The 2013 World Congress on
Advances in Structural Engineering and Mechanics (ASEM13)
Jeju, Korea, September 8-12, 2013

Nonlinear bending analysis of laminated composite stiffened plates

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ABSTRACT

This paper deals with the nonlinear bending analysis of laminated composite stiffened plates subjected to uniform transverse loading. The eight-noded degenerated shell element and three-noded degenerated curved beam element with five degrees of freedom per node is adopted in the present analysis to model the plate and stiffeners respectively. The Green-Lagrange strain displacement relationship is adopted and the total Lagrangian approach is taken in the formulation. The convergence study of the present formulation is carried out first and the results are compared with the results published in the literature. The effect of lamination angle on the bending response of the composite stiffened plates is considered and the results are discussed.

Keywords: Degenerated shell element, Degenerated curved beam element, Nonlinear analysis, Green-Lagrange nonlinearity, Stiffened plate and Laminated composite.

1. INTRODUCTION

With the increased application of fiber reinforced composites in various fields, research on their behaviour for different structural form has also increased. Most commonly used structural forms are plates, used in aircraft, ship and automotive industries. The performance, i.e. strength/stiffness to weight ratio of the plates is enhanced by adopting suitable stiffened forms. The literature dealing with the flexural behaviour of laminated composite stiffened plates is few and mostly restricted to small deflection. At higher loads the transverse deflection of plate is large compared to its thickness. The bending and stretching coupling comes into play and the load deflection behaviour becomes non-linear. At large deflection level, membrane stresses are produced which give additional stiffness to the structure. So a large deflection analysis provides accurate responses. For composite plates, coupling effects arising from ply orientation makes the matter more complicated.

The large deflection analysis of un-stiffened composite plates has been extensively studied in the recent years (Chia 1988).

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Some authors have undertaken the work of flexural behaviour of isotropic stiffened plate. Sheikh and Mukhopadhyay(2000) have performed the geometric nonlinear analysis of isotropic stiffened plates by the spline finite strip method. von Karman's nonlinear plate theory is adopted by them and the formulation is made in total Lagrangian coordinate system. Koko and Olson(1991) used a super-element approach for the large deflection and elastoplastic analyses of orthogonally stiffened plates. The super-elements are designed to contain all the basic mode of deformation so that only one plate element per bay and one beam element per span are needed to analyse a stiffened structure, therefore reducing the storage requirement and solution time. The geometrically non-linear analysis of isotropic stiffened plates with arbitrarily oriented stiffener was reported by Rao *et. al.* (1993). The authors presented the finite element static analysis of the large deflection response of isotropic stiffened plates using an isoparametric quadratic stiffened plate bending element. The stiffened element was a development of the linear formulation presented by Mukhopadhyay and Satsangi(1984). The authors excluded the contribution of the stiffener non-linearities in their formulation.

The literatures for nonlinear bending analysis of laminated composite stiffened plates are few. Chattopadhyay *et. al.* (1995) have performed the large deflections analysis of laminated composite stiffened plates using an eight noded isoparametric element. The element formulation is based on Reissner–Mindlin's hypothesis with a total Lagrangian description of motion. The nonlinear equilibrium equations are solved by the Newton-Raphson iteration procedure. Liao and Reddy(1989) have investigated the large deflection behaviour of composite stiffened plates and shells by the finite element method. They have used degenerated three dimensional shell elements and associated curved beam elements which have been derived from the degenerated elements by imposing appropriate kinematic constraints. The incremental equations of motion developed using the principle of virtual displacement of a continuum and the total Lagrangian concept have been solved by the Newton-Raphson iteration procedure. The experimental investigation on the flexural behaviour of composite stiffened plate in the nonlinear range has been carried out by Hyer *et. al.* (1990). They have used the STAGS (Almroth and Brogan 1978) computer code to compare the experimental results with the analytical ones. STAGS models the stiffened plate as shell branches. Recently, Ojeda *et.al*. (2007) have carried out the large deflection finite element analysis of isotropic and composite plates with arbitrary orientated stiffeners.

In the present analysis, the nonlinear bending analysis of laminated stiffened plate subjected to uniform transverse loading is carried out. The eight-noded degenerated shell element and three-noded degenerated curved beam element with five degrees of freedom per node is adopted in the present analysis to model the plate and stiffeners respectively. The Green-Lagrange strain displacement relationship is adopted and the total Lagrangian approach is taken in the formulation. The resulting nonlinear equations are solved by the Newton-Raphson iteration technique along with the incremental method. The effect of lamination angle on the bending response of the composite stiffened plates is considered and the results are discussed.

2. FORMULATION

The plate skin and the stiffeners are modeled discretely. The plate is modeled with Ahmad *et. al.'s* (1970) degenerated shell element with Green-Lagrange strain displacement relationship. The element contains five degree of freedom per node. A three noded degenerated curved beam element with five degree of freedom per node is taken to model the stiffeners. As the in-plane rigidity of the plate is very high the sixth d.o.f. θ_z in the beam element is also neglected. To consider the torsional rigidity of the beam adequately, a torsional correction factor is introduced in the formulation of the stiffener. The formulation of stiffener element is also based on the formulation of Ahmad *et. al.'s* (1970) degenerated shell element. The Green-Lagrange strain displacement relationship is adopted in this case also.

2.1. PLATE ELEMENT

The strain displacement relationship with Green-Lagrange strain of the plate element in local co-ordinate system can be expressed as,

$$\{\varepsilon'\} = \begin{Bmatrix} \varepsilon_{x'} \\ \varepsilon_{y'} \\ \gamma_{x'y'} \\ \gamma_{y'z'} \\ \gamma_{x'z'} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u'}{\partial x'} + \frac{1}{2}\left(\left(\frac{\partial u'}{\partial x'}\right)^2 + \left(\frac{\partial v'}{\partial x'}\right)^2 + \left(\frac{\partial w'}{\partial x'}\right)^2\right) \\ \frac{\partial v'}{\partial y'} + \frac{1}{2}\left(\left(\frac{\partial u'}{\partial y'}\right)^2 + \left(\frac{\partial v'}{\partial y'}\right)^2 + \left(\frac{\partial w'}{\partial y'}\right)^2\right) \\ \frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'} + \left(\frac{\partial u'}{\partial x'} \times \frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'} \times \frac{\partial v'}{\partial y'} + \frac{\partial w'}{\partial x'} \times \frac{\partial w'}{\partial y'}\right) \\ \frac{\partial v'}{\partial z'} + \frac{\partial w'}{\partial y'} + \left(\frac{\partial u'}{\partial y'} \times \frac{\partial u'}{\partial z'} + \frac{\partial v'}{\partial y'} \times \frac{\partial v'}{\partial z'} + \frac{\partial w'}{\partial y'} \times \frac{\partial w'}{\partial z'}\right) \\ \frac{\partial u'}{\partial z'} + \frac{\partial w'}{\partial x'} + \left(\frac{\partial u'}{\partial x'} \times \frac{\partial u'}{\partial z'} + \frac{\partial v'}{\partial x'} \times \frac{\partial v'}{\partial z'} + \frac{\partial w'}{\partial x'} \times \frac{\partial w'}{\partial z'}\right) \end{Bmatrix}$$

$$
= \begin{Bmatrix} \frac{\partial u'}{\partial x'} \\ \frac{\partial v'}{\partial y'} \\ \frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'} \\ \frac{\partial v'}{\partial z'} + \frac{\partial w'}{\partial y'} \\ \frac{\partial u'}{\partial z'} + \frac{\partial w'}{\partial x'} \end{Bmatrix} + \begin{Bmatrix} \frac{1}{2}\left(\left(\frac{\partial u'}{\partial x'}\right)^2 + \left(\frac{\partial v'}{\partial x'}\right)^2 + \left(\frac{\partial w'}{\partial x'}\right)^2\right) \\ \frac{1}{2}\left(\left(\frac{\partial u'}{\partial y'}\right)^2 + \left(\frac{\partial v'}{\partial y'}\right)^2 + \left(\frac{\partial w'}{\partial y'}\right)^2\right) \\ \left(\frac{\partial u'}{\partial x'} \times \frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'} \times \frac{\partial v'}{\partial y'} + \frac{\partial w'}{\partial x'} \times \frac{\partial w'}{\partial y'}\right) \\ \left(\frac{\partial u'}{\partial y'} \times \frac{\partial u'}{\partial z'} + \frac{\partial v'}{\partial y'} \times \frac{\partial v'}{\partial z'} + \frac{\partial w'}{\partial y'} \times \frac{\partial w'}{\partial z'}\right) \\ \left(\frac{\partial u'}{\partial x'} \times \frac{\partial u'}{\partial z'} + \frac{\partial v'}{\partial x'} \times \frac{\partial v'}{\partial z'} + \frac{\partial w'}{\partial x'} \times \frac{\partial w'}{\partial z'}\right) \end{Bmatrix} = \{\varepsilon'_0\} + \{\varepsilon'_{nl}\}
$$

$$
= [B'_0]\,\{\delta\} + \frac{1}{2}\,[A]\,[B'_G]\,\{\delta\}
$$

$$
= [B'_0]\,\{\delta\} + \frac{1}{2}\,[B'_{nl}]\,\{\delta\}
$$

Where, $\{\delta\}$ is the nodal displacement vector in global co-ordinate system and $[B_0]$ and $[B_{nl}]$ are strain-displacement matrices with respect to linear and nonlinear strain components respectively in local co-ordinate system. The normal strain $\varepsilon_{z'}$ along z' direction is neglected. After finding these two matrices the secant stiffness matrix can be expressed as,

$$
\begin{aligned}
[k]_S = & \int [B'_0]^T\,[D']\,[B'_0]\,dV \\
& + \frac{1}{2}\int [B'_0]^T\,[D']\,[B'_{nl}]\,dV + \int [B'_{nl}]^T\,[D']\,[B'_0]\,dV + \frac{1}{2}\int [B'_{nl}]^T\,[D']\,[B'_{nl}]\,dV
\end{aligned}
$$

This secant stiffness matrix is not symmetric. To efficiently use the storage scheme which is used in linear analysis, this non symmetric matrix scant stiffness matrix can be made symmetric(Wood and Schrefler 1978) as,

$$
\begin{aligned}
[k]_S = & \int [B'_0]^T\,[D']\,[B'_0]\,dV \\
& + \frac{1}{2}\int [B'_0]^T\,[D']\,[B'_{nl}]\,dV \\
& + \frac{1}{2}\int [B'_{nl}]^T\,[D']\,[B'_0]\,dV + \frac{1}{3}\int [B'_{nl}]^T\,[D']\,[B'_{nl}]\,dV + \frac{1}{2}\int [B'_G]^T\,[\tau]\,[B'_G]\,dV \\
& + \frac{1}{3}\int [B'_G]^T\,[\tau_{nl}]\,[B'_G]\,dV
\end{aligned}
$$

Where, $[D']$ matrix is stress-strain matrix in local co-ordinate system, and $[\tau]$ and $[\tau_{nl}]$

are stress matrix in local co-ordinate system for linear and nonlinear parts of the strain respectively.

The tangent stiffness matrix can be written as,

$$[k]_T = \int [B_0']^T\,[D']\,[B_0']\,dV + \int [B_0']^T\,[D']\,[B_{nl}']\,dV + \int [B_{nl}']^T\,[D']\,[B_0']\,dV + \int [B_{nl}']^T\,[D']\,[B_{nl}']\,dV + \int [B_G']^T\,[\tau]\,[B_G']\,dV$$

2.2. STIFFENER ELEMENT

The derivation of the stiffener element is based on the basic concept used to derive the shell element. In this case the stiffener element modeled with three dimensional solid element is degenerated with the help of certain extractions obtained from the consideration that the dimension across stiffener depth as well as breadth is small compared to that along the length. The strain displacement relationship with Green-Lagrange strain of the stiffener element in local co-ordinate system can be expressed as,

$$\{\varepsilon'\} = \begin{Bmatrix} \varepsilon_{x'} \\ \gamma_{x'z'} \\ \gamma_{x'y'} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u'}{\partial x'} + \dfrac{1}{2}\left(\left(\dfrac{\partial u'}{\partial x'}\right)^2 + \left(\dfrac{\partial v'}{\partial x'}\right)^2 + \left(\dfrac{\partial w'}{\partial x'}\right)^2\right) \\ \dfrac{\partial u'}{\partial z'} + \dfrac{\partial w'}{\partial x'} + \left(\dfrac{\partial u'}{\partial x'} \times \dfrac{\partial u'}{\partial z'} + \dfrac{\partial v'}{\partial x'} \times \dfrac{\partial v'}{\partial z'} + \dfrac{\partial w'}{\partial x'} \times \dfrac{\partial w'}{\partial z'}\right) \\ \dfrac{\partial u'}{\partial y'} + \dfrac{\partial v'}{\partial x'} + \left(\dfrac{\partial u'}{\partial x'} \times \dfrac{\partial u'}{\partial y'} + \dfrac{\partial v'}{\partial x'} \times \dfrac{\partial v'}{\partial y'} + \dfrac{\partial w'}{\partial x'} \times \dfrac{\partial w'}{\partial y'}\right) \end{Bmatrix}$$

$$= [B_{0s}']\,\{\delta_s\} + \frac{1}{2}\,[A_s]\,[B_{Gs}']\,\{\delta_s\}$$

$$= [B_{0s}']\,\{\delta_s\} + \frac{1}{2}\,[B_{nls}']\,\{\delta_s\}$$

The strains $\varepsilon_{y'}$ and $\gamma_{y'z'}$ are neglected in the stiffener element. The stress-strain matrix of stiffener element in local co-ordinate may be expressed as,

$$\begin{Bmatrix} \sigma_{x'} \\ \tau_{x'z'} \\ \tau_{x'y'} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{1m} & 0 & 0 \\ 0 & \beta_s\,\bar{Q}_{5m} & 0 \\ 0 & 0 & \beta_t\,\bar{Q}_{6m} \end{bmatrix} \begin{Bmatrix} \varepsilon_{x'} \\ \gamma_{x'z'} \\ \gamma_{x'y'} \end{Bmatrix} \qquad \text{or} \qquad \{\sigma'\} = [D_s']\,\{\varepsilon'\}$$

The parameters in the $[D_s']$ matrix can be found out from the lamina axis system of the stiffener. After finding$[B_{0s}']$, $[B_{nls}']$ and $[D_s']$ matrices the secant and tangent stiffness matrices of the stiffener element can be obtained by following the same procedure as followed in the plate element.

The secant and tangent stiffness matrices of all elements of the plate and stiffeners are calculated and assembled properly to form the global secant and tangent stiffness matrices of the structure. The load vector is calculated. The nonlinear equations are solved by the Newton-Raphson iteration technique along with the incremental method using Cholesky decomposition method. The tolerance is defined with respect to the residual load.

3. RESULTS

The definition of the problem for the present analysis is presented. The convergence and validation of the present formulation is tested considering the systems solved by previous researchers. The numerical results and the parametric study of the present system is reported.

3.1 Problem Definition

The basic configuration of the problem considered here(Fig.1) is a square laminated composite stiffened plate (1000mm×1000mm×10mm) with a central x-directional stiffener (d_s(depth)=20mm and b_s(breadth)=10mm) subjected to uniformly distributed transverse loading. The lamination scheme adopted is $(\theta/-\theta)$ for plate and $(-\theta/\theta)$ for the stiffeners. The numbering of layers starts from bottom to top in plate. The numbering of layers of stiffener starts from the layer which is nearer to the plate. The stiffeners are attached at the bottom of the plate. The angle θ varies from 0 to 90 degree with the interval of 15 degree.

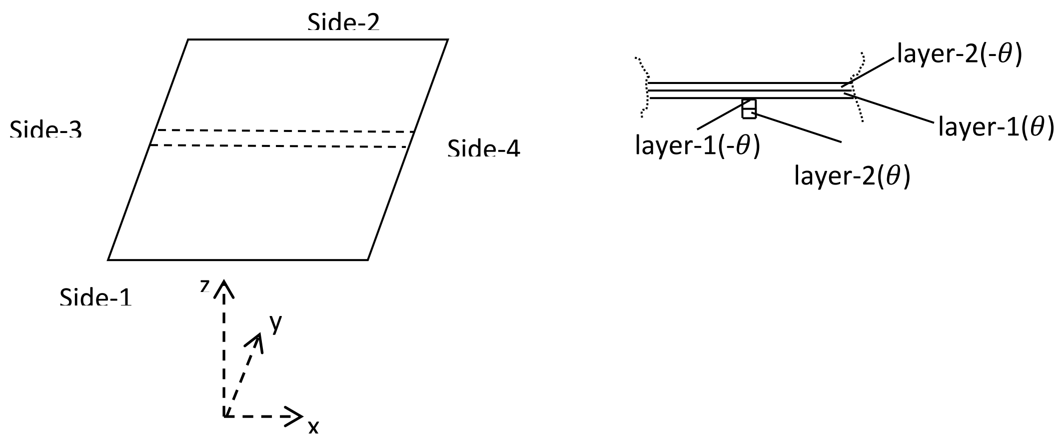

Fig.1. Stiffened plate with cross-section at stiffener with two layers of lamina

Simply supported boundary condition:

Side-1and Side-2, $\boldsymbol{u} = \boldsymbol{w} = \boldsymbol{\theta_x} = \mathbf{0}$, $\boldsymbol{\theta_x}$ is rotation along x-axis i.e. about y-axis.
Side-3and Side-3, $\boldsymbol{v} = \boldsymbol{w} = \boldsymbol{\theta_y} = \mathbf{0}$, $\boldsymbol{\theta_y}$ is rotation along y-axis i.e. about x-axis.

Fixed supported boundary condition: $u = v = w = \theta_x = \theta_y = 0$

3.2. Convergence and Validation

The convergence and validation of the present formulation is tested first. Then the results of the present problem are presented.

3.2.1.Two bay rectangular isotropic stiffened plate

An isotropic stiffened plate (1000mm×500mm×3mm) with a central stiffener (depth=18mm, breadth=10mm) along the shorter direction, clamped on all sides is considered for the convergence study and validation purpose of the present formulation. The Young's modulus of the plate is 71700 N/mm^2 and Poission's ratio is 0.33. The central deflection of the plate with different mesh size for two loading intensity is presented in Table.1. It is seen from the table that the results are converging quickly with the increase in the mesh size. The 8×8 mesh size of the full plate is sufficient to get the converged result.

Table.1. Central deflection of the stiffened plate

Mesh size (full plate)	Central deflection in mm	
	Load intensity(0.1 N/mm^2)	Load intensity(1.0 N/mm^2)
2×2	4.766	23.480
4×4	3.418	16.251
6×6	3.284	15.426
8×8	3.268	15.272
10×10	3.258	15.307

The result of this problem with 8×8 mesh of full plate is compared with the results of Sheikh and Mukhopadhyay(2000) by spline finite strip method, Ojeda *et.al.*(2007) by finite element method, Koko and Oslan(1991) by super element method as well as by semi-analytic finite strip method and ANSYS solved by Ojeda *et.al.*(2007) in Fig.2, to further validate the formulation. Sheikh and Mukhopadhyay(2000) and Ojeda *et.al.* (2007) have used the Green-Lagrange strain-displacement relationship with von Karman's assumption. However, in the present formulation, full Green-Lagrange strain-displacement relationship is used. It is seen from Fig.2 that the results obtained in the present formulation are matching well with others' results.

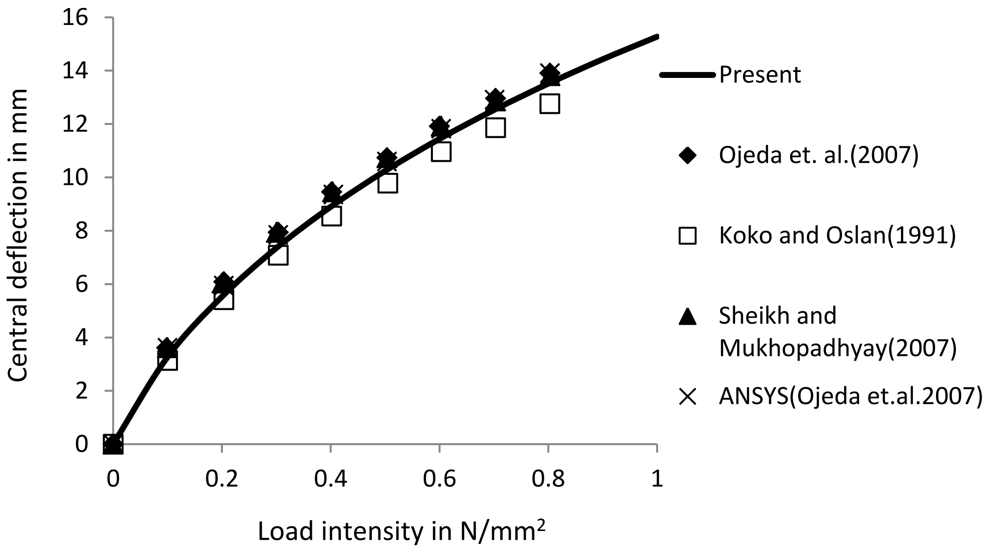

Fig.2. Central deflection of isotropic stiffened plate

3.2.2 Cross-stiffened laminated composite stiffened plate

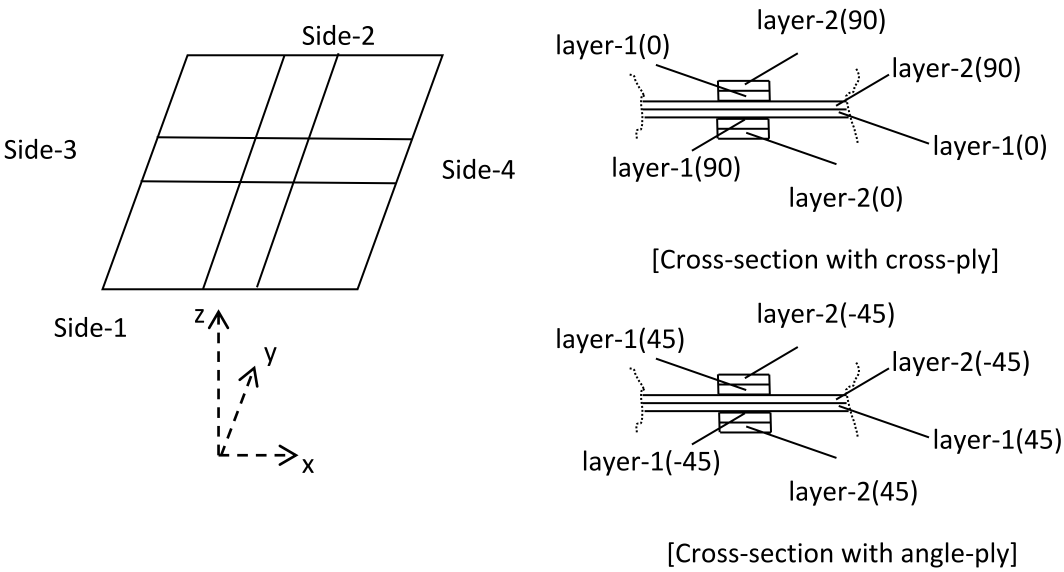

Fig.3. crossed-stiffened plate with cross-section at stiffener with two layers

A simply supported cross stiffened composite square plate (2438mm × 2438mm × 6.35mm) as shown in Fig.3 is taken to further validate the formulation. This plate has been analyzed for cross-ply and angle-ply lamination schemes. The stiffeners are attached on both sides of the plate. The stiffener depth, d_s=6.35mm and breadth, b_s=20mm. The material properties of the plate as well as stiffeners are E_1=25E_2, E_2=7031N/mm^2, G_{12}=G_{13}=0.5E_2, G_{23}=0.2E_2 and v_{12}=0.25.

The central deflection of the plate with different mesh size for two loading intensity for both, cross-ply and angle-ply lamination schemes are presented in Table.2. In this case also the results are converging quickly with the increase in the mesh size. The 8×8 mesh size of the full plate is sufficient to get the converged result. So in the present analysis 8×8 mesh of the full plate is adopted.

Table.2. Central deflection of the laminated composite crossed-stiffened plate

Mesh size (full plate)	Central deflection in mm			
	Cross-ply scheme		Angle-ply scheme	
	Load intensity (2.0×10^{-5} N/mm^2)	Load intensity (1.0×10^{-4} N/mm^2)	Load intensity (2.0×10^{-5} N/mm^2)	Load intensity (1.0×10^{-4} N/mm^2)
2×2	1.050	5.028	1.710	5.751
4×4	3.655	11.569	2.145	6.486
6×6	3.825	11.780	2.277	6.563
8×8	3.839	11.869	2.296	6.652

Results for this example have been previously reported by Chattopadhyay et. al.(1995), Liao and Reddy(1989) and Ojeda *et. al.*(2007). The result of this problem for angle-ply lamination with 8×8 mesh of full plate is compared with the finite element results of Chattopadhyay *et al.*, Liao and Reddy and Ojeda *et. al.* in Fig.4. The present results of angle-ply lamination are matching well with the published results and are more close to results of Liao and Reddy(1989). The present results along with the results of Chattopadhyay *et al.*, Liao and Reddy and Ojeda *et. al.* for cross-ply lamination is presented in Fig.5. It is seen that the present results are different from the published results. The results of Chattopadhyay *et al.*, Liao and Reddy and Ojeda *et. al.* for angle-ply and cross-ply lamination show almost equal central deflection of the plate. However, in the present formulation, the central deflections of the plate for angle-ply and cross-ply lamination are different and the angle-ply stiffened plate is stiffer for bending in comparison to the cross-ply stiffened plate.

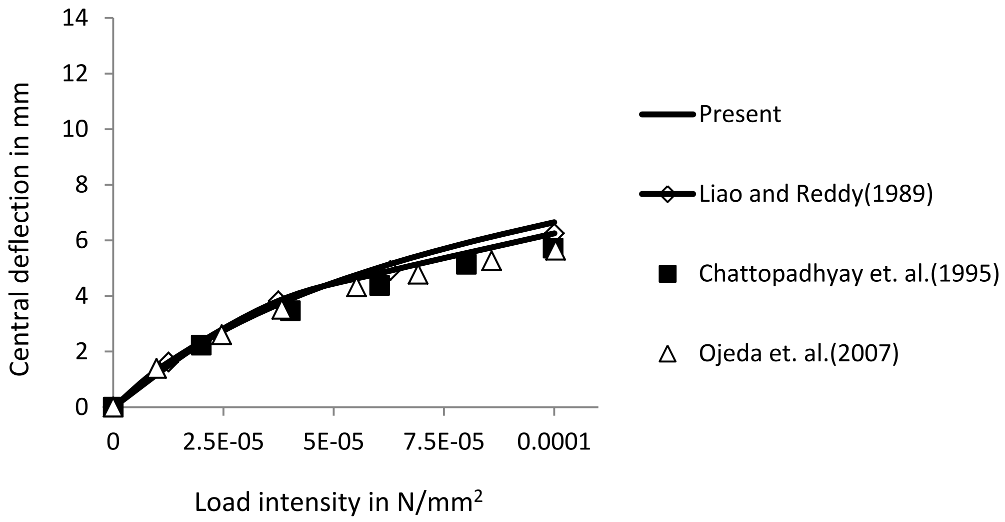

Fig.4. Central deflection of laminated composite stiffened plate with angle-ply

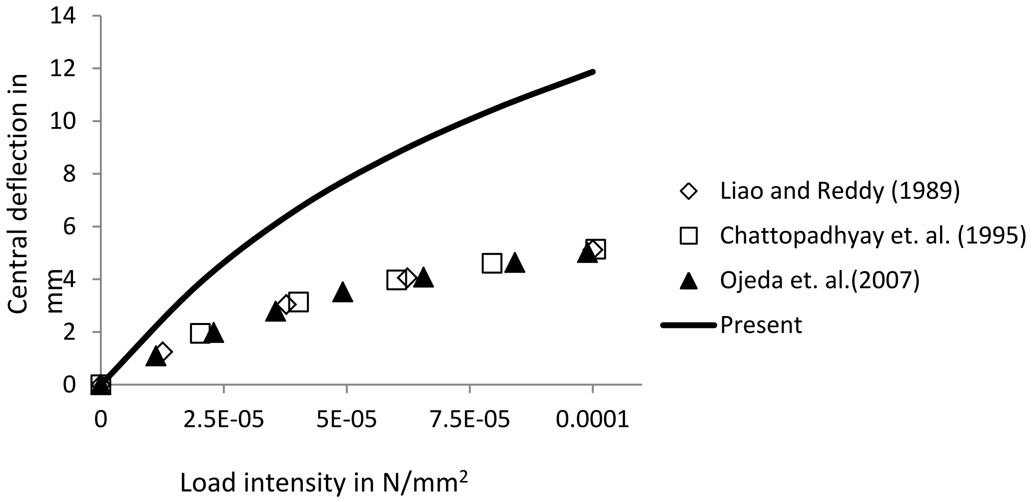

Fig.5. Central deflection of laminated composite stiffened plate with cross-ply

3.3. Numerical Results of the Present Problem

The central deflection of the simply supported laminated composite stiffened plate considering 8×8 mesh of the whole plate is computed for two layers $(\theta/-\theta)$ plate and two layers $(-\theta/\theta)$ stiffener, taking θ form 0^o to 90^o with the interval of 15^o. The material properties of the plate as well as stiffener are, E_1=25E_2, E_2=7031N/mm^2, G_{12}=G_{13}=0.5E_2, G_{23}=0.2E_2 and ν_{12}=0.25. The results are presented in Fig.6.

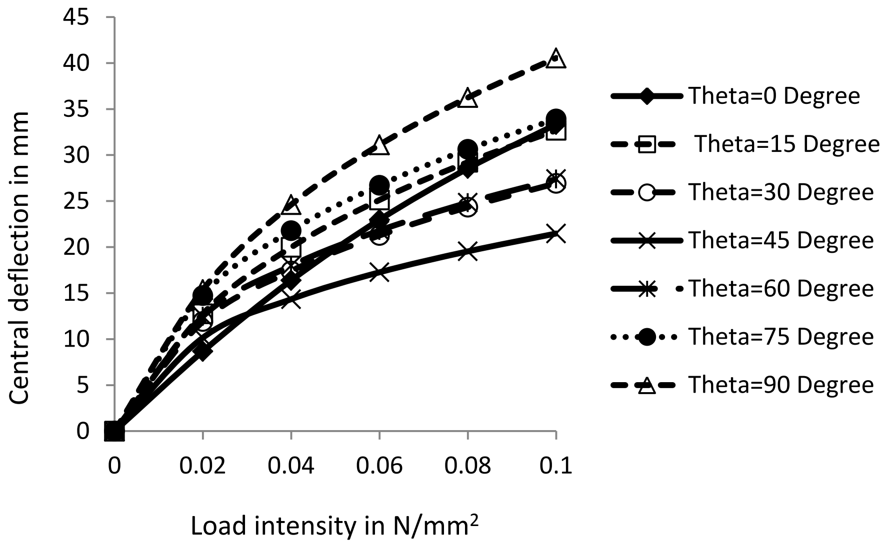

Fig.6. Central deflection of stiffened plate for different angle of lamination

It is observed that the stiffened plate with (45/-45) skin and (-45/45) stiffener is stiffer among all lamination angle and plate with (90/-90) skin and (-90/90) stiffener is weaker among all lamination angles. The central deflection for (45/-45) skin and (-45/45) stiffener plate is 21.474mm and for (90/-90) skin and (-90/90) stiffener plate is 40.551mm which is almost double of the previous result. The lamination scheme has great effect on the bending response of the stiffened plate.

4. CONCLUSIONS

The findings of the present investigation can be summarized as,

1. The formulation of geometrically nonlinear analysis of laminated composite stiffened plate with Green-Lagrange strain displacement relationship in total Lagrangian co-ordinate.
2. The deflection results are matching well with the previous results except for the cross-ply case.
3. The stiffened plate with (45/-45) skin and (-45/45) stiffener is stiffer among all lamination angles and with (90/-90) skin and (-90/90) stiffener is weaker among all lamination angles.
4. The present formulation can be extended to nonlinear dynamic, postbuckling and other aspects of analysis of composite stiffened plates.

REFERENCES

Ahmad,. S., Irons, B.M. and Zienkiewicz, O.C. (1970), "Analysis of thick and thin shell structures by curved finite elements." *Int. J. Numer. Methods Eng.*, Vol. **3**, 419-451.

Almroth, B. O. and Brogan, F. A. (1978), "The STAGS Computer Code. *NASA CR-2950*."

Chattopadhyay, B., Sinha, P.K. and Mukhopadhyay, M. (1995), "Geometrically nonlinear analysis of composite stiffened plates using finite elements." *Comput. Struct.*, Vol. **31**, 107-118.

Chia, C.Y. (1988), "Geometrically nonlinear behaviour of composite plates - a review." *Appl. Mech. Rev.*, Vol. **41**, 439-50.

Hyer, M. W., Loap, D. C. and Starnes, J. H. (1990), "Stiffener/skin interactions in pressure-loaded composite panels." *AIAA J*., Vol. **28**, 532-537.

Koko, T.S. and Olson, M.D. (1991), "Non-linear analysis of stiffened plates using super elements." *Int. J. Numer. Methods Eng.*, Vol. **31**, 319–343.

Liao, C. L. and Reddy, J. N. (1989), "Analysis of anisotropic stiffened composite laminates using a continuum-based shell element." *Comput. Struct.*, Vol. **34**, 805-8I5.

Mukhopadhyay, M. and Satsangi, S.K. (1984), "Isoparametric stiffened plate bending element for the analysis of ships' structures." *Trans. RINA*, Vol. 126. 144–151.

Ojeda, R., Prusty, B.G., Lawrence, N. and Thomas, G. (2007), "A new approach for the large deflection finite element analysis of isotropic and composite plates with arbitrary orientated stiffeners." *Finite Elem. Anal. Des*., Vol. **43**, 989-1002.

Rao, D.V., Sheikh, A.H. and Mukhopadhyay, M. (1993), "A finite element large displacement analysis of stiffened plates." *Comput. Struct*., Vol. **47**, 987–993.

Sheikh, A.H. and Mukhopadhyay, M. (2000), "Geometric nonlinear analysis of stiffened plates by the spline finite strip method." *Comput. Struct.*, Vol. **76**, 765-785.

Wood, R. D. and Schrefler, B. (1978), "Geometrically nonlinear analysis-a correlation of finite element methods." *Int. J. Numer. Methods Eng.*, Vol. **12**(4), 635–642.