The 2013 World Congress on
Advances in Structural Engineering and Mechanics (ASEM13)
Jeju, Korea, September 8-12, 2013

Effects of supported member on the nonlinear parametric vibration of a cable

*Ming-Hai Wei[1)] Yi-Qing Xiao[2)] Hai-Tao Liu[2)] and Du-Jian Zou[2)]

[1)] *Department of Construction and Engineering Management, Shenyang Jianzhu University, Shenyang 110168, People's Republic of China*
[1)] *hitsz.civil@gmail.com*
[2)] *Department of Civil and Environment Engineering, Shenzhen Graduate School, Harbin Institute of Technology, Shenzhen 518055, China*

ABSTRACT

In cable-supported structures, cables are easily subjected to potentially damaging large amplitude motion, mainly due to the parametric vibration on cables. This paper presents an analysis of the effects of support member on the nonlinear parametric vibration of a cable using a coupled cable-beam model. The model considers the parameters of the beam and geometric nonlinearities of the cable. First, the method of multiple scales is applied directly to the model, using the first-order equation, the frequency response and stability conditions are obtained. The effects of the mass ratio, stiffness ratio, and inclined angle of the coupled model are then evaluated. Then, the effects of these parameters on the parametric vibration characteristics of cable are investigated in terms of the maximum mid-span displacement and dynamic amplification factor. The results extend those of previous work and provide some useful insights into the design of cable-supported structures.

1. INTRODUCTION

Cable-supported structures, one of the most important components of engineering field, are used extensively throughout the world. The applications are extremely diverse, such as cable-stayed bridges, guyed towers, suspended roofs, cranes and many other applications in engineering and industry. However, the cables in these cable-supported structures are easily subjected to potentially damaging large amplitude motion, mainly due to the motion that comes from the small amplitudes of the moving deck or tower which the cables are attached. Though the cables in these cable-supported structures are connected with supported members, such as a continuous beam or tower,

---

[1)] Assistant Professor
[2)] Professor
[2)] Lab technician
[2)] Postdoctoral
Note: Copied from the manuscript submitted to "Structural Engineering and Mechanics, An International Journal" for presentation at ASEM13 Congress

numerous journal articles such as the reviews by Nayfeh and Pai (2004), Rega (2004 a, 2004 b) ignore the effects of the beam or tower's parameters, and simplify models of this behaviour to a bar cable with specified conditions. The failure to take into account the supported members may cause significant faults or undesirable vibrations in cable-supported structures. Thus, the effects of the supported members must be taken into consideration in the study of nonlinear parametric vibration of cables.

The research classes of parametric vibrations of a cable can be divided into two categories according to the different forms of excitation. The first category is an ideal incentive system, in which the forcing amplitude and frequency are constant or change in a specified manner. During the process of vibration, the effects of the cable vibrations on the deck or tower are ignored, under the assumption that the quality of the deck or tower is much higher than that of the cable. The second category is a non-ideal incentive system, in which the forcing amplitude and frequency are dynamic, changing during the process of vibration, and the vibrations of the cable and deck or tower are coupled together. Some researchers have been devoted to study the mechanisms with experimental or theoretical methods since parametric vibrations were discovered. Tagata (1977) studied the parametric vibration of a weightless string (excluding the impact of sag) and determined the unstable regions based on the dimensionless Mathieu equation. Perkins (1992) used a coupled two-degrees-of-freedom system to analyze the parametric vibrations of a cable under simultaneous parametric and external resonances, and the cable in the experiment exhibited behaviour similar to the predicted behaviour. Cai and Chen (1994) investigated the dynamic response of an inclined cable subjected to parametric and external resonances by applying the numerical method. Lilien and Pinto daCosta (1994) calculated the amplitudes and mechanical tension oscillations of cable, which resulted in a small amplitude motion along the cable axis. The oscillations of a cable due to periodic motions of the deck or tower have been investigated by daCosta et al (1996). Berlioz and Lamarque (2005) presented a nonlinear model to predict the nonlinear behaviours of an inclined cable subjected to the boundary motion condition and validated their work experimentally. Wang and Zhao (2009) used the shooting method and continuation technique to investigate the large amplitude of planar and non-planar motions of an inclined cable subject to the support motion. The chaotic dynamics of a suspended cable under combined parametric and external excitations were investigated by Zhang and Tang (2002) through 1:2 or 1:1 internal resonance cases between the in-plane and out-of-plane modes. The stability boundaries of parametric vibrations of multiple cable modes are analyzed by Macdonald et al. (2010), where support excitation close to any natural frequency. Their results were validated experimentally, and they identified a new nonlinear mechanism. These works deepen our understanding on the excitation mechanism of the large amplitude motion of an inclined cable caused by deck or tower motions.

The above researches all emphasized the fact that the excitations caused by support motion would make the analysis of parametric vibrations of a cable more approximating the real situations. However, previous studies on cable-supported structures ignored the effects of supported member motion. Nazmy and Abdel (1988) demonstrated the importance of cable-beam interactions by the nonlinear analysis of cable-stayed bridges. Fujino et al. (1993, 1995) presented a Ritz-type coupled model based on experiments to investigate the cable-beam interactions. Caetano et al. (2000 a, 2000 b) studied the dynamic interactions between the cable and deck or tower in cable-stayed bridges by physical modelling and experimental testing. Gattulli et al. (2002) used the Galerkin method to simplify the governing equations of cable-stayed beam to 1-DOF nonlinear equations. The effects of the mass ratio, stiffness ratio, and other parameters of the cable-stayed beam on the structural vibration modes have been investigated. The nonlinear interaction between the beam and the cable in a cable-stayed beam, including 1:2 and 2:1 internal resonance, were investigated and compared to the results by using a finite element model (Gattulli et al., 2003, 2005). Georgakis and Taylor (2005 a, 2005 b) presented an alternative cable-deck model to investigate the nonlinear dynamics of an inclined cable which both induced by sinusoidal and stochastic support excitations. The different cable-deck interactions of the Guadiana Bridge under environmental excitations investigated by Caetano et al. (2008) used the vibration data acquisitions and a refined finite element model. The results showed that the large amplitude vibration of a stay cable may be excited by its support motion, which was caused by the deck vibration.

Most of the above studies focused on the dynamic interaction between the cable and its supported member, and little attention has been devoted to investigate the effects of supported member parameters on the nonlinear parametric vibration of cables, i.e. the mass or stiffness parameter of supported member. In this paper, the nonlinear parametric vibrations of a cable are investigated by a coupled cable-beam model. The model includes the effects of beam and the geometric nonlinearities of cable. The method of multiple scales is directly applied to the model, and using the first-order equation, the frequency response and stability conditions are obtained. The effects of the beam parameters, i.e. mass ratio, stiffness ratio and inclined angle, on the nonlinear response curves are discussed below. Furthermore, the effects of the beam parameters on the parametric vibration characteristics of the cable are investigated by using the fourth-order Runge-Kutta algorithm. The individual parameters of the cable, i.e. the sag and tension, are not investigated in this paper because much of the literature has already investigated this area.

2. PROBLEM FORMULATION

Consider an essential simplification of a cable-support structure, a cable-beam coupled model with small sag of the cable (d/lc < 0.05), as shown in Fig. 1. The cable is supported at one end and attached by a beam at the other end. It is assumed that: (i) the beam and cable are homogeneous and only oscillate transversely on the in-plane; and (ii) the cable is taut and its mass is much smaller than that of the beam (ρ < 0.01), therefore, the role of the cable is that of an elastic support on the beam. Neglecting the bending, torsional and shear rigidities of the cable, the dimensionless equations of the in-plane motion can be expressed as (Wei et al., 2012).

$$\ddot{v}_1 + \xi_1 \dot{v}_1 + \frac{1}{\beta_b^4} v_1'''' + \frac{\rho}{\beta_c^2 \sin(\theta)} v_1'' + \frac{\rho \mu e}{\beta_c^2 \sin(\theta)} v_1'' = 0 \tag{1a}$$

$$\ddot{v}_2 + \xi_2 \dot{v}_2 - \frac{1}{\beta_c^2}[v_2'' + \mu e(\nu y'' + v_2'')] = 0 \tag{1b}$$

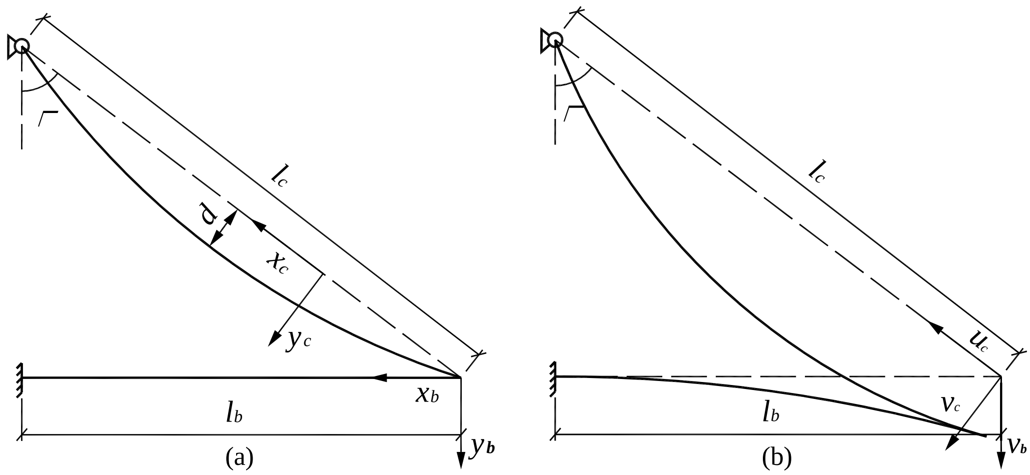

Fig. 1 Sketch of cable-beam structure: (a) the static model; (b) the dynamic model.

The boundary conditions are given by

$$v_1(1,t)=0, v_1(0,t)''=0, v_2(1,t)=0, v_2(1,t)'=0 \tag{2a}$$

$$v_2(0,t)-v_1(0,t)\sin(\theta)^2=0, \chi v_1(0,t)'''+\cos(\theta)^2\sin(\theta)v_1(0,t)=0 \tag{2b}$$

where ρ, χ, and θ are the mass ratio, stiffness ratio and inclined angle, respectively, which all relate to the beam.

In this paper, only the first-order modes of the beam and cable are considered. Assuming the transverse displacement v_1 has the following form

$$v_1(x,t)=\varphi_1(x)q_1(t) \tag{3a}$$

and substituting Eq. (3a) into Eq. (2b), the transverse displacement v_2 can be rewritten as

$$v_2(x,t)=\varphi_2(x)q_2(t)+\varphi_1(x)q_1(t)\sin(\theta)^2 \tag{3b}$$

where $\varphi_1(x)$ and $\varphi_2(x)$ are the mode shapes of the beam and the cable, respectively.

Then, according to the Galerkin method, the first-order modal equations of the cable-beam coupled model are obtained

$$\ddot{q}_1(t)+\xi_1\dot{q}_1(t)+a_1q_1(t)+a_{12}q_1(t)q_2(t)+a_{122}q_1(t)q_2(t)^2+a_{11}q_1(t)^2+a_{112}q_1(t)^2q_2(t) \\ +a_{111}q_1(t)^3=0 \tag{4a}$$

$$\ddot{q}_2(t)+\xi_2\dot{q}_2(t)+c_1\ddot{q}_1(t)+c_2\dot{q}_1(t)+b_2q_2(t)+b_{12}q_1(t)q_2(t)+b_{112}q_1(t)^2q_2(t)+b_{22}q_2(t)^2 \\ +b_{122}q_1(t)q_2(t)^2+b_{222}q_2(t)^3+b_1q_1(t)+b_{11}q_1(t)^2+b_{111}q_1(t)^3=f_{22}\cos(\Omega_2 t) \tag{4b}$$

where a_i, b_i, c_i, a_{ij}, b_{ij}, a_{ijk}, and b_{ijk} are the Galerkin coefficients of the beam and cable, which are derived in detailed and interpreted in the Appendix (Wei et al., 2012).

The beam motion under small harmonic excitation exhibits harmonic characteristic in mumerical analysis is reported in Wei et al. (2012) and confirmed in the experiment by Zhou et al. (2007). Thus, the beam motion under small harmonic excitation can be assumed to take the form of

$$q_1(t)=Q\cos(\omega_1 t) \tag{5}$$

where Q and ω_1 are the amplitude and frequency of the beam vibration, respectively.

Substituting Eqs. (4a) and (5) into Eq. (4b), the cable motion takes the following form:

$$\ddot{q}_2(t)+\xi_2\dot{q}_2(t)+(b_2+b_{12}\cos(\omega_1 t)Q-c_1a_{12}\cos(\omega_1 t)Q+b_{112}(\cos(\omega_1 t)Q)^2$$
$$-c_1a_{112}(\cos(\omega_1 t)Q)^2)q_2(t)+(b_{22}+b_{122}\cos(\omega_1 t)Q-c_1a_{122}\cos(\omega_1 t)Q)q_2(t)^2$$
$$+b_{222}q_2(t)^3=(c_2-c_1\xi_1)\sin(\omega_1 t)\omega_1 Q+(c_1a_1-b_1)\cos(\omega_1 t)Q \tag{6}$$
$$+(c_1a_{11}-b_{11})(\cos(\omega_1 t)Q)^2+(c_1a_{111}-b_{111})(\cos(\omega_1 t)Q)^3$$

Compared with the classical parametric vibration models used by Warnitchai et al. (1995) and Berlioz and Lamarque (2005), some main differences occur in the expression of Eq. (6). The model takes into account not only the contribution of individual parameters of a cable but also the parameters of the supported member. Moreover, the equivalent parametric excitations (on the right side of Eq. (6)) contain both the sinusoidal and cosine functions.

3. MATHEMATICAL ANALYSIS

3.1 Perturbation analysis

The multiple scales perturbation method (Nayfeh and Mook, 1979) is applied to Eq. (6) to asymptotically describe the slow dynamics of the model. To make the non-linear terms weak, one can substitute a_{ijk}, b_{ijk} and c_{ijk} with εa_{ijk}, εb_{ijk} and εc_{ijk}. Then, Eq. (6) can be rewritten as

$$\ddot{q}_2(t)+\varepsilon\xi_2\dot{q}_2(t)+(\omega_2^2+\varepsilon b_{12}\cos(\omega_1 t)Q-\varepsilon c_1a_{12}\cos(\omega_1 t)Q+\varepsilon b_{112}(\cos(\omega_1 t)Q)^2$$
$$-\varepsilon c_1a_{112}(\cos(\omega_1 t)Q)^2)q_2(t)+(\varepsilon b_{22}+\varepsilon b_{122}\cos(\omega_1 t)Q-\varepsilon c_1a_{122}\cos(\omega_1 t)Q)q_2(t)^2$$
$$+\varepsilon b_{222}q_2(t)^3-\varepsilon(c_2-c_1\xi_1)\sin(\omega_1 t)\omega_1 Q+\varepsilon(b_1-c_1\omega_1^2)\cos(\omega_1 t)Q \tag{7}$$
$$+\varepsilon(b_{11}-c_1a_{11})(\cos(\omega_1 t)Q)^2+\varepsilon(b_{111}-c_1a_{111})(\cos(\omega_1 t)Q)^3=0$$

A first-order approximation is given by

$$q_2(t,\varepsilon)=q_{20}(T_0,T_1)+\varepsilon q_{21}(T_0,T_1)+O(\varepsilon^2)+\cdots \tag{8}$$

where $T_0 = t$, $T_1 = \varepsilon t$ and $\varepsilon << 1$. Combining Eq. (22) and (19) and equating the power of ε, one has

$$D_{0,0}(q_{20})+q_{20}\omega_2^2=0 \tag{9a}$$

$$D_{0,0}(q_{21})+q_{21}\omega_2^2=-\xi_2D_0(q_{20})-2D_{0,1}(q_{20})-(b_{22}+b_{122}\cos(\omega_1 t)Q-c_1a_{122}\cos(\omega_1 t)Q)q_{20}^2$$
$$-b_{222}q_{20}^3-(b_{112}\cos(\omega_1 t)^2Q^2+b_{12}\cos(\omega_1 t)Q-c_1a_{12}\cos(\omega_1 t)Q-c_1a_{112}\cos(\omega_1 t)^2Q^2)q_{20} \tag{9b}$$
$$-(b_1-c_1\omega_1^2)\cos(\omega_1 t)Q-(b_{11}-c_1a_{11})\cos(\omega_1 t)^2Q^2-(c_1\xi_1-c_2)\sin(\omega_1 t)\omega_1 Q$$
$$-(b_{111}-c_1a_{111})\cos(\omega_1 t)^3Q^3$$

The general solution of Eq. (9a) can be rewritten as

$$q_{20}(T_0,T_1)=A(T_1)\exp(iT_0\omega_2)+cc \tag{10}$$

Substituting Eq. (10) into Eq. (9b) yields

$$
\begin{aligned}
&D_{0,0}(q_{21}) + q_{21}\omega_2^2 = -(\frac{1}{2}b_{112}AQ^2 - \frac{1}{2}c_1a_{112}AQ^2 + i\xi_2\omega_2 A + 2iD_1A\omega_2 + 3b_{222}A^2\overline{A})\exp(iT_0\omega_2) \\
&-(-\frac{3}{8}c_1a_{111}Q^3 + b_{122}A\overline{A}Q - \frac{1}{2}c_1\omega_1^2 Q - \frac{1}{2}ic_1\xi_1\omega_1 Q + \frac{1}{2}ic_2\omega_1 Q + \frac{1}{2}b_1 Q + \frac{3}{8}b_{111}Q^3 \\
&-c_1a_{122}A\overline{A}Q)\exp(iT_0\omega_1) - (\frac{1}{4}b_{11}Q^2 - \frac{1}{4}c_1a_{11}Q^2)\exp(iT_0 2\omega_1) \\
&-(\frac{1}{8}b_{111}Q^3 - \frac{1}{8}c_1a_{111}Q^3)\exp(iT_0 3\omega_1) - (\frac{1}{4}b_{112}\overline{A}Q^2 - \frac{1}{4}c_1a_{112}\overline{A}Q^2)\exp[i(2\omega_1 - \omega_2)T_0] \\
&-(-\frac{1}{2}c_1a_{12}\overline{A}Q + \frac{1}{2}b_{12}\overline{A}Q)\exp[i(\omega_1 - \omega_2)T_0] \\
&-(-\frac{1}{2}c_1a_{122}\overline{A}^2 Q + \frac{1}{2}b_{122}\overline{A}^2 Q)\exp[i(\omega_1 - 2\omega_2)T_0] - cc + \text{NST}
\end{aligned}
\tag{11}
$$

where *cc* and NST denote the parts of the complex conjugate of the function on the right side of Eq. (11) and the non-secular terms, respectively. Moreover, from Eq. (11), we can see that there are a variety of parametric resonance cases, i.e. $\omega_1 = 3\omega_2$, $\omega_1 = 2\omega_2$, $\omega_1 = \omega_2$, $\omega_1 = \omega_2/2$ and $\omega_1 = \omega_2/3$.

Compared with the classical parametric vibration models, this model can be more comprehensive and more accurately reflect the characteristics of the parametric vibration of a cable. There is a significant amount of literature on the cases where, $\omega_1 = \omega_2$ and $\omega_1 = 2\omega_2$ (Lilien and da Costa, 1994; Macdonald, et al., 2010; Takahashi et al., 2004), and little literature investigating the other parametric cases. Therefore, in this paper, the case in which, $\omega_1 = \omega_2/2$, will be investigated first in terms of nonlinear response amd parametric vibration.

To describe the closeness of the value ω_1 to the value of half of ω_2, a detuning parameter σ is defined by

$$\omega_1 = \frac{1}{2}\omega_2 + \varepsilon\sigma \tag{12}$$

Substituting Eq. (12) into Eq. (11) and eliminating the secular terms yields

$$
\begin{aligned}
2i\omega_2 D_1 A = \frac{1}{2}c_1a_{112}AQ^2 + \frac{1}{4}c_1a_{11}\exp(iT_1\sigma)^2 Q^2 - i\xi_2\omega_2 A - \frac{1}{2}b_{112}AQ^2 \\
-\frac{1}{4}b_{11}\exp(iT_1\sigma)^2 Q^2 - 3b_{222}A^2\overline{A}
\end{aligned}
\tag{13}
$$

Then, expressing the function $A(T_1)$ in the polar form as follows

$$A = \frac{1}{2}B(T_1)\exp(i\varphi(T_1)) \tag{14}$$

by substituting this transformation into Eq. (13), and separating the resulting real and imaginary parts followed, the following polar form of the modulation equations can be obtained

$$\frac{d}{dT_1}B(T_1) = \frac{1}{4}\frac{(c_1a_{11} - b_{11})\sin(\Phi(T_1))Q^2 - 2\xi_2\omega_2 B(T_1)}{\omega_2} \tag{15a}$$

$$B(T_1)\frac{d}{dT_1}\Phi(T_1) = \frac{1}{8\omega_2}\left[\begin{array}{l} 2(c_1 a_{11}\,\mathrm{c} - b_{11})\cos(\Phi(T_1))Q^2 + 2(c_1 a_{112} - b_{112})B(T_1)Q^2 \\ -3b_{222}B(T_1)^3 + 16\sigma\omega_2 B(T_1) \end{array}\right] \quad (15b)$$

where $\Phi(T_1) = -\varphi(T_1) + 2\sigma T_1$.

3.2 Stability of motion

Due to the parametric excitation and nonlinear coupling terms in the equations of the cable's motion, the response may be multi valued for the same force amplitude and frequency. Therefore, it is necessary to examine the stability of the steady state solutions because these solutions are not all stable. Consider $B(T_1)$ and $\Phi(T_1)$ in the following forms

$$B(T_1) = B_0(T_1) + B_1 \quad (16a)$$

$$\Phi(T_1) = \Phi_0(T_1) + \Phi_1 \quad (16b)$$

Here, subscript 1 denotes the steady-state responses and subscript 0 denotes small perturbations of the steady-state solution. Substituting Eq. (16) into Eq. (14) and applying the solvability condition (Nayfeh and Pai, 2004), the eigenvalues are then given by the solution of the characteristic equation

$$\lambda^2 + \frac{1}{2}\frac{(B_1+1)\omega_2^2\xi_2\lambda}{\omega_2^2} + \Gamma = 0 \quad (17)$$

in which

$$\begin{aligned}\Gamma = \frac{1}{64}\frac{1}{\omega_2^2}(&256\sigma^2\omega_2^2 B_1 + 16\omega_2^2\xi_2^2 B_1 - 64\sigma\omega_2 b_{112}B_1Q^2 + 64\sigma\omega_2 c_1 a_{112}B_1Q^2 + 4b_{112}^2B_1Q^4 \\ &+ 4c_1^2a_{112}^2B_1Q^4 - 8c_1a_{112}b_{112}B_1Q^4 - 192\sigma\omega_2 b_{222}B_1^3 + 24b_{112}b_{222}B_1^3Q^2 \\ &- 24c_1a_{112}b_{222}B_1^3Q^2 + 27b_{222}^2B_1^5)\end{aligned} \quad (18)$$

On the stability boundary, the larger real parts of the two eigenvalues are zero, hence, the stability boundary is given by

$$\sigma_1 = 1 + \frac{1}{16}\frac{-2c_1a_{112}Q^2 + 2b_{112}Q^2 + 6b_{222}B_1^2 + \sqrt{9b_{222}^2B_1^4 - 16\omega_2^2\xi_2^2}}{\omega_2} \quad (19a)$$

$$\sigma_2 = 1 + \frac{1}{16}\frac{-2c_1a_{112}Q^2 + 2b_{112}Q^2 + 6b_{222}B_1^2 - \sqrt{9b_{222}^2B_1^4 - 16\omega_2^2\xi_2^2}}{\omega_2} \quad (19b)$$

4. NONLINEAR RESPONSE ANALYSIS

In this section, the nonlinear responses of the model are investigated and the results are presented graphically in Figs. 2-4. Because the parameters of beam and cable in the model both have effects on the response of the cable, we first chose the dimensional parameters, ρ = 0.01, χ = 0.02, μ = 1000, v = 0.03 and $\theta = \pi/3$, as a basic case. Then, based on this case, the effects of the mass ratio, stiffness ratio, and inclined angle are investigated, respectively.

Fig. 2 shows the effect of the mass ratio both on the frequency-response and the unstable region of the cable as a function of the detuning parameter σ in the neighbourhood of the parametric resonance ($\omega_1 = \omega_2/2$) with $Q = 0.01$. It is shown that the frequency response always exhibits hardening with an increase in the mass ratio, while the handing trend and unstable region are not always increase monotonically. Referring to Fig. 2(a), the frequency response initially tend to soften when the mass ratio increases from 0.002 to 0.004, tends to harden when the mass ratio increases from 0.004 to 0.007, and shifts to softening again when the mass ratio increases from 0.007 to 0.01. Fig. 2(b) shows that there is a similar change for the effect of the mass ratio on the unstable region. The width of the unstable region of different mass ratios are arranged as $\rho = 0.01$, $\rho = 0.004$, $\rho = 0.002$ and $\rho = 0.007$. The phenomena indicate that the cable in the cable-supported structure is prone to parametric vibration when there is a larger mass ratio between the cable and supported member.

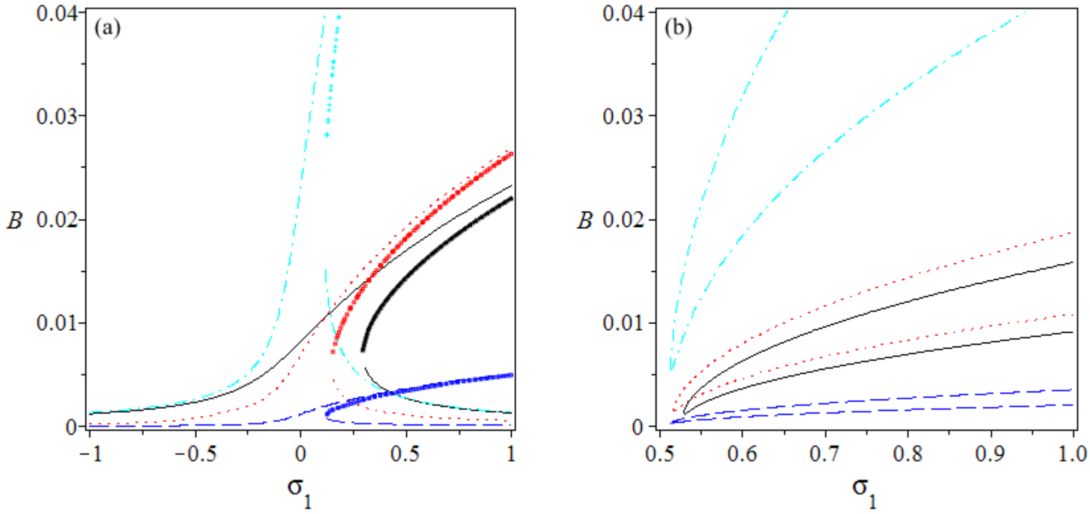

Fig. 2 Effects of the mass ratio on the nonlinear response of cable: (a) frequency-amplitude, (b) unstable region, (—— $\rho = 0.002$, ········ $\rho = 0.004$, - - - - $\rho = 0.007$, ·-·-·- $\rho = 0.01$).

Fig. 3 shows the effect of the stiffness ratio on both the frequency-response and the unstable region of the cable with $Q = 0.01$. It is shown that the stiffness ratio has a more significant effect compared with that described in Fig. 2. As seen in Fig. 3(a), the frequency response always tend to soften with the stiffness ratio under a value of 0.02, but tends to harden when the stiffness ratio between the value 0.02 and 0.2. As seen in Fig. 3(b), the cable under parametric excitation becomes unstable at a larger disturbing frequency with a small stiffness ratio, i.e. $\chi = 0.002$. When the stiffness ratio increases, the unstable region is initially shifted left in parallel along the frequency ratio axis, and then shifted up in parallel along the amplitude axis, at lastly, shifted down. Furthermore, the width of the unstable regions of varying stiffness ratios are arranged as $\chi = 0.02$, $\chi = 0.01$, $\chi = 0.002$ and $\chi = 0.2$. This phenomenon indicates that a suitable stiffness ratio can easily elicit parametric vibration in the cable up to a certain point; ratios higher than the optimum value prevent such behaviour.

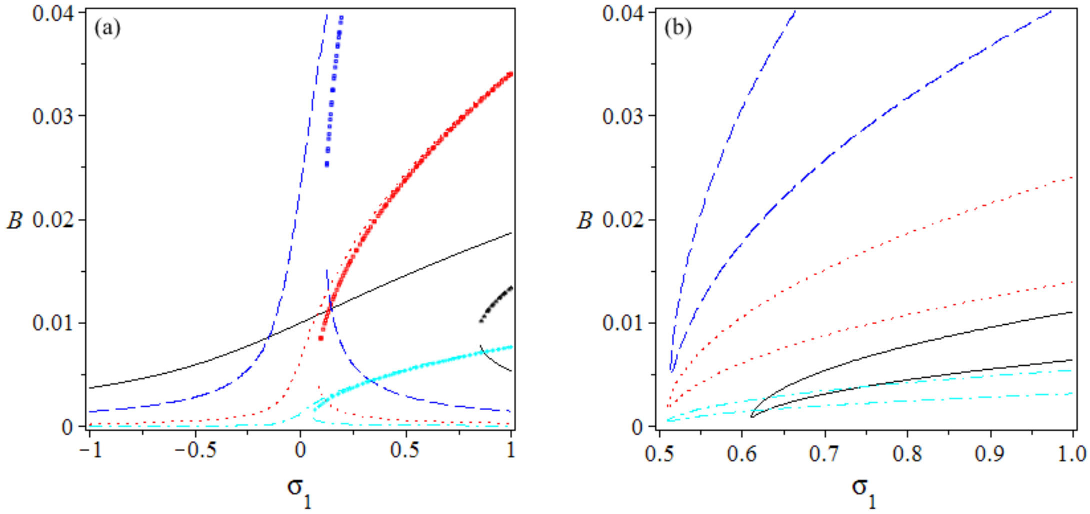

Fig. 3 Effects of the stiffness ratio on the nonlinear response of cable:
(a) frequency-amplitude, (b) unstable region,
(—— χ = 0.002, ········ χ = 0.01, - - - - χ = 0.02, -·-·-· χ = 0.2).

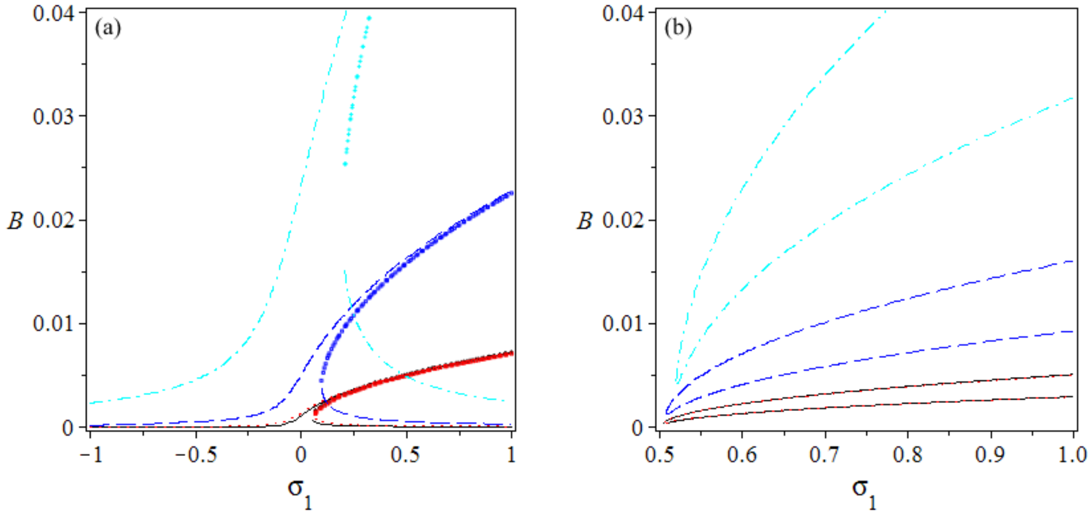

Fig. 4 Effects of the varying angle on the nonlinear response of cable:
(a) frequency-amplitude, (b) unstable region,
(—— θ = π/6, ········ θ = π/5, - - - - θ = π/4, -·-·-· θ = π/3).

The effect of the inclined angle on both the frequency-response and unstable region of the cable are shown in Fig. 4. Contrary to the effects of the first two parameters, the effect of inclined angle monotonically increases as the inclined angle increases. However, there is only a small change when the inclined angle increases from π/6 to π/5, both on the frequency-response and unstable region.

5. PARAMETRIC VIBRATION ANALYSIS

In this section, applying the fourth order Runge-Kutta method to the first-order Galerkin model (Eq. (6)), the transient nonlinear parametric vibrations of the model subjected to the parametric excitation are investigated and the effects of the mass ratio, stiffness ratio, and inclined angle, which are all related to the support member, are evaluated. Note that the time series data of the first 3800 solutions of the model are deliberately excluded from the transient nonlinear parametric vibration investigation to ensure that the data used corresponded to the steady state.

In the process of parametric vibrations, the parameters of the cable-beam model can be designed across a wide range of technical values. In order to determine their effects on the vibration of the cable, the relationship between the dynamic response and the parametric excitation is defined as the dynamic amplification factor

$$f = \frac{V_{d,\max}}{Q} \tag{20}$$

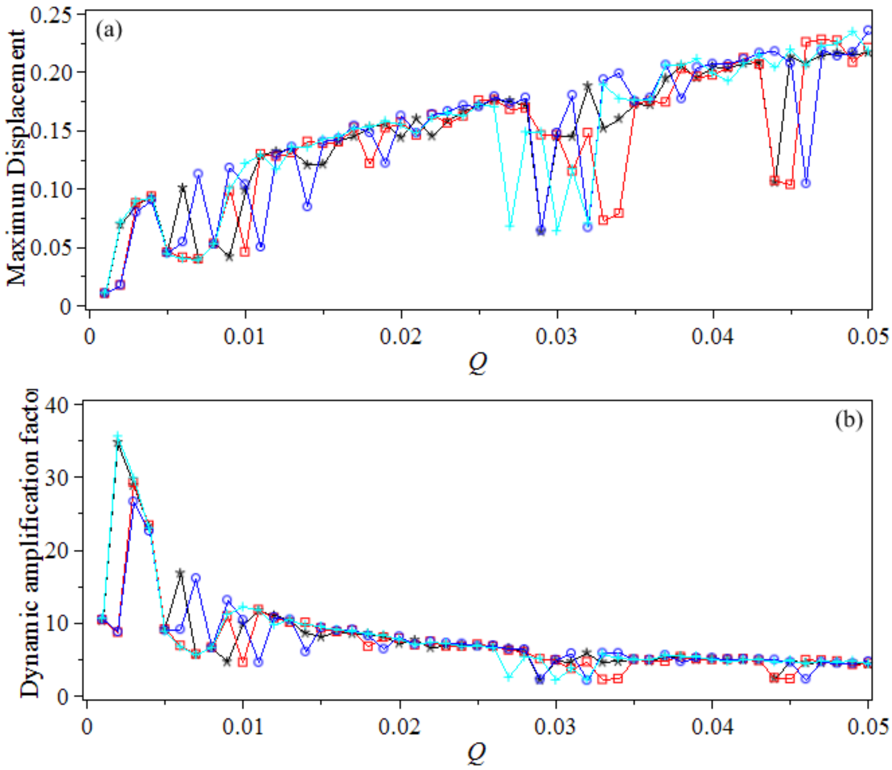

Fig. 5 Effects of the mass ratio on the nonlinear parametric vibration of the cable: (a) maximum mid-span displacement, (b) dynamic amplification factor, (ρ = 0.002, ρ = 0.004, ρ = 0.007, ρ = 0.01).

Fig. 5 shows the effects of the mass ratio on the parametric vibration of the cable versus the parametric excitation Q. It shows that there are some effects on the maximal mid-span displacement and dynamic amplification factor of the cable when the mass ratio increases. As seen in Fig. 5(a), the maximal mid-span displacements are not always increased as the parametric excitation Q increases, and some ranges have large fluctuations, i.e. $0.004 \le Q \le 0.012$, $0.026 \le Q \le 0.035$, and $0.043 \le Q \le 0.047$, in which the effect of the mass ratio may have led to a reduction in the maximal mid-span displacement. As expected, there are also similar changes in the dynamic amplification factor, which is shown in Fig. 5(b). Referring to the figure, the effect of the mass ratio changed as the parametric excitation Q increases. If not in the ranges of the large fluctuations, the effect can be neglected as expected. It is interesting to note that the larger dynamic amplification factor always occurs with a smaller parametric excitation Q, i.e. $Q \leq 0.005$, no matter how the mass ratio changes.

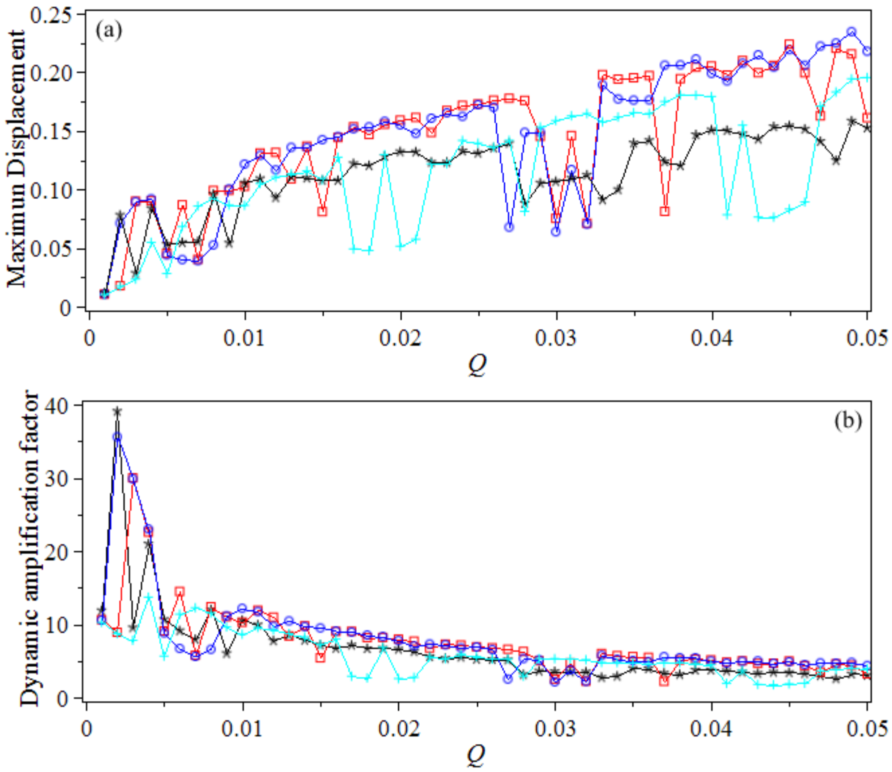

Fig. 6 Effects of the stiffness ratio on the nonlinear parametric vibration of the cable: (a) maximal mid-span displacement, (b) dynamic amplification factor, (—☆— χ = 0.002, ····□···· χ = 0.01, - -○- - χ = 0.02, -·-○-·- χ = 0.2).

The parametric vibrations of the cable versus the parametric excitation Q for the various stiffness ratios are shown in Fig. 6. With the increasing stiffness ratio, the maximal mid-span displacement is significantly increased along the parametric excitation Q axis, in addition to the value χ = 0.2, where the effect on the maximal mid-span displacement is reduced and even less than χ = 0.002 at some ranges of Q. On the other hand, the maximal dynamic amplification factor f, as seen in Fig. 6(b), is arranged as 39.12, 35.59, 29.86 and 13.69 for χ = 0.002, χ = 0.02, χ = 0.01 and χ = 0.2, respectively. The phenomenon indicates that the cable with a smaller stiffness ratio in the cable-support structure under parametric excitation can easily produce larger amplitude.

The effect of the inclined angle on the parametric vibration of the cable is shown in Fig. 7. As seen in Fig. 7(a), the increasing inclined angle led to an increase in the maximal mid-span displacement, in addition to adding some fluctuation ranges. Also, for the larger values of the parametric excitation, the sensitivity of the displacement to this type of change increases. As seen in Fig. 7(b), the maximal dynamic amplification factor increases as the inclined angle increase. However, the difference between the θ = π/4 and θ = π/3 can be neglected when the parametric excitation Q is greater than 0.01.

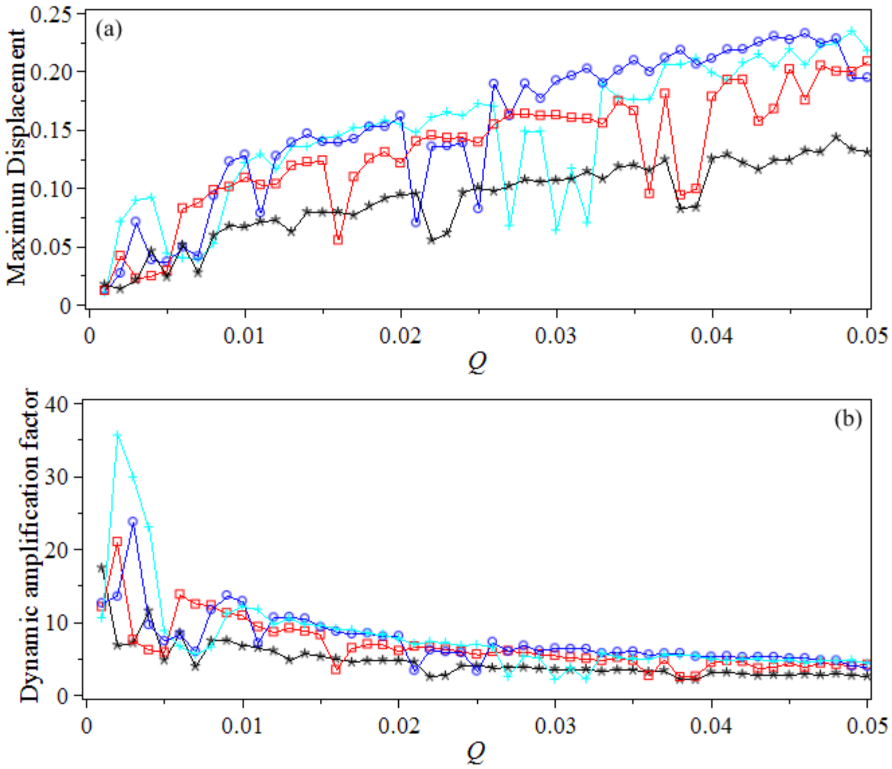

Fig. 7 Effects of the inclined angle on the nonlinear parametric vibration of the cable: (a) maximal mid-span displacement, (b) dynamic amplification factor, (—☆— $\theta = \pi/6$, ····□···· $\theta = \pi/5$, --○-- $\theta = \pi/4$, -·✚·- $\theta = \pi/3$).

6. CONCLUSIONS

A coupled cable-beam model describing the effects of the supported member on the nonlinear parametric vibration of a cable has been presented. Assuming that the modal amplitude of the beam is a cosine function, the independent first-order mode Galerkin model of the cable is obtained. Then, by directly applying the multiple scales method and using the first-order approximation to analyze the model, the results show that there are a variety of parametric resonance cases, i.e. $\omega_1 = 3\omega_2$, $\omega_1 = 2\omega_2$, $\omega_1 = \omega_2$, $\omega_1 = \omega_2/2$ and $\omega_1 = \omega_2/3$. Lastly, in order to investigate the effects of the parameters related to the beam on the nonlinear parametric vibration of the cable, the case of $\omega_1 = \omega_2/2$, which has never be studied is analyzed by the two aspects of the nonlinear response and parametric vibration.

In the nonlinear response analysis, the frequency response and stability conditions of the cable are obtained. The nonlinear response analysis shows that the frequency response always exhibits hardening. As the mass ratio increases, the handing tendency of the frequency response and the unstable region are not always increase monotonically. As the stiffness ratio increases to a certain critical value, the frequency response and the unstable region tends to soften and increases, respectively, however, when the stiffness ratio exceeds the critical value, the response starts to decrease. As the inclined angle increases, the softening tendency of the frequency response and the unstable region increases monotonically, respectively.

In the parametric vibration analysis, the maximal mid-span displacement and the dynamic amplification factor of the cable are obtained. It is shows that the maximal mid-span displacements and dynamic amplification factor do not always increases with an increase in parametric excitation increasing, with some ranges exhibiting large fluctuations. As the mass ratio increases, the effect on the nonlinear parametric vibration of the cable changes, as it does with the increase in parametric excitation, but the change is not very obvious. As the stiffness ratio increases to a certain critical value, the maximal mid-span displacements increases significantly along the parametric excitation axis, however, when the stiffness ratio exceeds the critical value, the effect on the maximal mid-span displacement reduced. As the inclined angle increases, the maximal mid-span displacement also increases. In addition, for the larger value of the parametric excitation, the sensitivity of the displacement to this type of change increases. The effect on the dynamic amplification factor shows a similar way.

REFERENCES

Abdel-Ghaffer, A.M. and Nazmy, A.S. (1988), "3-D nonlinear seismic behavior of cable-stayed bridges," Journal of Structural Engineering **117**(11), 3456-3476.

Berlioz, A. and Lamarque, C.H. (2005), "A non-linear model for the dynamics of an inclined cable," *J. Sound Vib.,* **279**(3-5), 619-639.

Cai, Y. and Chen, S.S. (1994), "Dynamics of elastic cable under parametric and external resonance," *J. Eng. Mech.*, *ASCE*, **120**(8), 1786-802.

Caetano, E., Cunha, A. and Taylor, C.A. (2000), "Investigation of dynamic cable–deck interaction in a physical model of a cable-stayed bridge. Part I: modal analysis," *Earthquake Eng, Struc.,* **29**(4), 481-498.

Caetano, E., Cunha, A. and Taylor, C.A. (2000), "Investigation of dynamic cable–deck interaction in a physical model of a cable-stayed bridge. Part II: seismic response," *Earthquake Eng, Struc.,* **29**(4), 499-521.

Cactano, E., Cunha, A., Gattulli, V., et al. (2008), "Cable-deck dynamic interactions at the International Guadiana Bridge: On-site measurements and finite element modeling," *Struct. Control Hlth.,* **15**(3), 237-264.

Dacosta, A.P. (1996), "Oscillations of Bridge Stay Cables Induced by Periodic Motions of Deck and/or Towers," *J. Eng. Mech.*, *ASCE*, **122**(7), 613-622.

Fujino, Y., Warnitchai, P. and Pacheco, B.M. (1993), "An experimental and analytical study of autoparametric resonance in a 3-DOF model of cable-stayed-beam," *Nonlinear Dynam.,* **4**(2), 111-138.

Gattulli, V., Morandini, M. and Paolone, A. (2002), "A parametric analytical model for non-linear dynamics in cable-stayed beam," *Earthquake Eng, Struc.,* **31**(6), 1281-1300.

Gattulli, V. and Lepidi, M. (2003), "Nonlinear interactions in the planar dynamics of cable-stayed beam," *Int. J. Solids Struct.,* **40**(18), 4729-4748.

Gattulli, V., Lepidi, M., Macdonald, J.H.G., et al. (2005), "One-to-two global-local interaction in a cable-stayed beam observed through analytical, finite element and experimental models," *Int. J. Nonlin. Mech.* **40**(4), 571-588.

Georgakis, C.T. and Taylor, C.A. (2005), "Nonlinear dynamics of cable stays. Part 1: sinusoidal cable support excitation," *J. Sound Vib.,* **281**(3-5), 537-564.

Georgakis, C.T. and Taylor, C.A. (2005), "Nonlinear dynamics of cable stays. Part 2: stochastic cable support excitation," *J. Sound Vib.,* **281**(3-5), 565-591.
Lilien, J.L. and da Costa, A.P. (1994), "Vibration Amplitudes Caused by Parametric Excitation of Cable Stayed Structures," *J. Sound Vib.,* **174**(1), 69-90.
Macdonald, J.H.G., Dietz, M.S., Neild, S.A., et al. (2010), "Generalised modal stability of inclined cables subjected to support excitations," *J. Sound Vib.,* **329**(21), 4515-4533.
Nayfeh, A.H. and Mook, D.T. (1979), "Nonlinear oscillations," New York, Wiley Publisher.
Nayfeh, A.H. and Pai, P.F. (2004), "Linear and nonlinear structural mechanics," Hoboken NJ, Wiley-Interscience.
Perkins, N.C. (1992), "Modal interactions in the non-linear response of elastic cables under parametric/external excitation," *Int. J. Nonlin. Mech.* **27**(2), 233-250.
Rega, G. (2004), "Nonlinear vibrations of suspended cables---Part I: Modeling and analysis," Applied Mechanics Reviews **57**(6), 443-478.
Rega, G. (2004), "Nonlinear vibrations of suspended cables---Part II: Deterministic phenomena," Applied Mechanics Reviews **57**(6), 479-514.
Tagata, G. (1977), "Harmonically forced, finite amplitude vibration of a string," *J. Sound Vib.,* **51**(4), 483-492.
Takahashi, K., Wu, Q. and Nakamura, S. (2004), "Non-linear response of cables subjected to periodic support excitation considering cable loosening," *J. Sound Vib.,* **271**(1-2), 453-463.
Wang, L.H. and Zhao, Y.Y. (2009), "Large amplitude motion mechanism and non-planar vibration character of stay cables subject to the support motions," *J. Sound Vib.,* **327**(1-2), 121-133.
Warnitchai, P., Fujino, Y. and Susumpow, T. (1995), "A nonlinear dynamic model for cables and its application to a cable-structure system," *J. Sound Vib.,* **187**(4), 695-712.
Wei, M.H., Xiao, Y.Q. and Liu, H.T. (2012), "Bifurcation and chaos of a cable–beam coupled system under simultaneous internal and external resonances," *Nonlinear Dynam.,* **67**(3), 1969-1984.
Zhang, W. and Tang, Y. (2002), "Global dynamics of the cable under combined parametrical and external excitations," *Int. J. Nonlin. Mech.,* **37**(3), 505-526.