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Plastic Failure of locally supported Silos with U-shaped longitudinal Stiffeners

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ABSTRACT

For practical considerations, thin-walled steel silos are often supported by a limited number of discrete equidistant supports around their circumference. In such cases, large loads are transferred to the limited number of supports, causing locally high axial compressive stress concentrations. A possible solution is to add a partial-height U-shaped longitudinal stiffener above each support. Such stiffeners create a more gradual transmission of vertical loads to the supports, increasing the maximum failure load. This paper aims to map the influence of the dimensions of such longitudinal stiffeners (i.e. the parameters of the cross-section and the height) on the failure behaviour of a thick-walled silo. All the results and the findings are based on geometrically and material nonlinear analyses - GMNA - performed with finite element software. The simulations indicate that correctly, for thick-walled silos, failure will always occur by yielding, but the location of yielding depends on the cross-section of the longitudinal stiffeners. Yielding will occur in the stiffened zone of the silo just above the supports for silos with longitudinal stiffeners with a small cross-section, whereas for stiffeners with larger cross-sections, yielding occurs in the unstiffened zone just above the terminations of the longitudinal stiffeners. In addition the stiffener's width in circumferential and radial direction have respectively an advantageous and a disadvantageous influence on the failure load. Finally, the stiffener's height only has a positive impact on the failure load when that failure occurs in the unstiffened silo wall. This can be addressed to the distribution of the supporting force over the entire circumference with higher stiffeners.

KEYWORDS: cylindrical Shell; Silo; longitudinal Stiffener; plastic; yielding

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1. INTRODUCTION

For practical reasons, a steel cylindrical silo is frequently located in elevated position in order to enable easy access beneath the cylindrical barrel (Cornelia Doerich, 2007; C. Doerich & Rotter, 2008; C. Doerich, Vanlaere, Lagae, & Rotter, 2009; Guggenberger, Greiner, & Rotter, 2004; Vanlaere, 2006). In this way, the contents of the silo can easily be discharged by gravity flow into trains, trucks, etc. Ground support can be provided by discrete equidistant column supports around the circumference. These columns can be engaged into the silo wall, or can be extended to the bottom of the silo wall. The last possibility will be addressed in this paper.

Given that such silo structures are discretely supported and are for the most part exposed to vertical load, subjecting the silo wall to axial compression, peak stresses are introduced in the silo wall above the local supports. These disadvantageous stress concentrations cause premature failure of the silo structure, either by plastic yielding, by elastic buckling, or a combination of both phenomena. In this paper, thick-walled silos are considered, which will reasonably always fail by pure plastic yielding.

A possible solution to further increase the failure load is the addition of a longitudinal U-shaped stiffener above each support. In this way, the ground reaction force is transferred more gradually into the silo wall. Consequently, the reaction force of the local supports and thus the stresses are more spread in circumferential direction, and as a result of which those disadvantageous stress concentrations occur by an increased load.

Furthermore, a base ring stiffener is used at the bottom of the barrel, and a transition ring stiffener is situated at the top of longitudinal stiffeners. These stiffeners prevent (large) out-of-roundness deformations and to a lesser extent, the axial stresses are spread more in circumferential direction.

For the purpose to obtain the maximum failure load of a specific silo, failure should occur in the silo wall above the terminations of the stringer stiffeners, and premature failure in the stiffened zone should be prevented. From this perspective, these stringer stiffeners should satisfy some essential requirements in order that the aforementioned condition is met. First, the stiffener's cross-section should be sufficiently large to prevent plastic yielding just above its base. Second, the stiffener should have a minimum moment of inertia resulting in a sufficiently small inward oriented deformation at its top since the stiffener has the tendency to undergo inward oriented deformations. Both basic conditions should be satisfied, but these will be discussed more in detail in this paper based on results of numerical simulations with the finite element package ABAQUS (Simulia).

2. GEOMETRY

The geometrical parameters of the locally supported silo, the longitudinal stiffeners, and the ring stiffeners with the corresponding symbols are illustrated in Fig. 1.

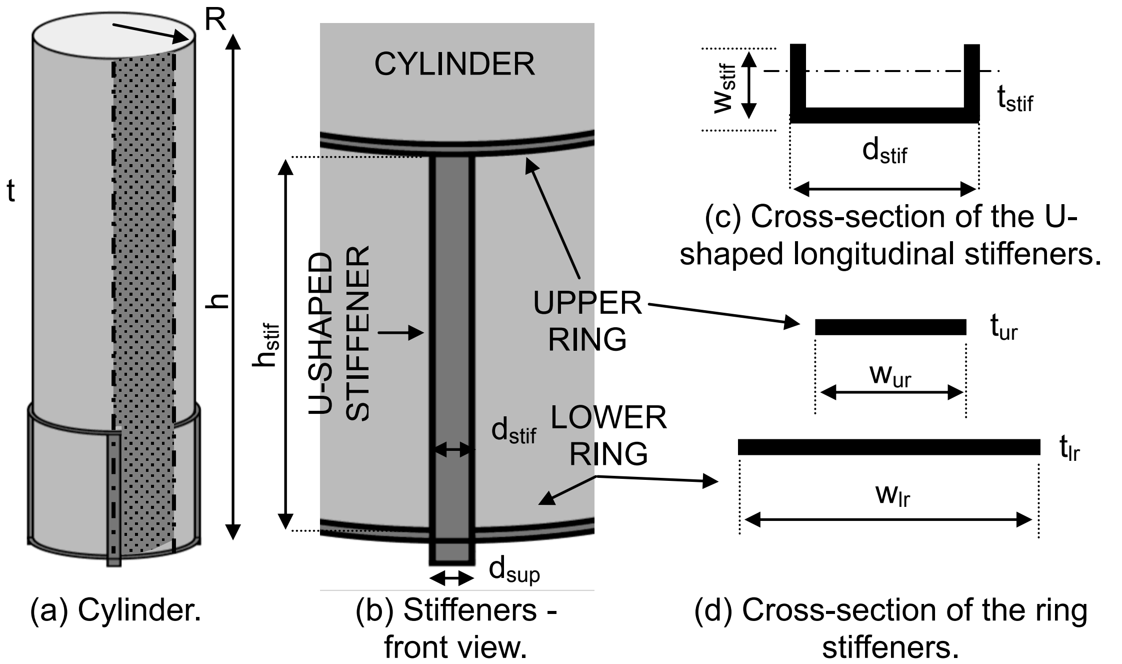


Fig. 1 Geometrical parameters of a locally supported cylinder.

2.1 Silo geometry

In Table 1, the geometrical parameters of the cylindrical barrel are given, using the symbols of Fig. 1. Only the cylinder radius R has a constant value, while all other geometrical parameters, both of the silo and the stiffeners, are relative to this radius R. In this paper, thick-walled silos are considered with a radius-to-thickness ratio R/t of 200 (i.e. relative thick silo wall). To exclude the effect of the cylinder height (Jansseune, De Corte, Vanlaere, & Van Impe, 2012), only high cylindrical barrels are considered.

Table 1 Geometrical parameters of the cylinder.

Parameter	Dimension	Value(s)
R	m	1.0
R/t	-	200
h/R	-	8.0

2.2 Geometry of the supports

In the current study, only discretely supported silos are considered with a limited number of supports. The number of supports n_{sup} is always equal to 4. These local supports are distributed over the circumference with equally spaced intervals. The ratio of the width in circumferential direction to the cylinder radius d_{sup}/R is varied between 0.05 and 0.30, corresponding with a total degree of support of respectively 3.18% and 12.7% of the circumference. In radial direction, the width of the support w_{sup} is equal to the maximum width of the stiffener $(w_{stif})_{max}$.

Table 2 Geometrical parameters of the supports.

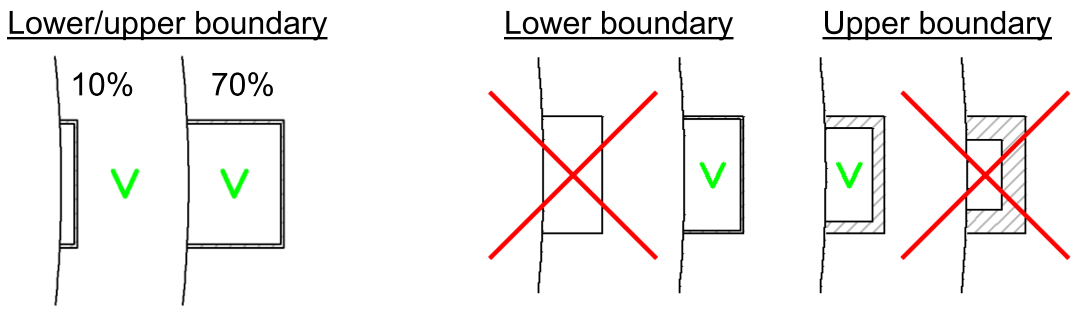
Parameter	Dimension	Value(s)
n_{sup}	-	4
d_{sup}/R	-	0.05; 0.10; 0.15; 0.20; 0.25; 0.30
w_{sup}/R	-	$(w_{stif}/R)_{max}$ = 70% · d_{stif}/R

2.3 Geometry of the longitudinal stiffeners

Above each support, a U-shaped longitudinal stiffener is placed with an equal width in circumferential direction d_{stif} as the width of the support d_{sup}. The width in radial direction w_{stif} is varied between 10% and 70% of the width in circumferential direction d_{stif} (see Fig. 2 (a)). The thickness of the stiffener should fulfil two conditions. Firstly, a maximum value of the ratio of the thickness of the stiffener t_{stif} to the thickness t of the silo is imposed (Eq. (3)), because of the necessity to weld the stiffener to the silo wall. Secondly, the stiffener may not be too thin or too thick compared to its circumferential width d_{stif} (Eq. (4)). The latter, based on local compression induced by local buckling considerations (Standardisation, 2005), is presented in Fig. 2 (b). The height of the longitudinal stiffeners h_{stif} is varied between 0.5 and 2.0 times the cylinder radius R. Table 3 gives an overview of all geometrical parameters of the longitudinal stiffeners, including the imposed restrictions.

Table 3 Geometrical parameters of the U-shaped stiffeners.

Parameter	Dimension	Value(s)	
d_{stif}/R	-	d_{sup}/R	
w_{stif}/R	-	$10\% \cdot d_{stif} \leq w_{stif} \leq 70\% \cdot d_{stif}$ $0.02 \cdot R \leq w_{stif}$	(1) (2)
t_{stif}/t	-	$1.0 \leq \frac{t_{stif}}{t} \leq 5.0$ $10 \leq \frac{d_{stif}}{t_{stif}} \leq 40$	(3) (4)
h_{stif}/R	-	0.50; 1.00; 1.50; 2.00	



(a) Width of the stiffener in radial direction w_{stif} (Eq. (1)).

(b) Stiffener's thickness t_{stif} (Eq. (4)).

Fig. 2 Graphical representation of the restrictions to the dimensions of the stringer stiffeners.

3. NUMERICAL MODEL

The commercial finite element package ABAQUS (Simulia) was used to perform numerical analyses in order to map the structural behaviour of such locally supported silo with U-shaped longitudinal stiffeners. The Riks algorithm (Systèmes) was used to obtain the maximum load and to have the possibility to examine the behaviour before, during and after failure. To reduce the size of the computations, symmetry conditions were exploited down the longitudinal edges through the centre of the support and the midplane between the supports. In this way, only a segment of 45° (grey shaded area in Fig. 1) was modelled (see Fig. 3). The results of one eighth of the shell were verified with results of the model of a complete shell (Jansseune, et al., 2012). The reader should notice that the figures in this paper were obtained by mirroring the 45 degree segment in circumferential direction. The connection of the conical roof at the top of the barrel has been replaced by boundary conditions preventing out-of-round deformations.

At the bottom of the barrel, it is the lower ring which brings about a reduction of the deformations in a horizontal plane. A uniform axial load is applied at the upper edge of the silo, subjecting the silo wall to (vertical) compression (Nielsen, 2008; Rotter, 2004, 2009).

On the one hand, shell elements (S8R5 - a 8-node doubly curved shell element) were used for the silo wall, the longitudinal stiffener, and the ring stiffeners (Systèmes). On the other hand, volume elements (C3D20R) were used for the local supports.

In this paper, the results of GMNA analyses (Standardisation, 2007a) are used, including geometric and material nonlinearity, but without imperfections. The material behaviour for all the parts of the model consists of an elastic perfectly plastic material behaviour (without strain hardening) with a Young's modulus E of 210 GPa, a Poisson's ratio v of 0.3, and a yield stress f_y of 235 MPa (ECCS, 2008; Standardisation, 2007b).

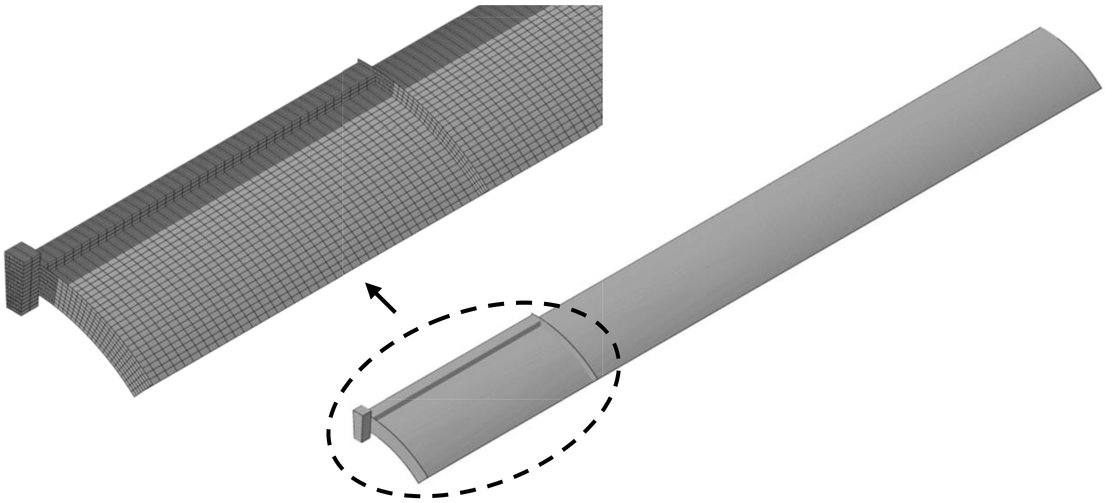


Fig. 3 Numerical model (45 degrees).

4. RESULTS AND DISCUSSION

The failure phenomenon to be feared in thick-walled silos is always plastic yielding, irrespective of the dimensions of the longitudinal stiffeners. However, as will be demonstrated in this paper, the dimensions of the longitudinal stiffeners play a major role in the structural behaviour. Indeed, the failure load and the location of the plastic yielding region is highly dependent on how much and where exactly material is added to the stringer stiffeners.

In the following, the dimensions of the stringer stiffeners will be discussed one by one. The parameters of its cross-section are discussed in Section 4.1, while the stringer's height is considered in Section 0.

4.1 Influence of the parameters of the longitudinal U-shaped stiffener's cross-section

The cross-section   In Fig. 4, the failure load of the silo structure is plotted against the ratio of the stiffener's cross-section to the cross-section of the silo wall (for a constant stiffener's height). Obviously, the cross-section of the stringer stiffeners has a significant impact on the maximum load of such a silo. In general, the curve consists of a rapidly increasing branch followed by a slowly rising branch. In what follows, these branches will considered separately.

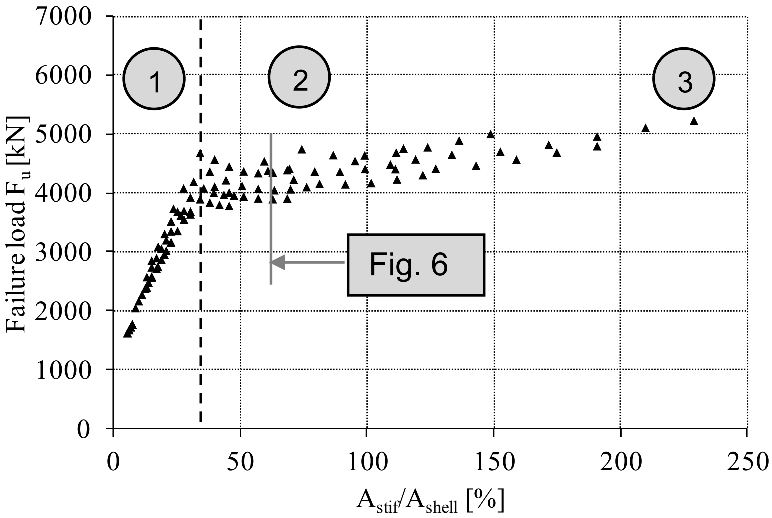
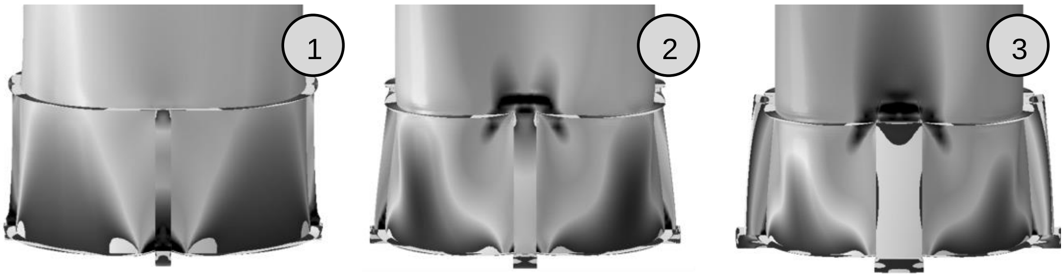


Fig. 4 Failure load F_u in function of the ratio of the cross-section of the longitudinal stiffeners to the cross-section of the silo wall (h_{stif}/R = 1.0).

In the first branch (i.e. the rapidly rising curve in Fig. 4), the failure load F_u increases consistently when adding material to the stiffener's cross-section. This can be attributed to the location where yielding occurs. Indeed, it turns out that the silo wall and the longitudinal stiffeners just above the discrete supports fail by plastic yielding, while the axial stresses in the silo wall above the stiffeners are below the yielding stress (see Fig. 5 (a)). In other words, the material in the stiffened region is completely exhausted, while the material of the cylindrical barrel (in the unstiffened zone) is not yet fully utilized. In conclusion, a small increase of the stiffener's cross-section means a rapid increase of the failure load due to the fact that more material can yield in the stiffened zone above the supports.

However, as more and more material is added to the stiffeners, there is a certain turning point at which the plastic yielding region moves to the silo wall above the terminations of the U-shaped stringer stiffener (see Fig. 5 (b)). From this turning point, which corresponds with the beginning of the second curve in Fig. 4, the maximum load continues to increase less rapidly due two reasons. When comparing Fig. 5 (b) with Fig. 5 (c), the axial stresses are slightly better distributed in circumferential direction, and the yielding region is extended in circumferential direction. Both can be attributed to, for example, an increase of the width of the support d_{sup} (and stiffener) in circumferential direction. The influence of the width in circumferential direction will discussed further in detail.



(a) Small cross-section. (b) Average cross-section. (c) Large cross-section.

Fig. 5 Contourplot of the axial stresses at the moment of buckling (yielding corresponds with the black regions).

<u>The optimum cross-section</u> For the stringer stiffener's with a small cross-section A_{stif} (i.e. first branch in Fig. 4), all dimenions of the cross-section have an advantegeous effect on the failure load because of the increase in material that can yield in the stiffened zone just above the supports.

However, the situation for a stringer stiffener with an average or large cross-section A_{stif} (i.e. the second branch in Fig. 4) is slightly different. In Fig. 6, the failure load is depicted for a range of geometries of the longitudinal stiffener with a constant cross-section in order to determine its optimal dimensions. For a constant cross-section (A_{stif}/A_{shell} = 60%) and a constant number of supports ($n_{stif} = n_{sup} = 4$), the total length of the stiffener L_{stif} and its thickness t_{stif} are calculated using Eqs. (5)-(6). Furthermore, each total length of the stiffener L_{stif} can be divided in several combinations of the width in circumferential direction d_{stif} and the width in radial direction w_{stif} (see Eq. (7)).

$$A_{stif} = \mathrm{n}_{\mathrm{stif}} \cdot L_{stif} \cdot \mathrm{t}_{stif} \tag{5}$$

$$L_{stif} \cdot \mathrm{t}_{stif} = \left(\frac{A_{stif}}{A_{shell}}\right) \cdot \frac{A_{shell}}{n_{stif}} \tag{6}$$

$$L_{stif} = d_{stif} + 2 \cdot w_{stif} \tag{7}$$

It should be noticed that only those combinations are calculated for which the restrictions mentioned in Section 0 (Eqs. (2)-(4)) are valid.

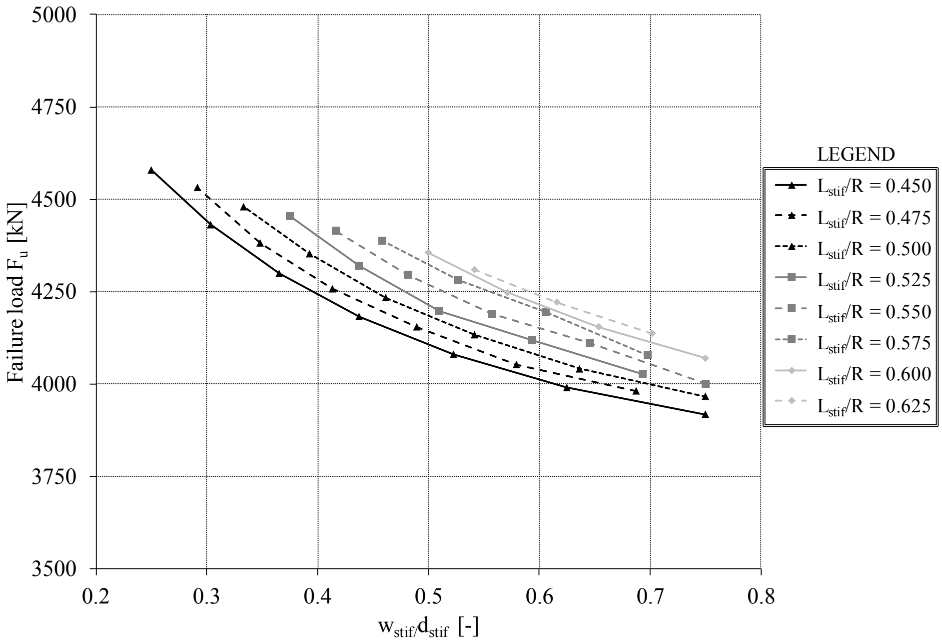
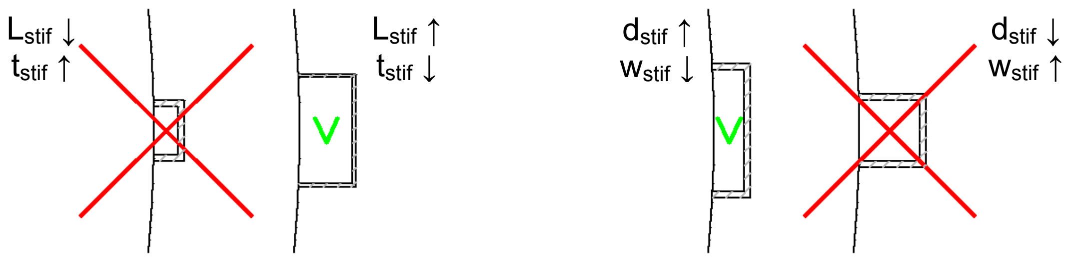


Fig. 6 Determination of the optimum dimensions for a constant quantity of material (A_{stif}/A = 60%; h_{stif}/R = 1.0).

From the above graph, consisting of parallel descending curves, one can formulate some interesting findings. First of all, the curve moves upward as the total length of the stiffener L_{stif} increases and thus the thickness t_{stif} decreases. In other words, it is better to increase the length of the U-shaped profile instead of increasing its thickness (Fig. 7 (a)). Secondly, the ratio of the width in radial direction to the width in circumferential direction should be kept as low as possible to maximize the failure load (Fig. 7 (b)).



(a) Constant cross-section, variable length and thickness.

(b) Constant cross-section, length, and thickness.

Fig. 7 Graphical representation of the conclusion of Fig. 6.

To conclude, the most efficient way to add material to the stringer stiffener's is by increasing its length, in particular the width in circumferential direction d_{stif} (see also Paragraph *The width in circumferential direction d_{stif}*). Furthermore, it is preferable not to increase the width in radial direction w_{stif} more than necessary (see further in Paragraph *The width in radial direction w_{stif}*).

<u>The width in circumferential direction d_{stif}</u> Since the silos are discretely supported, the degree of support is a very important geometrical parameter. It was chosen to give the stringer stiffener an equal width in circumferential direction as the support (or d_{stif} = d_{sup}). This corresponds with the most optimum way to maximize the failure load for a specific degree of suppor

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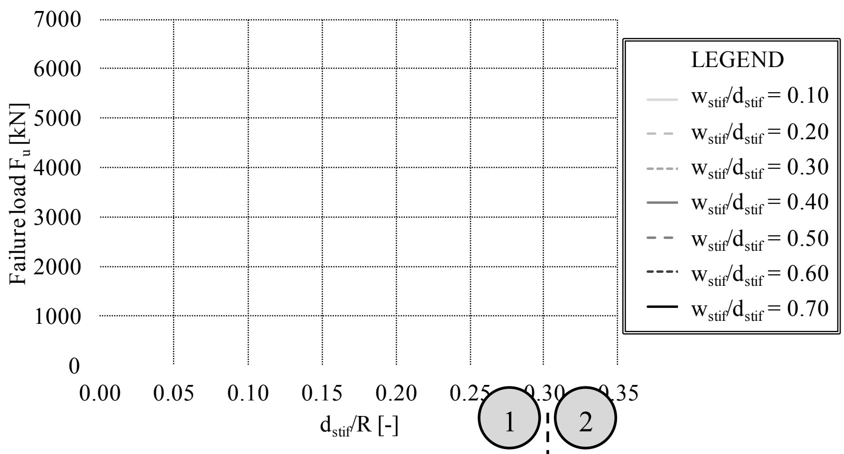
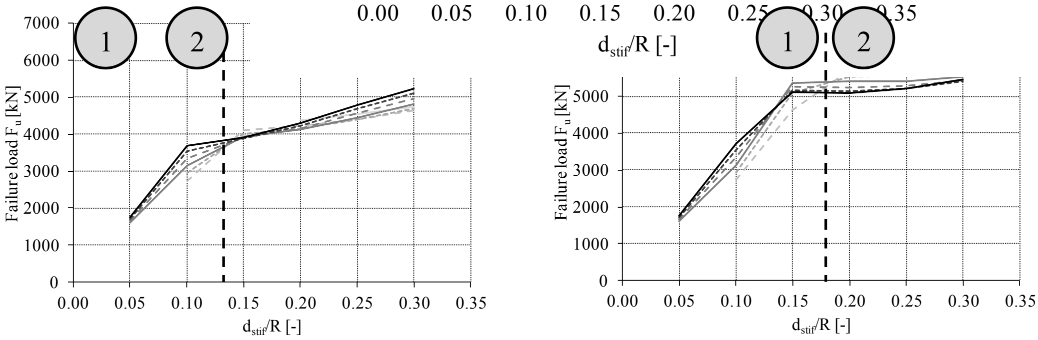




(a) Short stringer stiffener (h_{stif}/R = 1.0).

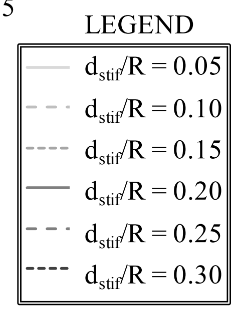
(b) High stringer stiffener (h_{stif}/R = 2.0).

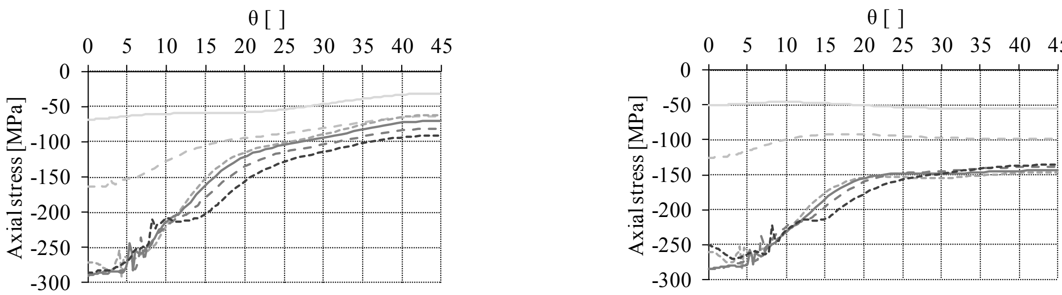
Fig. 8 Failure load F_u in function of the ratio of the width of the longitudinal stiffeners in circumferential direction to the cylinder radius.

However, from a particular width d_{stif} (i.e. black dashed line in Fig. 8), the slope of the curve reduces significantly, and thus reducing the advantegous effect of the circumferential width d_{stif} on the failure load F_u. Moreover, the area of failure moves to the unstiffened silo wall above the terminations of the U-shaped longitudinal stiffeners. As will be shown below, the effect of the width d_{stif} in the second branch largely depends on the height of stiffener h_{stif}. Indeed, by comparing the second branch of Fig. 8 (a) and Fig. 8 (b), the circumferential width d_{stif} is more favourable for shorter than for higher longitudinal stiffeners.

The above mentioned finding will be illustrated in Fig. 9. In this figure, the distribution of the axial stresses is plotted along a circumferential path in the silo wall just above the upper ring, which corresponds with the height in the unstiffened silo wall where yielding occurs. The circumferential angle θ is between 0° and 45°, which corresponds respectively with the vertical line through the centre of the support and the midplane between two supports. Furthermore, compressive stresses are negative.

Clearly, the width d_{stif} has a significant advantegous effect on the failure load in the case of shorter longitudinal stiffeners (Fig. 8 (a)). This can be addressed to the fact that, within the limited height of the stiffened zone, the axial stresses could not be maximal spread over the circumference of the silo wall. By increasing the circumferential width d_{stif}, the circumferential distance/angle over which the stresses should be distributed is reduced, increasing the stress level between the supports (See stress level between the supports in Fig. 9 (a)). In other words, the stresses are spread more quickly over the circumferential for a constant small stiffener height. In contrast, for silos with higher longitudinal stiffener's (Fig. 8 (b)), this beneficial effect does not occur, because the stresses can perfectly be spread in circumferential direction within the height of the stiffened zone (See stress level between the supports in Fig. 9 (b)). In this case, an additional width in circumferential direction is no longer necessary.





(a) Short stringer stiffener ($h_{stif}/R = 1.0$).

(b) High stringer stiffener ($h_{stif}/R = 2.0$).

Fig. 9 Axial stress distribution along a circumferential path just above the upper ring.

Independent of the stringer's height, a second but less pronounced effect of the width d_{stif} on the maximum load can be observed. By increasing the circumferential width d_{stif}, the plastic yielding zone is slighty extended in circumferential direction. This is negligible with respect to the first-mentioned finding.

The width in radial direction w_{stif} In Fig. 8, the width of the longitudinal stiffener in radial direction w_{stif} is varied between 10 and 70 percent of the width in circumferential direction d_{stif}. The limited spread of the curves in Fig. 8 shows that the radial width w_{stif} has a less important effect on the failure load F_u (especially compared to the circumferential width d_{stif}).

If plastic yielding occurs in the stiffened zone just above the local supports (i.e. the first branch in Fig. 8 (a) and (b)), the radial width w_{stif} has an advantageous effect, since in that case, more material can yield before the silo fail by plastic yielding in the stiffened region of the silo just.

For silos which fail by plastic yielding in the unstiffened silo wall (i.e. the second branch in Fig. 8 (a) and (b)), the influence of the radial width w_{stif} depends on the cross-section (d_{stif}) and the height h_{stif} of the stiffener. Indeed, the radial width w_{stif} is slightly beneficial for short stringer stiffeners with a small cross-section, but in contrast, disadvantageous for higher longitudinal stiffeners with a large cross-section. In what follows, the latter unfavourable effect will be explained.

In Fig. 10, the centre of gravity of the support reaction force, with respect to the silo wall, is plotted. A positive value corresponds with a reaction force on the exterior of the silo wall.

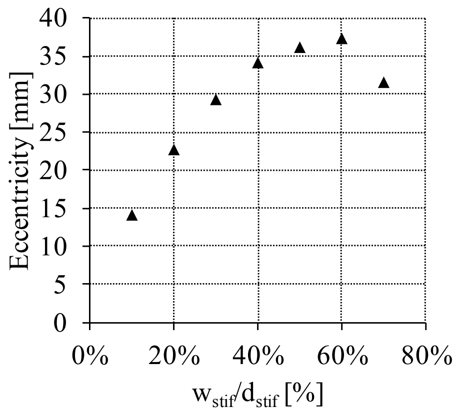
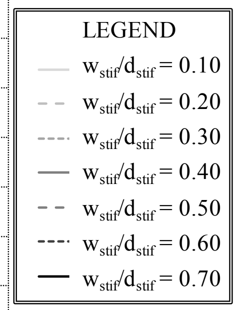
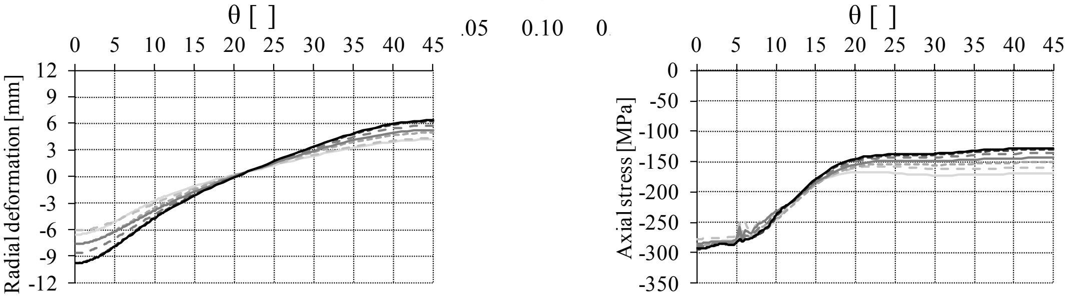


Fig. 10 Eccentricity of the support reaction force in function of the ratio of the width in radial direction w_{stif} to the width in circumferential direction d_{stif} (d_{stif}/R = 0.20; h_{stif}/R = 2.0).

For stiffeners with a small radial width w_{stif}, the eccentricity of the reaction force is limited, since all the material of the stiffener has been added in the vicinity of the silo wall. The reaction force will be largely absorbed by the web and less by the flanges of the stiffener. By increasing the radial width w_{stif}, the eccentricity of the reaction force increases (i.e. the reaction force moves in outward direction), resulting in the tendency of the longitudinal stiffener to deform more in inward direction. In Fig. 11 (a), the radial deformations are plotted are plotted for a circumferential path in the silo wall just above the upper ring. The inward oriented deformations above the terminations of the longitudinal stiffener (0° ≤ θ ≤ 22°) are increasing as the width w_{stif} (and the eccentricity) increase. Clearly, the upper ring cannot prevent these inward oriented deformations of the silo wall in its vicinity.

In addition, the increased value of the eccentricity means that the reaction force is absorbed by more distant material of the stiffener with respect to the silo wall. The flanges will absorb more force, the web less. For a stiffener with a constant height, the reaction force is thus less rapidly transferred from the stiffener to the silo wall. Indeed, the latter is confirmed by Fig. 11 (b). In this figure, the compressive stress in axial direction is plotted along a circumferential path in the silo wall just above the stiffeners. Clearly, the axial stresses in the region between the supports (15° ≤ θ ≤ 45°) are lower as the radial width w_{stif} increases. In other words, the longitudinal stiffener becomes less efficient in the transfer of the support load to the silo wall as the radial width increases, assuming that the stiffener's height remains constant.





(a) The radial deformations. (b) The axial stress distribution.

Fig. 11 The distribution of deformations and stresses along a circumferential path just above the upper ring (d_{stif}/R = 0.20; h_{stif}/R = 2.0).

4.2 Influence of the height of the longitudinal U-shaped stiffener

Fig. 12 shows the failure load for different stiffener geometries (d_{stif}/R = variable; w_{stif}/R = 0.06; t_{stif}/t = maximum according to Eqs. (3)-(4)). The horizontal axis in Fig. 12 is the height of the longitudinal stiffener. The figure shows that the stiffener's height h_{stif} has no effect to the failure load if the circumferential width d_{stif} is limited, corresponding to the situation where the silo fails by yielding in the stiffened area above the local supports. In contrast, if the silo fails by yielding in the unstiffened silo wall above the terminations of the stiffener (i.e. for stiffeners with larger circumferential width d_{stif}), the stiffener's height has a significant influence on the failure behaviour. Indeed, as the height of the stiffeners increases, the failure load increases substantially due to a more gradual transfer of the reaction force from the stiffener to the silo wall. This results in a better distribution of the axial compressive stresses over the entire circumference of the silo wall.

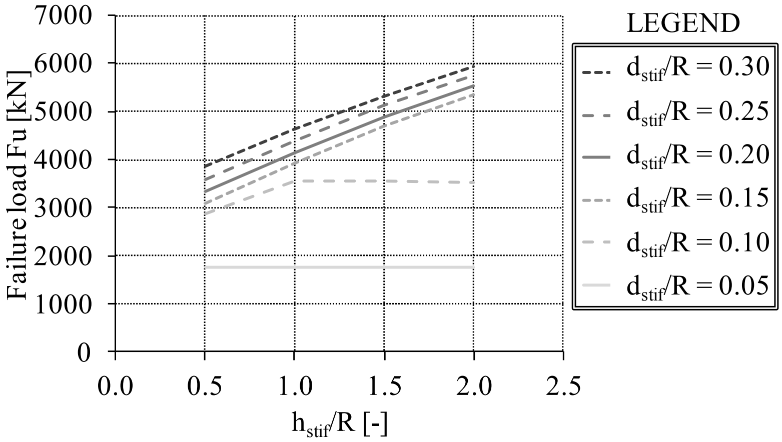


Fig. 12 Failure load F_u in function of the ratio of the height of the longitudinal stiffeners to the cylinder radius ($w_{stif}/R = 0.06$; t_{stif}/t = max.).

Fig. 13 is similar to Fig. 4 showing for a range of stiffener's heights the influence of the stiffener's cross-section to the failure load. For each stiffener's height, the curve consists of a rapidly rising curve (corresponding with yielding of the stiffened zone) and a slowly increasing/decreasing curve (corresponding with yielding of the unstiffened zone). The transition between those branches (and the failure location) moves to the right as the height of the stiffener increases. Indeed, more material should be added to the stiffener's cross-section in order that the stiffener can absorb a sufficiently high force, causing that yielding occurs in the unstiffened silo wall.

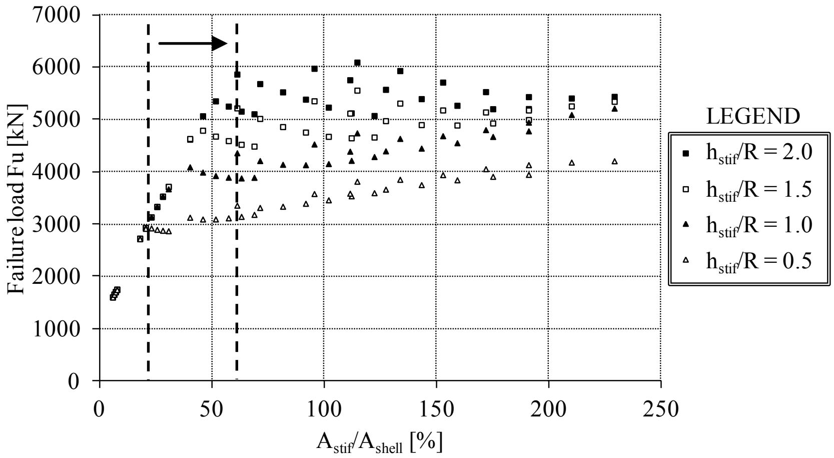


Fig. 13 Failure load F_u in function of the ratio of the cross-section of the longitudinal stiffener to the cross-section of the silo wall for different stringer's heights.

5. CONCLUSIONS

For thick-walled cylindrical silos, the failure behaviour has been explored for a wide range of U-shaped longitudinal stiffeners. The comprehensive study has been demonstrated that the dimensions of such a longitudinal stiffener have an important influence on the failure load and the failure behaviour (i.e. yielding). The following conclusions may be drawn.

If the longitudinal stiffeners have a small cross-section, plastic yielding will occur in the stiffened region just above the local supports, in the silo wall as well as in the stiffeners itself. In other words, the thick-walled silo will fail prematurely due to the fact that the stiffeners can only absorb a limited supporting load.

From a clear turning point, the region of plastic yielding will shift to the unstiffened silo wall if the longitudinal stiffeners have a large cross-section. This larger cross-section can be obtained by an increased value of either the stiffener's perimeter or the stiffener's thickness. It has been demonstrated that it is mainly the circumferential length of the U-shaped stiffener that is important, and to a lesser extent the stiffener's thickness. The most optimum way to increase the stiffener's length, is by increasing the width in circumferential direction, while the width in radial direction is slightly disadvantageous. Further, the longitudinal stiffener's height has a beneficial influence, because the stiffener will distribute the stresses better in circumferential direction. However, the latter is only valid if plastic yielding region occurs in the unstiffened region above the terminations of the stiffeners.

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