

## Online Walking Pattern Generation through Model Predictive Control of Capture Point

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### ABSTRACT

We propose a method of generating a walking pattern based on a capture point dynamics by applying a model predictive control to determine optimal control inputs considering constraints. Predicted trajectory of capture point from capture point dynamics over the prediction horizon is used for minimizing the cost function that is consist of reference capture point tracking term. The optimization variable ZMP and foot variation are constrained to remain within the support polygon and within the kinematic limits, respectively. It is formulated as quadratic programming form from cost function and constraint. The sub-sampling method is applied so that quadratic programming can be solved in real-time while ensuring stability of model predictive control using long prediction horizon.

### 1. INTRODUCTION

In order for humanoid robots to coexist with human beings and provide various services and perform tasks that are difficult for human beings, the ability of humanoid robots to move stably even in unexpected disturbances and uneven grounds is a prerequisite. In order to ensure stable mobility, many walking pattern generation methods using ZMP (Vukobratović,2004), which is an index indicating the stability of biped robot, have been studied. A method of generating a stable CoM trajectory by minimizing the jerk input while the system output servos the reference ZMP using a linear quadratic regulator has been proposed (Kajita,2003). Using this method, walking pattern generation methods were developed by reflecting the current state of the robot continuously (Nishiwaki(2006, 2010)).

In addition, walking pattern generation method using model predictive control is proposed by Wieber(2006). This method generate optimal control input considering the constraints imposed on the system state or input or output. Diedam(2008) and Herdt (2010) expended the above model predictive control scheme by adding the footstep adjustment as the optimization variable.

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It is necessary to consider the state space variables used in pattern generation in aspect of generating a pattern reflecting the current state of the robot. The state space dynamics using the CoM acceleration as the state space variable is not suitable for generating the gait pattern through the state feedback because it is difficult to estimate the CoM acceleration(Wieber(2006),Diedam(2008), Herdt(2010)). The method of generating a gait pattern in which the ZMP is included as a state space variable instead of the CoM acceleration is limited in the uneven terrain (Urata 2010). However, because the capture point is calculated from the CoM position and the CoM velocity, it is more suitable for generating the gait pattern through real-time state feedback (Englsberger (2011),Pratt (2012)).

In this study, we propose a method of generating a gait pattern based on a capture point dynamics by applying a model predictive control to determine the optimal control inputs considering constraints.

## 2. Method

### 2.1 Capture Point Dynamics

In (Englsberger,2011), the capture point  $\xi_x$  is defined as

$$\xi_x = x + \frac{\dot{x}}{\omega}$$

where  $\omega = \sqrt{g/z}$ .  $g$  and  $z$  are gravity acceleration and height of CoM respectively. Detailed derivation is found in (Englsberger,2011). In this paper, for concise formula, we represent all the formula about sagittal plane.

Capture point can be controlled by ZMP. Control input and state are ZMP and capture point respectively. The capture point behavior can be predicted by the following capture point dynamics.

$$\begin{aligned}\xi_{k+1} &= A\xi_k + Bu_k \\ y_k &= C\xi_k + Du_k \\ A &= e^{\omega T}, B = 1 - e^{\omega T}, C = 1, D = 0,\end{aligned}$$

Where T is the sampling time. Using this dynamics, future capture point behavior is like below:

$$\Omega = A_\xi \xi_k + B_u P$$

Where

$$A_\xi = \begin{bmatrix} CA \\ \vdots \\ CA^N \end{bmatrix} \quad B_u = \begin{bmatrix} CA^0 B & \cdots & 0 \\ \cdots & \ddots & \vdots \\ CA^{N-1} B & \cdots & CA^0 B \end{bmatrix}$$

## 2.2 Objective Function

Predicted capture point trajectory over the prediction horizon (NT) is used for minimizing the cost function that is consist of reference capture point trajectory tracking term and control input. Optimal control inputs are the ZMP  $P \in R^N$  and the footstep variation  $\Delta F \in R^m$ . Note that we added the footstep variation as optimal control input. If the control input ZMP exceed the support polygon, footstep is adjusted by footstep variation. This makes the walking pattern to be more robust to large disturbance.

This cost function is formulated as quadratic program like below

$$\min_{P, \Delta F} \frac{1}{2} \begin{bmatrix} P \\ \Delta F \end{bmatrix}^T \begin{bmatrix} H & 0 \\ 0 & wI \end{bmatrix} \begin{bmatrix} P \\ \Delta F \end{bmatrix} + \begin{bmatrix} P \\ \Delta F \end{bmatrix}^T \begin{bmatrix} f \\ 0 \end{bmatrix}$$

Where

$$H = R + B_u^T Q B_u, f = B_u^T Q (A_\xi \xi_k - \Omega_{ref})$$

$Q$  and  $R$  is N by N weight matrix for input ZMP like below.

$$Q = \begin{bmatrix} q & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & q \end{bmatrix}, R = \begin{bmatrix} r & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & r \end{bmatrix}$$

$w$  is scalar weight for input footstep variation.

In order to generate a real-time gait pattern generation from the robot state, the computation time of the QP must be minimized. The sub-sampling method is applied so that QP can be performed in the control period while ensuring stability of model predictive control using long prediction horizon. We set the sampling time differently. First sampling time is same with control frequency and the others sampling time are set the more roughly to reduce the number of optimization variable.

## 2.3. Constraint

The optimization variable ZMP is constrained to stay within the support polygon and optimization variable foot variation is also constrained for kinematic limit like reachable work space.

$$S_{\min} \leq P_k \leq S_{\max}$$

$$L_{\min} \leq \Delta F_k \leq L_{\max}$$

$S_{\min}$  and  $S_{\max}$  are the minimum and maximum value of support polygon area respectively. This constraints can be represented as following linear inequality form.

$$A_c \begin{bmatrix} P \\ \Delta F \end{bmatrix} \leq b_c$$

### 3. Simulation

We conducted the two simulations of the proposed walking pattern using MATLAB. In first simulation, we apply the small disturbance and in second simulation, we apply the large disturbance. Dynamics properties used in simulation is same with DRC-HUBO2 developed in our lab. Table 1 lists the parameters used in simulation.

Table 1 Parameters used in simulation

Parameter	Value
h	0.77m
T	0.005s
Step period	Ssp time:0.7s Dsp time:0.1s
q	1.0
r	0.0005
w	1
N	10
m	2

Figure 1 shows result of simulation about lateral plane. From time 2.1s, constant velocity disturbance was applied for the time 0.25s. Because the small disturbance is applied, footstep position is not changed. The robot recover the balance only using the ZMP and we can see that the ZMP is maintained within the support polygon (green line).

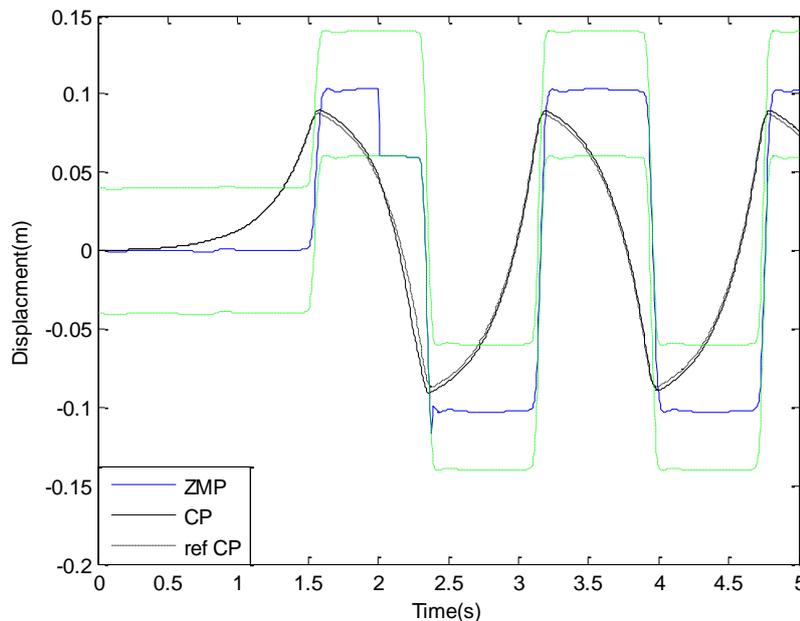


Figure 1 Simulation result of a proposed walking pattern under the small disturbance

Figure 2 shows the simulation result under the 1.5 times larger disturbance than the first simulation. Because the robot cannot recover the balance from the disturbance using only ZMP, right foot of footstep position is automatically changed from -0.1m to -0.14m. It can be confirmed that the position of the right foot has shifted outward by about 0.04m in order to cope with the disturbance. We can confirm that the capture point follows the reference capture point trajectory again.

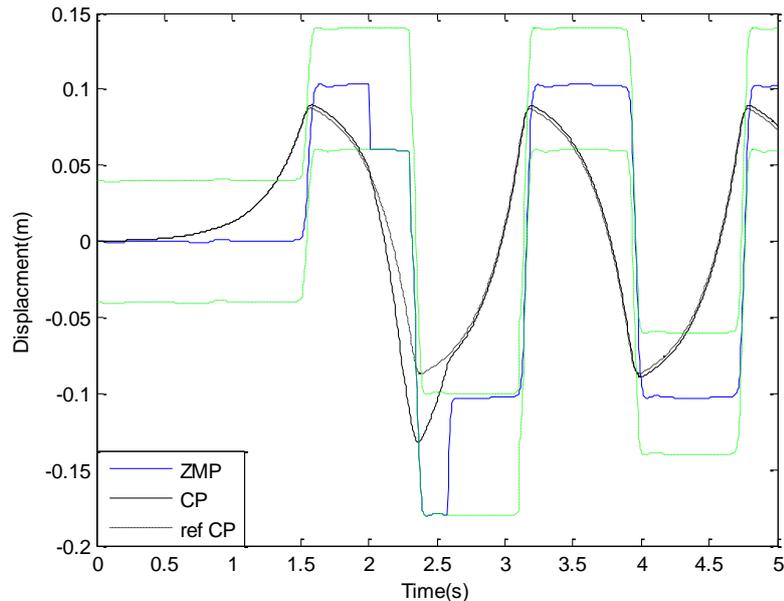


Figure 2 Simulation result of a proposed walking pattern under the large disturbance

#### 4. Conclusion

We proposed the walking pattern generation method using capture point dynamics. For ensuring the dynamic and kinematic feasibility, we introduce the model predictive control, which determines the optimal control input considering the constraints. From the simulation, we verified the effectiveness of proposed algorithm. If the robot can not recover the balance using ankle torque(ZMP), the robot automatically changes the footstep positions to cope with disturbance. The control input ZMP and footstep position are also constrained for feasible solution. For future work proposed algorithm will be implemented to real humanoid robot 'DRC-HUBO'.

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