the attitude and the position of a HAUV during hovering. A relatively simple controller for regulating the attitude and position subsystems for the highly non-linear HAUV system has been presented. Simulation results demonstrate the controller performance.

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