The 2013 World Congress on
Advances in Structural Engineering and Mechanics (ASEM13)
Jeju, Korea, September 8-12, 2013

Analysis of post-elastic damage measures in Nonlinear Dynamic Analyses of RC structures

Cristina Cantagallo[1)], Guido Camata[2)] and Enrico Spacone[3)]

[1), 2), 3)] *Department of Engineering and Geology, University "G. D'Annunzio" Chieti-Pescara, Italy*
[3)] *espacone@unich.it*

ABSTRACT

The use of Nonlinear Dynamic Analyses, although commonly accepted as the most suitable method for determining the seismic demand on structures with non-linear behavior, provides a significant increase of the uncertainties on the structural demand. These uncertainties generally vary depending on the damage measures considered for the evaluation of the seismic demand. In order to evaluate the variability of the seismic demand, this study considers two damage indicators, including a local (the maximum section curvature) and a global measure (the maximum inter-story drift ratio). This analysis was performed by subjecting a reinforced concrete structure to different groups of spectrum-compatible real records (according to the Eurocode 8 provisions). The comparison of the different Engineering Demand Parameters permits to estimate the damage measure most representative of the seismic demand obtained from a Nonlinear Dynamic Analysis. The current seismic codes prescribe that if the response is obtained from at least 7 nonlinear time-history analyses the average of the response quantities should be used. However, when a structure is subjected to seismic records, the response quantity is characterized by a high variability and the average structural response could underestimate the seismic demand. For this reason a measure of the structural demand depending on its variability based on the statistical concept of upper tolerance limit L_u is proposed and compared with the arithmetical and geometrical mean of response quantities. The tolerance limit L_u is determined depending on two tabulated parameters obtained in function of the coefficients of variation of the structural response. The proposed method allows to consider a higher value of the response quantity when the output variability is high and the number of records is low.

1. INTRODUCTION

As Non-Linear Time History Analyses (NLTHA) become more prevalent in practice, there is the need to better understand what type of damage indicator and response

[1)] PhD
[2)] Assistant Professor
[3)] Professor

quantity should be considered for the evaluation of the seismic demand. Thus, the aim of this paper is to analyse two fundamental issues related to NLTHA: the choice of the post-elastic damage measure and the response quantity to use as seismic demand.

Specifically, two of the most commonly used damage measures are analyzed, according to their significance and practical use: the maximum section curvature and the maximum inter-story drift ratio. This analysis is performed by subjecting a reinforced concrete structure to different groups of spectrum-compatible real records (according to the Eurocode 8 provisions).

When a structure is analyzed using non-linear dynamic analyses, the current seismic codes (Eurocode 8, ASCE standards 7-10, FEMA, etc.) require that the demand generated from the seismic input must be obtained by using at least three pairs of accelerograms. Eurocode 8, Part 1 prescribes that if the response is obtained from at least 7 non-linear time-history analyses the average of the response quantities from all of these analyses should be used as the design value of the action effect E_d. Otherwise, the most un-favorable value of the response quantity among the analyses should be used as E_d.

However, when a structure is subjected to more than one seismic records, the seismic demand can be characterized by a high variability and in this case the average value of the structural demands resulting from all analyses is not able to reliably represent the demand generated from the seismic input. Furthermore, the maximum value of the structural responses could be too high and lead to an oversized and costly design. This study proposes a parameter representing the variability of the structural responses resulting from all time history analyses and investigate two response quantities suitable to characterize the structure dynamic behavior.

2. CASE STUDY

The structure under study is a 1-storey rectangular multi-bay reinforced concrete structure. It can be generally defined as regular, but due to the column geometry, the structure has a longitudinal stiffness much higher than the transversal one. The building plan is 15 by 3 meters. The beam and column cross sections are 30x60 *cm*. The columns are reinforced with four 14 *mm* diameter rebars.

The Non-Linear Time History Analyses (NLTHA) on the structural model shown in Fig. 1 are carried out with the commercial computer software Midas Gen 7.21 using a force-based fiber-section beam model (Spacone et al. 1996) for the columns (with four Gauss-Lobatto integration points) and linear elastic elements for all beams, as the building is designed as older structures that fail in the columns only. Column sections are subdivided into 6x12 concrete fibers. Floor diaphragm is used. The concrete is modeled with the Kent and Park constitutive law (Kent and Park 1971) with f_{ck} = 20 *MPa*, strain at maximum compressive strength ε_{c0} = 0.003 and ultimate strain ε_{cu} = 0.0165. The Menegotto and Pinto constitutive law (Menegotto and Pinto 1973) is used for the reinforcing steel, with f_{yk} = 430 *MPa*, *E* = 200 *GPa* and strain hardening ratio *b* = 0.02. The structure is subjected to permanent gravity loads G_k = 3 kN/m^2 and live load Q_k = 2 kN/m^2, both applied with a two-way distribution. The gravity loads are applied statically before the ground motion records are dynamically applied to the structure' base.

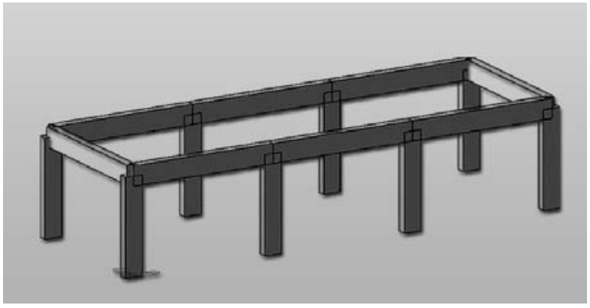

Fig. 1. Structural configuration of the case study

3. SELECTION AND PROCESSING OF RECORDS

The record selection is based on the Probabilistic Seismic Hazard Analysis (PSHA) derived from an Italian study carried out by the National Institute of Geophysics and Volcanology (INGV) and the Civil Protection Department (DPC). This work (http://esse1.mi.ingv.it/) provides the PSHA and the disaggregation for each point of a regular grid covering the entire Italian territory.

Records are selected using an earthquake scenario based on moment magnitude M_w, epicentral distance R and class soil A. The reference site for the analyses carried out in this work is located on rock soil in Sulmona (AQ-Italy) - 42.084° latitude and 13.962° longitude. The M_w-R bins providing the larger contribution to the seismic hazard at a specified probability of exceedance (Spallarossa and Barani 2007) are derived from the seismic hazard disaggregation (Bazzurro and Cornell 1999). The target scenario examined in this work corresponds to a probability of exceedance of 10%in 50 years and is characterized by M_w ranging from 5.5 to 6.5 and R from 15 to 30 km. Epicentral distances smaller than 15 km are not considered to avoid "near-field" effects. Based on the above earthquake scenario, 61 ground motion records (each consisting of two orthogonal horizontal components) are selected from two databases: the European Strong-motion Database (ESD) and the ITalian ACcelerometric Archive (ITACA). The ground motion components are correlated since they are recorded along random directions and do not generally coincide with the ground motion principal directions (Penzien and Watabe 1975). Therefore, all selected records are uncorrelated using a coordinate transformation (Lopez and Hernandez 2004).

Following a previous study by Cantagallo et al. (2012), the spectra corresponding to the 61 selected records are all scaled to the target spectral acceleration $S_a(T^*)$ corresponding to the "non-linear period" T^*. This approach follows the study by Shome et al. (1998), which proposed scaling the ground motions to the target spectral acceleration $S_a(T_1)$, where T_1 is the fundamental period of the elastic structure, and refines it by proposing the “non-linear period” T^* that accounts for the elongation of the fundamental period during the non-linear analyses. Cantagallo et al. (2012) show that $S_a(T^*)$ is well correlated with the deformation demand (MIDR in Cantagallo et al. 2012) and produces the lowest variability in structural demand among several intensity measures that were investigated in the study. The “non-linear period” T^* represents the period corresponding to the initial branch of the bilinear idealized capacity curve obtained from the non-linear static (pushover) analysis, according to Eurocode 8 (UNI EN 1998-1:2005: Annex B). The T^* values of the structure under study in the longitudinal and transverse directions are respectively 0,20 sec and 0,49 sec. In this study the T^* value corresponding to the direction of the first linear translational period was used to obtain the recorded ground motion scaling factors (T^* = 0,49 sec).

The scaling procedure is as follows. For each record, a single response spectrum is obtained: for all periods T, a single spectral acceleration $S_a(T)$ is obtained as geometric mean of the two corresponding horizontal spectral components:

$$S_a(T) = \sqrt{S_{a\mathrm{X}}(T) \cdot S_{a\mathrm{Y}}(T)} \tag{1}$$

As stated in Beyer and Bommer (2006), the geometric mean is the most widely used definition of the horizontal component of motion. More specifically, the spectral acceleration corresponding to period T^* is defined as:

$$S_a\left(T^*\right) = \sqrt{S_{a\mathrm{X}}\left(T^*\right) \cdot S_{a\mathrm{Y}}\left(T^*\right)} \tag{2}$$

For each record, a scale factor SF is used so that:

$$\mathrm{SF} \cdot S_a\left(T^*\right) = S_{a,UHS}\left(T^*\right) \tag{3}$$

where $S_{a,UHS}$ (T^*) is the spectral acceleration at T^* on the Uniform Hazard Spectrum. For each recorded ground motion, both ground motion acceleration components are scaled by SF.

After the above described scaling procedure, three different groups (combinations) of records are selected adding an additional spectrum-compatibility criterion. These groups of ground motions are named Comb 1, Comb 2 and Comb 3. Comb 1 and Comb 2 contains twenty records, while Comb 3 contains seven records. The records are selected so that in the 0.2T^*-2T^* spectrum-compatibility range of Eurocode 8, the mean elastic spectrum calculated from all time histories is within the 90% to 110% window of the uniform hazard spectrum. Basically, the group of records are obtained using a mixture between the scaling criterion by Shome et al. (1998), as modified by

Cantagallo et al. (2012) and part of the Eurocode 8 spectral compatibility criterion. Eurocode 8 requires: a) scaling to the same peak ground acceleration a_g obtained from the PSHA; b) spectrum compatibility in the range of periods $0{,}2T_1$ - $2T_1$, where no value of the elastic spectrum calculated from all time histories should be less than 90% of the corresponding value of the elastic response spectrum. In the current study, only condition b) is imposed, adding a 110% upper bound to the existing 90%lower bound.

The selection the different spectrum-compatible record groups is based on the average value δ_{mean} of the deviations of the single spectrum from the target spectrum, whose definition is derived from Iervolino et al. (2008):

$$\delta_{mean} = \underset{in\ a\ set}{\text{mean}} \left\{ \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{Sa_o(T_i) - Sa_s(T_i)}{Sa_s(T_i)}\right)^2} \right\} \tag{4}$$

where $Sa_o(T_i)$ represents the pseudo-acceleration spectrum of a single recorded accelerograms at period T_i, $Sa_s(T_i)$ is the target spectrum value at period T_i, and N is the discrete number of structural periods considered in the $0.2T^*$-$2T^*$ spectrum-compatibility range.

Table 1 describes, for each combination and structure, the maximum, minimum and average values of the Scale Factors SF, Peak Ground Accelerations PGA and deviations from the target spectrum δ. Fig. 2 shows the recorded ground motions of the three combinations Comb 1, Comb 2 and Comb 3 scaled to $S_a(T^*)$ corresponding to T^* of the case study.

Table 1. Maximum, minimum and average values of SF, PGA and δ of the three record combinations.

Comb	**SF$_{max}$**	**SF$_{min}$**	**SF$_{mean}$**	**PGA$_{max}$** (m/sec^2)	**PGA$_{min}$** (m/sec^2)	**PGA$_{mean}$** (m/sec^2)	$\boldsymbol{\delta_{max}}$	$\boldsymbol{\delta_{min}}$	$\boldsymbol{\delta_{mean}}$
Comb 1	9.52	0.76	4.61	4.56	1.25	2.73	0.66	0.13	0.33
Comb 2	8.98	0.76	4.27	4.56	1.25	2.78	0.66	0.13	0.34
Comb 3	5.22	1.51	3.16	4.33	1.87	2.74	0.65	0.13	0.30

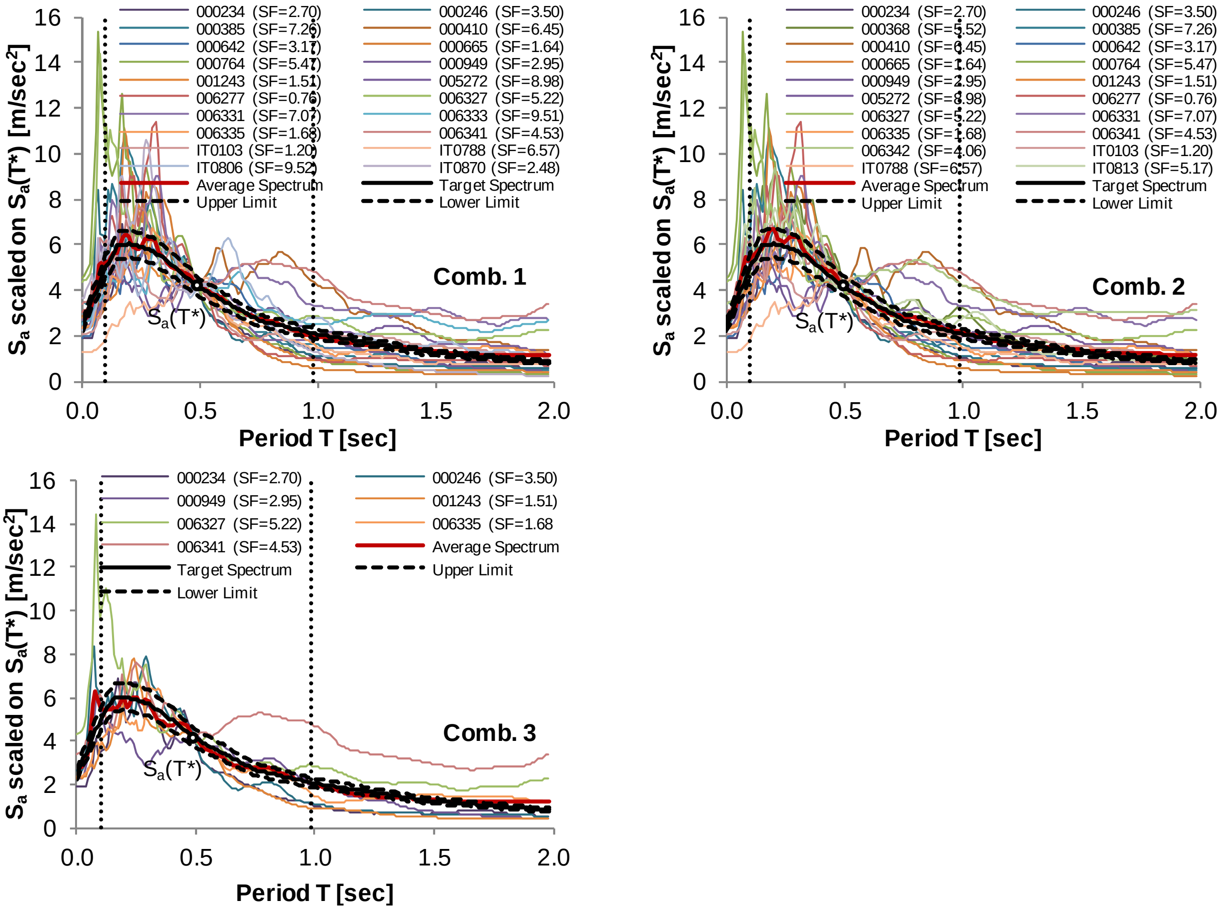

Fig. 2. Combinations of spectrum-compatible spectra scaled to $S_a(T^*)$

4. POST-ELASTIC DAMAGE MEASURES

In the structural engineering literature various damage measure are analyzed to explain damages observed in structures subjected to real earthquake ground motions. This study considers two damage indicators, including a local and a global measure, corresponding respectively to the maximum section curvature and the maximum inter-story drift ratio. The following paragraphs explain how these measures are obtained and processed in this study.

4.1 Maximum Section Curvature

From each record belonging to the analyzed structure, the Maximum Section Curvature (RZ) is computed as the maximum curvature RZ(t) over time (the record duration), that is RZ = max|RZ(t)|. The maximum value between all the RZ values computed from all columns belonging to the structure is used for the processing data.

4.3 Maximum Inter-story Drift Ratio

The Maximum Inter-story Drift Ratio (MIDR) is computed as the maximum percentage interstory drift DXY(t) over time (the record duration), that is MIDR = max|DXY(t)|. For each record, the interstory drift ratio at an instant t is computed as:

$$\mathrm{DXY}(t) = \sqrt{\mathrm{DX}(t)^2 + \mathrm{DY}(t)^2} \quad (5)$$

where DX(t) and DY(t) are the instantaneous interstory drifts in the X and Y directions, respectively, between the centers of mass of two adjacent floors.

5. DEFINITION OF RESPONSE QUANTITIES

Three different definitions of response quantity are used in this study: the maximum response, the average response and a response based on the upper tolerance limit L_u:

- *Maximum response.* For each record, the maximum response from all 9 ground-motion axes orientations is determined. The corresponding damage measures are referred to as the response for θ_{max}.
- *Average response.* For each record, the average response from all 9 ground-motion axes orientations is determined. The corresponding damage measures are referred to as the response for θ_{avg}.
- *Response based on the upper tolerance limit $L_{u.}$* Since the use of different ground motion records produces a variability of the response quantities, the use of the average response could significantly underestimate the seismic demand. Moreover, the use of the maximum response is overly conservative when using a large group of records. For this reason, a measure of the structural demand depending on its variability is searched. The proposed method is based on the statistical concept of upper tolerance limit, which is calculated in function of both the dispersion of the Engineering Demand Parameter (EDP) and the considered number of ground motion records.

The proposed method is also defined Tolerance Factor Method being based on the concept of tolerance interval. In general, a tolerance interval is a statistical interval within which, with some probability, a specified proportion of results falls. From an engineering point of view, this interval defines a range of allowable variation for the structural response variable. The endpoints of a tolerance interval are also called tolerance limits; they provide the upper and lower limits between which it confidently expects to find a prescribed proportion of the structural response (Natrella, 1963). The tolerance limits define the two values A and B between which at least a proportion β% of results will lie with a prescribed confidence level, (two-sided limits), or the value A above (or upper) which at least a proportion β% will lie (one-sided limit).

When the structural responses can be represented by a normal probability distribution the extremes of the tolerance intervals can be easily determined by using a parametric approach. If X is a Gaussian random variable with mean μ and variance σ^2

both unknowns and *m* and s^2 are their estimates based on a sample of *n* independent observations, to determine the tolerance limits that includes with probability *γ* at least the *β*% of the data, the following relationship is defined:

$$\Pr\left[\Phi\left(L_u\right)-\Phi\left(L_i\right)\geq\beta\right]=\gamma \quad (6)$$

where Φ(x) is the area under the Gaussian density function in the interval]-∞, *x*], L_u is the upper tolerance limit and L_i is the lower tolerance limit. The solution for the L_i and L_u extremes is given by Wald and Wolfowitz (1946):

$$\begin{cases} L_i\left(\gamma,\beta\right)=m-zs \\ L_u\left(\gamma,\beta\right)=m+zs \end{cases} \quad \text{with} \quad z=\tau_{n,\beta}=\left[\sqrt{\frac{n-1}{\chi^2_{n-1,\gamma}}}\right] \quad (7)$$

where $\chi^2_{n-1,\gamma}$ is the quantile of order $(1 - \gamma)$ of the chi-square distribution with $(n - 1)$ degrees of freedom and $\tau_{n,\beta}$ derives from the solution of the equation:

$$\Phi\left(\frac{1}{\sqrt{n}}+\tau_{n,\beta}\right)-\Phi\left(\frac{1}{\sqrt{n}}-\tau_{n,\beta}\right)=\beta \quad (8)$$

The *z* values derived from Eq. (7) are tabulated by Natrella (1963) in function of the proportion of data *β*% and the probability *γ*. The specific objective of this study is to determine a measure of structural response depending on its variability; this measure can be the upper tolerance limit L_u defined in Eq. (7).

6. DISCUSSION AND RESULTS

Fig. 3 and Fig. 4 show the trend of the maximum curvatures RZ(θ) and the maximum inter-story drift ratios DXY(θ) for the selected combinations of records. These curvatures are different in function of the considered record and incidence angle at which the input is applied. The maximum inter-story drift ratios DXY(θ) obtained from NLTHAs vary from 0.27% and 1.76%, while the maximum curvatures from 0.0069 rad/m and 0.0629 rad/m. Both the maximum curvature and maximum inter-story drift ratio occurs when the record 000410 is applied at an incident angle of 67.5°. This result suggests that the two selected post-elastic damage measures provides similar results.

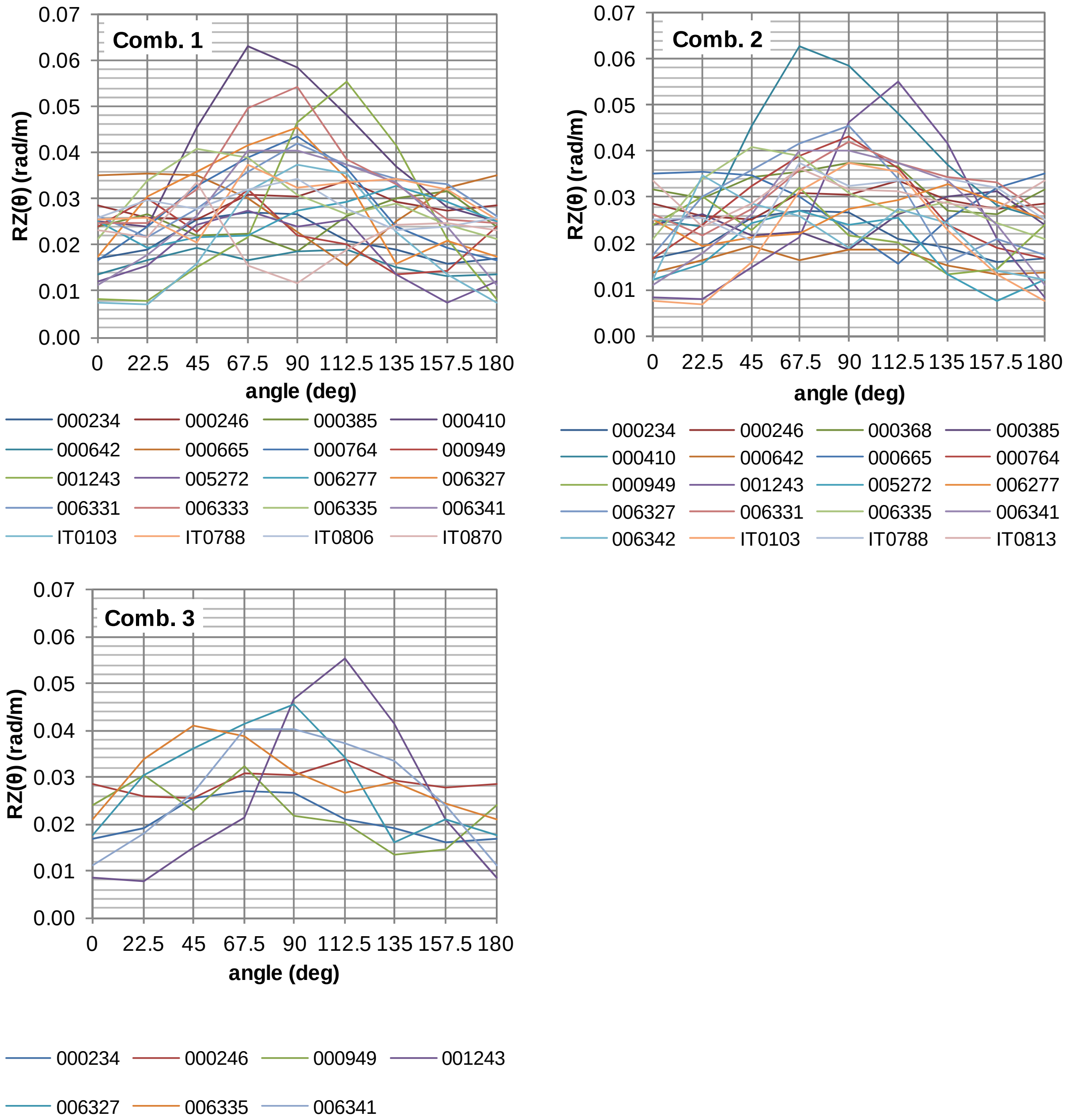

Fig. 3. Variation of Maximum Section Curvature RZ obtained from the three combinations of spectrum-compatible records orientated along 9 different incident angles

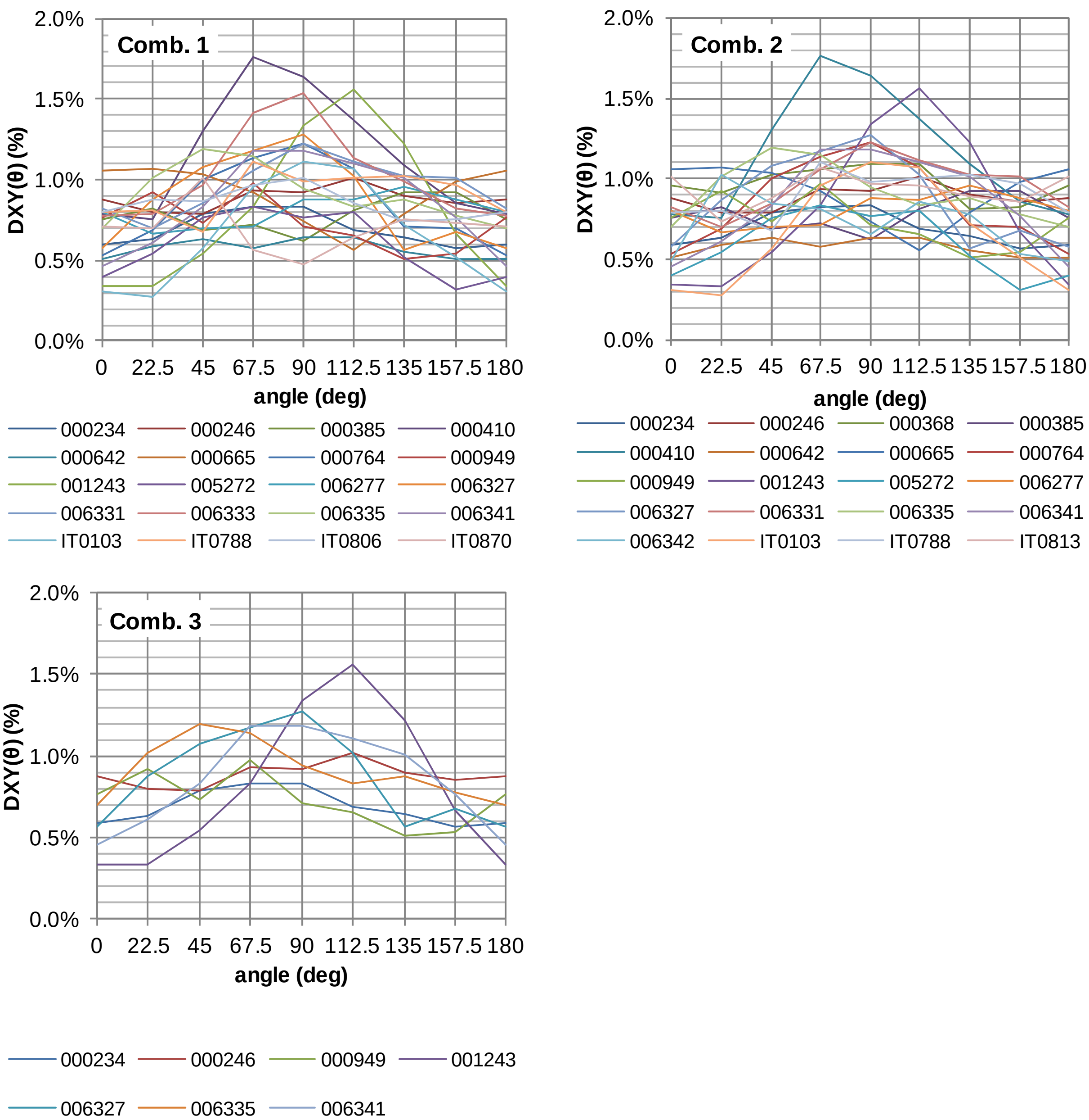

Fig. 4. Variation of Maximum Inter-story Drift Ratios DXY(%) obtained from the three combinations of spectrum-compatible records orientated along 9 different incident angles

The variability of the structural response obtained from structure under study by using three different sets of spectrum-compatible pairs of accelerograms orientated along different incident angles is significantly high, as suggests the coefficient of variations (CV) of the maximum inter-story drift ratios DXY(%) and the maximum curvatures obtained from the three spectrum-compatible combinations orientated along 9 different incident angles (Fig. 5).

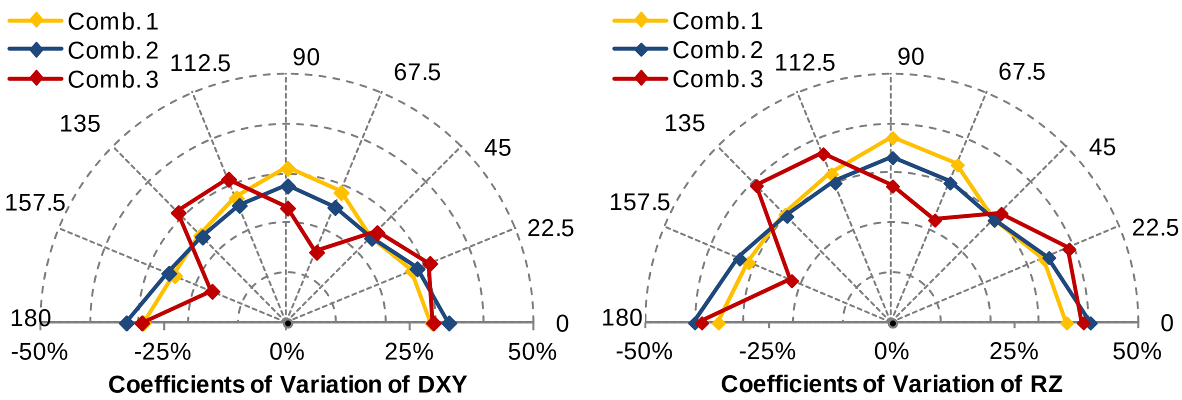

Fig. 5. Coefficient of Variations of the maximum inter-story drift ratios DXY(%) and the maximum curvatures obtained from the three spectrum-compatible combinations orientated along 9 different incident angles

The high dispersion of the damage measures confirms that the mean of the results does not describe properly the structural behavior, but it is necessary to consider the uncertainties the different inputs produce. To this end, a method to determine the measure of the structural response depending on its variability is here proposed. This method uses the upper tolerance limit as response quantity. The tolerance limit L_u is determined depending on the two parameters γ and β% established a-priori. These parameters, representing respectively the probability and the percentage of the considered structural responses, can be obtained from statistical tables (Natrella, 1963) according to the their coefficients of variation CV. Table 2 shows the method used for the evaluation of the two parameters γ and β% for the sets of records examined in this work consisting of 20 and 7 pairs of accelerograms. The measure of the response quantity depending on its variability is calculated by applying Eq. (7) where the z values are obtained depending on the coefficient of variation CV of the structural responses.

Table 2. Determination of the two parameters γ and β% in function of the coefficients of variation CV of the structural responses

	γ	Β		***z* (*n* = 20)**	***z* (*n* = 7)**
CVmax ≥ 80%	0.75	0.95	⇨	1.933	2.250
40% ≤ CVmax < 80%	0.75	0.9	⇨	1.528	1.791
0% ≤ CVmax < 40%	0.75	0.75	⇨	0.865	1.043

Fig. 6 and Fig. 7 show a comparison between the average values of the structural responses obtained for each combination at the nine considered incident angles and the corresponding tolerance limits (m + zs). The mean deformation demands are always smaller than the corresponding upper tolerance limits. In particular, the ratio between the tolerance limits and the average values of the structural responses are maximum when the structure is subjected to Comb 1 and Comb 2 at angle of incidence θ = 67.5° and it is equal to 1.33.

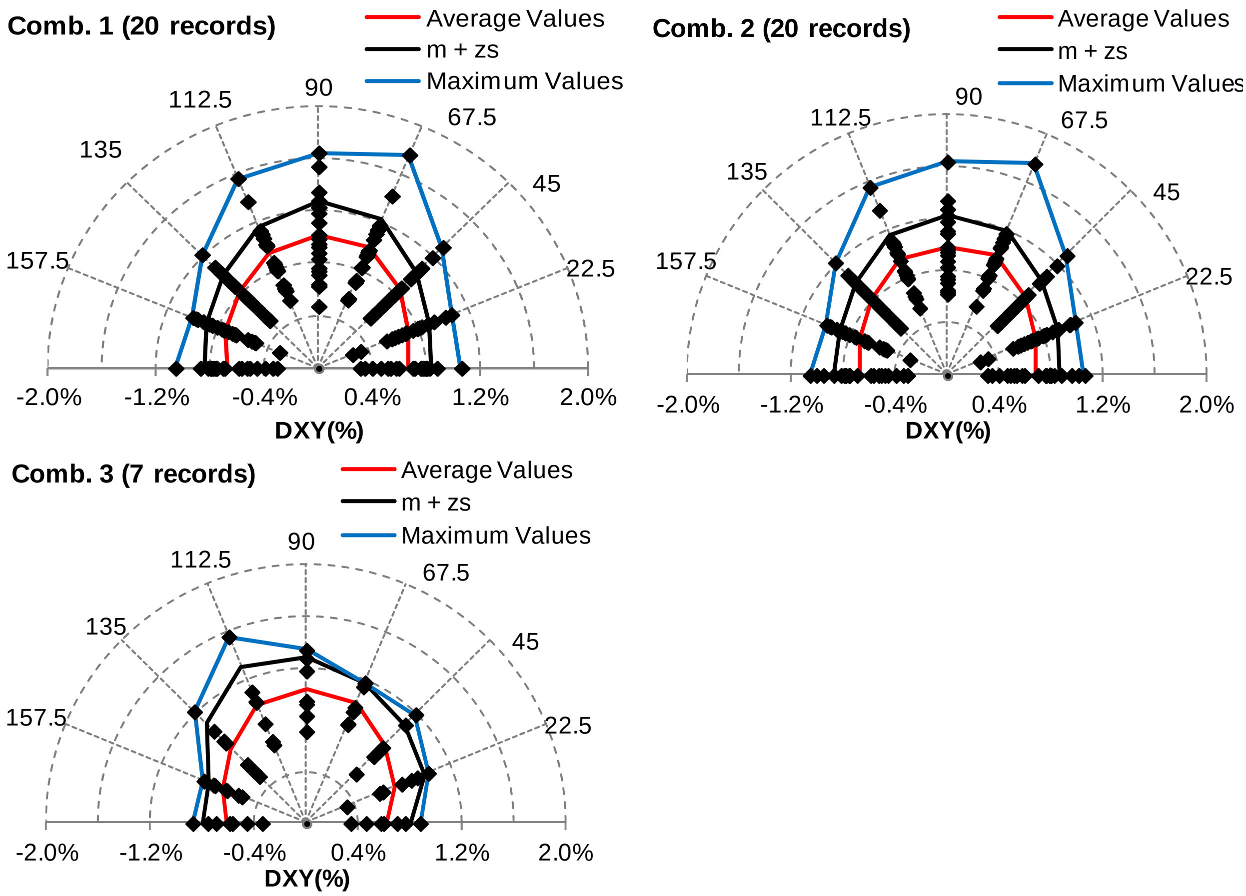

Fig. 6. Maximum values, average values and upper tolerance limits (m + zs) of the maximum inter-storey drift ratios DXY obtained from the sets of scaled and spectrum-compatible records Comb. 1, Comb. 2 and Comb. 3 orientated along 9 different incident angles

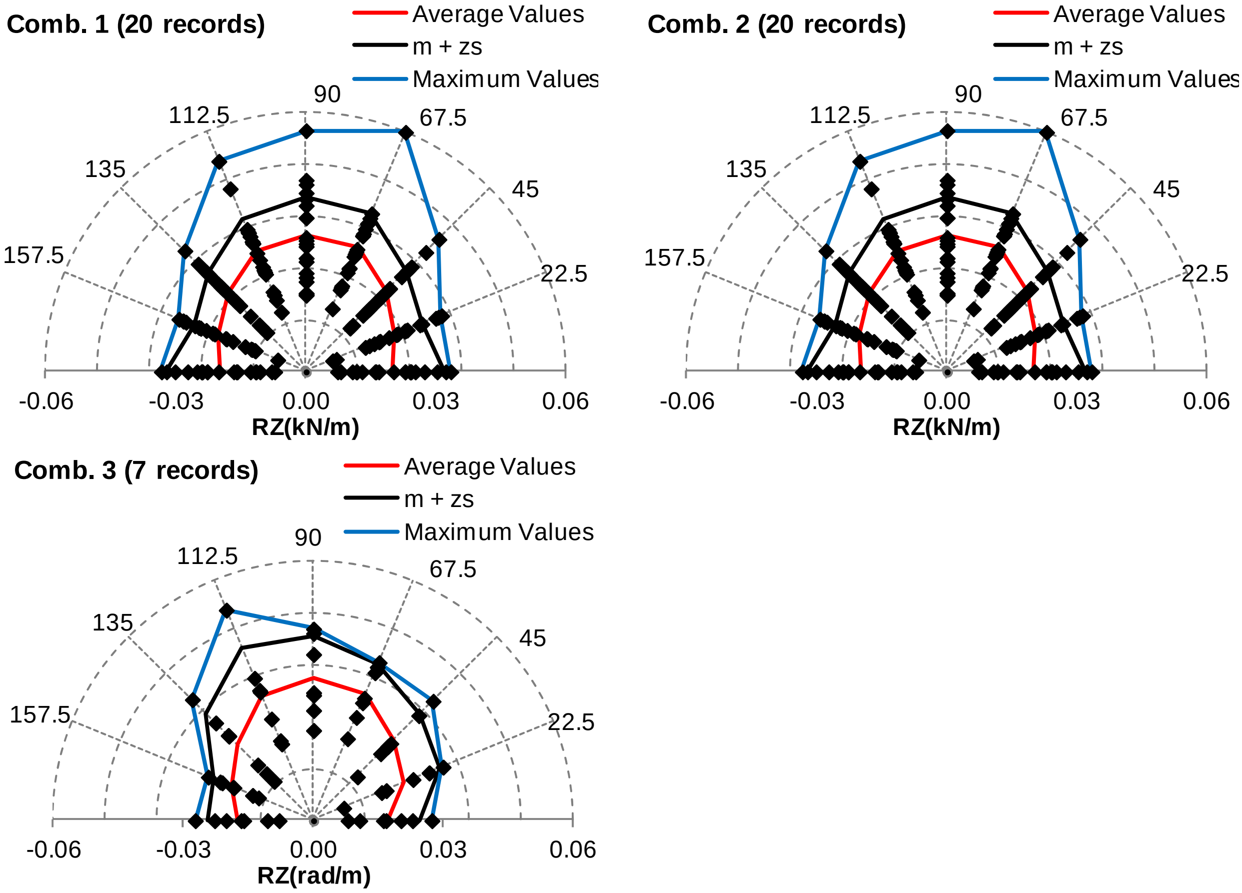

Fig. 7. Maximum values, average values and upper tolerance limits (m + zs) of the maximum curvatures RZ obtained from the sets of scaled and spectrum-compatible records Comb. 1, Comb. 2 and Comb. 3 orientated along 9 different incident angles

7. CONCLUSIONS

This article addresses two fundamental issues related to ground-motion input for bi-directional analysis. These issues concern the choice of the post-elastic damage measure and the response quantity to use in Nonlinear Dynamic Analyses of RC structures.

The two damage measure used in this study (the maximum inter-story drift ratio and the maximum section curvature) provides very similar results at the nine incident angles analyzed. Since the maximum section curvature may be difficult to obtain in practice, the authors suggest using the maximum inter-story drift ratio as damage measure, if the horizontal diaphragm can be considered rigid. This paper is part of an ongoing work and other damage measures are under investigation including the maximum fiber strains and the residual displacements.

A further issue addressed in this work is related to the measure of the response quantity to use when non-linear dynamic analyses are performed using different inputs.

The current seismic codes prescribes that if the response is obtained from at least 7 nonlinear time-history analyses the average of the response quantities should be used. However, when a structure is subjected to more than one seismic record, the response quantity can be characterized by a high variability and for this reason the average structural response is not able to represent reliably the demand generated from the seismic inputs. Thus, a measure of the structural response depending on its variability based on the statistical concept of upper tolerance limit L_u was proposed. The tolerance limit L_u is determined depending on two tabulated parameters obtained in function of the coefficients of variation CV of the structural response. With the proposed method a high variability of the response and a low number of records provide a higher response quantity.

When non-linear dynamic analyses provide structural responses with high spreads, the structure should be verified by using a measure of structural response higher than the average one. In addition, a greater number of records should produces a lower response quantity measure. The analyzes carried out in this paper show that the use of the upper tolerance limit as response quantity satisfy both these requirements.

ACKNOWLEDGMENTS

The authors would like to acknowledge the financial support from the ReLUIS program of the Italian Civil Protection Agency (DPC), task 1.1.2, contract n. 823, 24/09/2009.

REFERENCES

Bazzurro, P., and Cornell, C. A., (1999). “Disaggregation of seismic hazard”, *Bulletin of the Seismological Society of America*, **89**(2), 501-520.

Beyer K. and Bommer J. J., (2006). “Relationships between median values and between aleatory variabilities for different definitions of the horizontal component of motion”, *Bulletin of the Seismological Society of America*, **96**(4A), 1512–1522.

Cantagallo, C., Camata, G., Spacone, E. and Corotis, R., (2012). “The Variability of Deformation Demand with Ground Motion Intensity”, *Probabilistic Engineering Mechanics*, **28**, 59-65.

EN 1998-3: 2005, Eurocode 8: “*Design of structures for earthquake resistance - Part 3: Assessment and retrofitting of buildings*”, EN1998-3, Brussels, 2005.

European Strong-motion Database (ESD). http://www.isesd.cv.ic.ac.uk/ESD/frameset.htm.

Iervolino, I., Maddaloni, G. and Cosenza, E., (2008). “Eurocode 8 Compliant Record Sets for Seismic Analysis of Structures”, *Journal of Earthquake Engineering*, **12**, 54–90.

ITalian ACcelerometric Archive (ITACA). *http://itaca.mi.ingv.it/ItacaNet/*.

Kent, D.C., Park, R., (1971). “Flexural members with confined concrete”, *Journal of the Structural Division*, **97**(7), 1969–1990.

López, A., Hernández, J. J, 2004. “Structural Design for Multicomponent Seismic Motion”, Vancouver, B.C., Canada, in *Proceedings, 13th World Conference on Earthquake Engineering*, paper 2171.

Menegotto, M., Pinto, P. E., (1973). “Method of analysis for cyclically loaded reinforced concrete plane frames including changes in geometry and non-elastic behavior of elements under combined normal force and bending”, Zurich, in *Proceedings, IABSE Symposium on Resistance and Ultimate Deformability of Structures Acted on by Well Defined Repeated Loads*, pp. 112-123.

MIDAS/Gen, ver. 7.2.1 (2007), *http://www.cspfea.net/midas_gen.php*.

Natrella, M., (1963). Experimental Statistics; NBS Handbook 91. US Department of Commerce.

Penzien, J., and Watabe, M., (1975). “Characteristics of 3-dimensional earthquake ground motion”, *Earthquake Engineering and Structural Dynamics*, **3**(4), 365-374.

Shome, N., Cornell, C. A., Bazzurro, P., and Carballo, J. E., (1998). “Earthquakes, records and nonlinear responses”, *Earthquake Spectra*, **14**(3), 469–500.

Spacone, E., Filippou, F. C., Taucer, F., (1996). “Fiber Beam-Column Model for Nonlinear Analysis of R/C Frames: I. Formulation”, *Earthquake Engineering and Structural Dynamics*, **25**(7), 711-725.

Spallarossa, D., and Barani, S., (2007). “Progetto DPC-INGV S1, Deliverable D14”, *http://esse1.mi.ingv.it/d14.html*.

Wald A., Wolfowitz J. (1946). Tolerance limits for a normal distribution. Annals of Mathematical Statistics, 17, 208-215.